The twisted wing

In order to obtain aerodynamic force and moment distributions along the span, the wing is often twisted geometrically or aerodynamically or both.

1. Geometric twist is achieved by twisting the axis of the wing so that the geometric angle of attack $\alpha$ varies spanwise.

   Geometric washout: $\alpha$ decreases from root to tip.

2. Aerodynamic twist is achieved by changing the airfoil section from root to tip, effectsing a spanwise variation of camber and position of maximum camber. These variations affect the spanwise on the variation of the absolute angle of attack and center of pressure.

For aerodynamic washout could be achieved by decreasing the camber from root to tip.
For a given twist and planform in steady motion the absolute \( a \) and chord \( c \) are known. To determine numerically the spanwise distribution of sectional lift and drag, we select \( K \) spanwise stations \( \theta_i \), \( \theta_1 \), \( \theta_2 \), \( \theta_3 \), \( \theta_4 \), \( \theta_5 \), at which wing chord \( c_1 \), \( c_2 \), \( c_3 \), \( c_4 \), \( c_5 \), then after changing the upper limits of the summations from \( \infty \) to \( K \).

\[
\frac{\sum_{j=1}^{K} \frac{An. \sin \theta_j \cdot \cos \theta_j \cdot \sum_{n=1}^{K} \frac{An. \sin n \theta_j}{\sin \theta_j}}{\sum_{j=1}^{K} A_n \cdot \sin n \theta_j}}{A_0 \cdot \sin \theta_j}
\]

In general for accuracy calculation, \( K \) is increased or by decreasing the relative spanwise interval.

\[
Y_{i+1} - Y_i = \frac{1}{2} b (\cos \theta_{i+1} - \cos \theta_i)
\]

Properties will be changed by the section, such as near wing tips and in the vicinity of nacelles or engine pods.

The above equation will yield \( K \) simultaneous equations, which is solved for the coefficient \( A_1 \), \( A_2 \), \( A_3 \), \( A_4 \), \( A_5 \).

The local values of the effective and induced angles of attack at the \( j \)th station are.
\[
de_{ij} = \frac{\cos C_s}{a_{ij} C_s} \sum_{n=1}^{K} a_n \sin (n \theta_j) \quad (3)
\]

\[
d_{ij} = d_{ij}' - d_{ij} = \frac{\cos C_s}{4b} \sum_{n=1}^{K} n a_n \frac{\sin (n \theta_j)}{\sin \theta_j} \quad (4)
\]

The local slope of the \( C_L \) vs. \( \Delta a \) curve is given by

\[
d_{ij}' = \frac{a_{ij}}{1 - d_{ij}/d_{ij}'} \quad (5)
\]

These equations give the slope at each of these stations for the \( \Delta a \) distribution, represent by the equation of \( \theta_j' \), that is, for one given attitude of the wing relative to the flight path.

The above calculations outlines the procedure for finding the distribution of aero dynamic characteristics for a given \( \Delta a (y) \). We assume the wing is installed at every section.

\[2.L.L.W \text{ (Zero lift line wing)}\]
In the above figure, angles are expressed relative to the direction designated E.E.L.W., that is the direction of \( V_{\infty} \) for which the total lift of the wing is zero.

\[ \alpha = \alpha_b + \alpha_w. \]

\( \alpha_b \): basic absolute angle of attack, is the local absolute angle of attack for zero total lift of the wing and thus depends at a given station only on the wing twist.

\( \alpha_w \): additional absolute of attack, is measured from E.E.L.W. to the flight path.

