Topic 6. Radiation Fundamentals

Telecommunication Systems Fundamentals

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Academic year 2013-2014

Concepts in this Chapter

- Antennas: definitions and classification
- Antenna parameters
- Fundamental Theorems: uniqueness and reciprocity. Images’ method
- Friis’ equation
- Link Budget of a Radio-Link

Theory classes: 1.5 sessions (3 hours)
Problems resolution: 0.5 session (1 hours)
Introduction: Radio-Telecommunication Systems

- Info transmission implies to transmit a signal (with a given energy) through a radio-channel
Introduction: Transmitting and Receiving Antenna

• An antenna can either transmit (radiate) energy in Transmission

• Or capture energy in Reception

Radiation Performance of an Antenna

• Radiation is the electromagnetic energy flux outward form a source
• Basic Problem in electromagnetic theory:
  – Calculus of the electromagnetic field produced by a structure in any given space point

From Electrical Currents within the Tx’ing antenna structure

From the Electromagnetic Field distribution along a closed surface that surrounds the Tx’ing antenna
Efficiency as Main Objective in Antennas

- Efficiency is the main objective when designing/selecting an antenna
  - Maximize the electromagnetic field power in a given point given an amount of power provided to the antenna

- Which antenna parameters should we consider
  - Phase Center
  - Power Parameters
    - Radiated power flux density
    - Radiation intensity
    - Directivity
    - Power Gain
  - Gain diagram
  - Polarization
  - Bandwidth

Power Parameters: Poynting’s Theorem

- **Complex Poynting’s Vector**: electromagnetic energy flux density through a given surface
  \[ S = \frac{1}{2} \text{Re}\{\vec{E} \times \vec{H}^*\} \]

- **Average Power**: Poynting’s vector flux
  \[ P_{\text{media}} = \int \int S \cdot dS \ [\text{W}] \]
Power Parameters: Radiation Density

- Average radiated power **per surface unit** in a given direction

\[
\phi(\theta, \varphi) = \frac{1}{2} \text{Re} \left\{ \mathbf{E} \times \mathbf{H}^* \right\} \left[ \frac{W}{m^2} \right]
\]

\[
\phi(\theta, \varphi) = \frac{\Delta p(\theta, \varphi)}{\Delta S}
\]

Power Parameters: Radiated Power

- **Sum up** radiated flux density along a sphere surface that circumscribe the antenna

\[
\Pr = \int \int \phi(\theta, \varphi) \cdot dS \ [W]
\]
Power Parameters: Radiation Intensity

- Average radiated power **per solid angle unit** in a given direction

\[ i(\theta, \varphi) = \frac{\Delta p(\theta, \varphi)}{\Delta \Omega} \left[ \frac{W}{\text{esteroradian}} \right] \]

\[ \mathbf{P}_r = \int \int i(\theta, \varphi) \cdot d\Omega \]

- Independently of the distance from the antenna

\[ \Delta S = r^2 \Delta \Omega \Rightarrow \phi(\theta, \varphi) \cdot r^2 = i(\theta, \varphi) \]

- The Power Flux Density decreases with distance inversely proportional to the area of the spherical solid angle

\[ r(\theta, \varphi) = \frac{i(\theta, \varphi)}{i_{\text{max}}} \]

- Radiation Diagram (power-wise)
Power Parameters: Radiation Intensity

Omnidirectional (on Azimuth)

Dipolo: typical on cellular terminals

Directive

Yagi: typical for television receivers

Power Parameters: Isotropic Antenna

- **Ideal** point source that radiates uniformly in all directions

\[ i_{\text{iso}} = \frac{P_r}{4\pi} \]

\[ \phi_{\text{iso}} = \frac{P_r}{4\pi \cdot r^2} \]

Reference to compare rest of Antennas
Power Parameters: Isotropic Antenna

Assuming the same transmitted power by both antennas…which focalizes better?

