Fundamental Concepts of Statistics

Exercise session 1

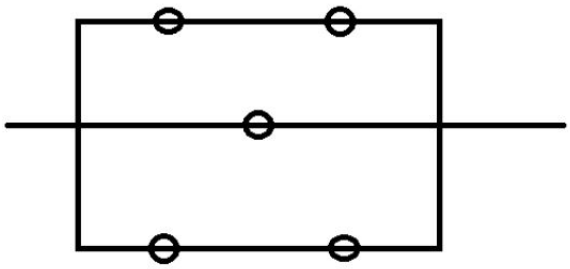
1. Derive the following inequality (Bonferroni inequality)

$$P(A \cap B) \geq 1 - P(A^c) - P(B^c)$$

Can you generalize for more than 2 events?

2. A fire insurance company has high-risk, medium-risk and low-risk clients who have, respectively, probabilities 0.02, 0.01 and 0.0025 of filing claims within a given year. The proportions of the numbers of clients in the three categories are 0.10, 0.20 and 0.70, respectively.
What proportions of the claims filed each year come from high-risk clients?

3. What is the probability that the following system works if each unit fails independently with probability p ? (see Figure 1)



4. This problem deals with an elementary aspect of simple branching processes. A population starts with one member; at time $t = 1$ it either divides with probability p or dies with probability $1-p$. If it divides, then both of its children behave independently with the same two alternatives at time $t = 2$. What is the probability that there are no members in the 3rd generation? For what value of p is this probability equal to 0.5?

5. The following table shows the cumulative distribution function of a discrete random variable. Find the pdf.

k	$F(k)$
0	0
1	0.1
2	0.3
3	0.7
4	0.8
5	1.0

6. If X has a geometric random variable, show that for any positive integers n, k (using the definition of conditional probability)

$$P(X > n + k - 1 | X > n - 1) = P(X > k)$$

Given the construction of a geometric random variable from a sequence of independent Bernoulli trials, explain this property directly.

7. If f and g are densities, show that $\alpha f + (1 - \alpha)g$ with $0 < \alpha < 1$ is a density too.

8. Let T be an exponential random variable with parameter $\lambda > 0$. Let X be a dicrete random variable defined as $X = k$ if $k \leq T < k + 1$, $k = 0, 1, 2, \ldots$. Find the pdf of X.

9. T is an exponential random variable and $P(T < 1) = 0.05$. What is λ?

10. Let $f(x) = (1 + \alpha x)/2$ for $-1 \leq x \leq 1$, and 0 otherwise. We further assume that $|\alpha| \leq 1$. Show that f is a density and find the cumulative distribution function F and quantile function Q.