The corresponding spanwise circulation contributions designated, respectively \( \Gamma_b \) and \( \Gamma_a \)

\[ \Gamma = \Gamma_a + \Gamma_b \]

They satisfy the relations

\[ \int_{-b/2}^{b/2} \Gamma_b \, d\gamma = \int_{-b/2}^{b/2} \Gamma_a \, d\gamma = 0 \]

\[ L = \Gamma_a + L_b = \int V_{\infty} \, (\Gamma_b + \Gamma_a) \]

Corresponding lift coefficients are defined by

\[ C_{L_b} = \frac{L_b}{\frac{1}{2} \rho V_{\infty}^2 C} \quad C_{L_a} = \frac{L_a}{\frac{1}{2} \rho V_{\infty}^2 C} \]
The local lift coefficient $C_L$ is

$$C_L = C_{L0} + C_{L1}$$  \(6\)

where $C_{L0}$ depends on $\alpha$, that is, on the twist of the wing, and is thus independent of $\alpha$; $C_{L1}$ (the additional lift coefficient) is dependent on $\alpha$ and is thus independent of wing twist.

$$C_L = C_{L0} + C_{L1} \quad \text{(7)}$$

The equation (3) is solved for two angles of attack, that is, for two sets of $\alpha_j$ designated $\alpha_{j1}$ and $\alpha_{j2}$ to obtain two sets of coefficients $A_{j1}$, $A_{j2}$, and $A_{j3}$ with the limitations of the lifting line representation.

Equation (5)

$$a_j = \frac{dC_{Lj}}{d\alpha_{j}}$$

is a constant at a given station, independent of $\alpha$, but varying station to station.

We approximate $a_{j1} = a_{j2} = a_j$ and

$$C_{Lj1} = a_j \cdot \alpha_{j1} \quad C_{Lj2} = a_j \cdot \alpha_{j2}$$  \(8\)
If these coefficients are integrated spanwise with respect to y, we obtain

\[ C_{l1} = \frac{1}{5} \int_{-b/2}^{b/2} e_{11} \cdot c \, dy = \frac{a_0 s C_s b A_1}{45} \]  

\[ C_{l2} = \frac{1}{5} \int_{-b/2}^{b/2} c_{l2} \cdot c \, dy = \frac{a_0 s C_s b A_{12}}{45} \]  

Then we write equations (7) for two angle of attack

\[ e_{ij} = C_{ij} + C_{i} c \quad (9\alpha) \]

\[ C_{ij} = C_{ij} + C_{i} c \]

Substituting (8) and (9), the solution of simultaneous equations (10) yield that \( C_{cb} \) and \( C_{ca} \) as function of \( y \). The equations can be solved for these coefficients and \( C_{c} \) versus \( \Delta \alpha \) determined throughout the range of angle of attack.

The spanwise distribution of induced drag can be found

\[ C_{di} = C_{di} \quad (11) \text{ and } d_i = \text{drag} - d_a = \frac{C_{d} - C_{i}}{\langle a \rangle} \]

So that

\[ C_{di} = \frac{e_{i}^2 (a_0 - a)}{a_0 a} \quad (12) \]
We may also define a "weighted mean slope"

\[ \bar{a} = \frac{1}{S} \int_{-b/2}^{b/2} ac \, dy \]

so that

\[ C_L = \bar{a} \cdot C_{aw}. \]

**Example 6.4**

This example demonstrates the use of the analysis for computing the performance of a twisted wing of finite span. The symmetrical trapezoidal wing shown in Fig. 6.16 is considered, which has the following properties:

- Aspect ratio: \( AR = 6. \)
- Taper ratio: \( \lambda = \frac{\text{tip chord } c_t}{\text{root chord } c_r} = 0.55. \)
- The wing has a geometric twist that varies linearly from zero at the root to \(-4^\circ\) at the tip.
  - A negative twist denotes washout.
- There is no aerodynamic twist.
- Airfoil shapes are identical and \( m_0 \) has a constant value of \( 2\pi \) per radian along the span.
(a) Compute sectional aerodynamic properties $c_{h}$ and $c'_{s}$ for this wing.
(b) When flying at sea level at speed $V$ of 250 km/hr, the wing loading $W/S$ is 800 N/m². Compute $c_{l}$ and $c_{d}$ along the span and the wing characteristics for this flight condition.

As shown in Fig. 6.16, eight spanwise stations are selected on one side of the wing, so that $k = 8$ in Eq. (6.40). We defined dimensionless spanwise distance $Y$ as $y/0.5b$ and dimensionless chord $C$ as $c/c_{s}$. At the $j$th station on a linearly tapered wing,

$$
\theta_{j} = \frac{\pi}{2}\left(\frac{j}{k}\right); \quad Y_{j} = \cos \theta_{j}; \quad \text{and} \quad C_{j} = 1 - (1 - \lambda) \cos \theta_{j}
$$

It can also be verified that

$$
\frac{m_{0}C_{s}}{4b} = \frac{\pi}{\mathcal{A}(1 + \lambda)}
$$

Because the coefficients $A_{n}$ vanish for even values of $n$ for a symmetrical wing, it is more convenient to replace $A_{n}$ by $A_{h}$ in Eq. (6.40) and to replace $n$ by $2N - 1$ elsewhere in that equation, where $N = 1, 2, \ldots, k$. Thus, Eq. (6.40) can be written in a simplified form

$$
\sum_{n=1}^{k} D_{jN} A_{N} = \alpha_{h j} ; \quad j = 1, 2, \ldots, k \quad (6.40a)
$$

in which, for a uniform distribution of $m_{0} = 2\pi$,

$$
D_{jN} = \left[ \frac{1}{C_{j}} + \frac{(2N-1)\pi}{\mathcal{A}(1 + \lambda) \sin \theta_{j}} \right] \sin(2N-1) \theta_{j}
$$

$D_{jN}$ is a function of geometry only and can readily be computed for given values of $j$ and $N$.

Similarly, Eqs. (6.35), (6.41), (6.42), and (6.53) are rewritten as

$$
C_{D} = \frac{\pi^{3}}{\mathcal{A}(1 + \lambda)^{2}} \sum_{N=1}^{k} (2N-1) A_{N}^{2} \quad (6.35a)
$$

$$
c_{l} = \frac{2\pi}{C_{j}} \sum_{N=1}^{k} A_{N} \sin(2N-1) \theta_{j} \quad (6.41a)
$$

$$
\alpha_{l} = -\frac{\pi}{\mathcal{A}(1 + \lambda)} \sum_{N=1}^{k} (2N-1) A_{N} \frac{\sin(2N-1) \theta_{j}}{\sin \theta_{j}} \quad (6.42a)
$$

$$
\overline{m} = \frac{1}{1 + \lambda} \int_{0}^{1} m \text{d}Y
$$

The integral in the last equation can be evaluated numerically using the trapezoidal rule:

$$
\overline{m} = \frac{1}{2} \frac{1}{(1 + \lambda)} \sum_{j=1}^{k-1} \left( m_{j} C_{j} + m_{j+1} C_{j+1} \right) (Y_{j} - Y_{j+1}) \quad (6.53a)
$$
\[ A_1 = 0.0425 \quad A_6 = 0.0006 \]
\[ A_2 = -0.0076 \quad A_7 = -0.0003 \]
\[ A_3 = 0.0027 \quad A_8 = -0.0002 \]

\[ V = 250 \text{ km/hr} \quad \text{Wing Loading} = 800 \text{ N/m}^2 \]

\[
C_{D\alpha} = \frac{\pi^3}{6 (1 + 0.55)^2} \left[ (0.0425)^2 + 3 (0.0006)^2 + 5 \times (0.0022)^2 \\
+ 7 \times (0.0008)^2 + 9 \times (0.0006)^2 \\
+ 11 \times (0.0003)^2 + 13 \times (0.0003)^2 \\
+ 15 \times (0.0002)^2 \right]
\]

\[ C_{D\alpha} = 0.00435 \]

\[
C_L = 0.269 = \frac{kN/m^2}{900} = \frac{800 \text{ N/m}^2}{1 \times 1.23 \times (69.40)^2}
\]

\[ C_{D\alpha} = \frac{(0.018)^2 (1 + 0.023)}{\pi \times 6} \]

\[ \left[ C_{p\alpha} = 0.00386 \right] \quad 0.00400 \]

\[ C_{Lw} = 0.221 \]