Power Parameters: Directivity (function of direction)

- Ratio between the power density flux an antenna radiates and the one an isotropic (omnidirectional) antenna would do, as a function of the radiating direction

\[
D(\theta, \varphi) = \frac{i(\theta, \varphi)}{i_{iso}} = 4\pi \frac{i(\theta, \varphi)}{Pr}
\]

\[
i(\theta, \varphi) = \frac{Pr}{4\pi} D(\theta, \varphi)
\]

\[
\phi(\theta, \varphi) = \frac{Pr}{4\pi \cdot r^2} D(\theta, \varphi)
\]
Power Parameters: Directivity

• Directivity is defined as the maximum value of the Directivity function

\[
D = 4\pi \frac{i_{\text{max}}}{P_r}
\]

– Because \( P_r = \int \int i(\theta, \phi) \cdot d\Omega \) \( r(\theta, \phi) = \frac{i(\theta, \phi)}{i_{\text{max}}} \)

\[
D = \frac{4\pi}{\Omega_A} \quad \text{being} \quad \Omega_A = \int \int r(\theta, \phi) d\Omega
\]

Power Parameters: Directivity

• When the Beam is narrow

\[
D \approx \frac{4\pi}{\theta_1 \theta_2}
\]

• Conclusions
  – Directivity provides information about how the radiated power is distributed with direction (elevation and azimuth)
  – Directivity does not provide information about the actual transmitted power
Power Parameters: Gain Function

- **Ratio** between the power intensity radiated in a direction and the radiated intensity of an isotropic antenna, given a power available to the antenna

\[
G(\theta, \varphi) = 4\pi \frac{i(\theta, \varphi)}{P_{in}}
\]

being \( P_{in} \) the power available at the antenna input

Power Parameters: Gain

- Gain is the maximum value of the Gain Function

\[
G = 4\pi \frac{i_{\text{max}}}{P_{in}}
\]

- Because it is a ratio, the units are dBs
**Power Parameters: Examples of Gain**

<table>
<thead>
<tr>
<th>ANTENNA TYPE</th>
<th>GAIN (dBi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isotropic</td>
<td>0,0</td>
</tr>
<tr>
<td>Ground Plane 1/4 wavelength</td>
<td>1,8</td>
</tr>
<tr>
<td>Dipole 1/2 wavelength</td>
<td>2,1</td>
</tr>
<tr>
<td>Monopole 5/8 wavelength</td>
<td>3,3</td>
</tr>
<tr>
<td>Yagui 2 elements</td>
<td>7,1</td>
</tr>
<tr>
<td>Yagui 3 elements</td>
<td>10,1</td>
</tr>
<tr>
<td>Yagui 4 elements</td>
<td>12,1</td>
</tr>
<tr>
<td>Yagui 5 elements</td>
<td>14,1</td>
</tr>
</tbody>
</table>

**Power Parameters: Efficiency**

- $P_{in}$ and $Pr$ are related to each other around radiating Efficiency of the antenna.

\[
\eta = \frac{R_s + R_L}{R_L} \quad \eta_{cd} \quad \eta_d = 1 - |\Gamma|^2 \quad \Gamma = \frac{Z_a - Z_s}{Z_a + Z_s}
\]

\[
\begin{align*}
Z_a &= R_s + R_L + jX_a \\
V_s
\end{align*}
\]
Power Parameters: Efficiency

- From the above definition of Efficiency, the relationship between Gain and Directivity of an antenna can be derived

\[ G = \eta \cdot D \]

- Can the Gain of an antenna be increased by increasing the Directivity?

Example

- A dipole of half wavelength without losses, with input impedance of 73Ω is connected to a transmission line with characteristic impedance of 50Ω. Assuming the radiating intensity of the antenna is

\[ i(\theta) = B_0 \text{sen}^3(\theta) \]

Compute the Gain of the Antenna

\[
\begin{align*}
\eta_{\text{max}} &= i_{\text{max}}(\theta) = B_0 \\
P_r &= \int \int D(\theta) \text{sen} \theta = 2B_0 \int_0^\pi \text{sen}^3 \theta d\theta = B_0 \left( \frac{3\pi^2}{4} \right) \\
D &= 4\pi \eta_{\text{max}} / P_r = 1.697 \\
G &= \eta \cdot D = \left[ \frac{P_r}{1 + \frac{1}{\sqrt{1 + \frac{73}{50}}} \left( 1 - \frac{73 - 50}{73 + 50} \right)^4} \right] \text{[dB]} \\
\end{align*}
\]

\[ 1.697 = 0.965 \quad 1.697 = 1.638 = 2.14\text{dB} \]
Example Answer

\[ i_{\text{max}} = i_{\text{max}}(\theta) = B_0 \]
\[ P_r = \int_{0}^{\pi} D(\theta) \sin \theta = 2\pi B_0 \int_{0}^{\pi} \sin^4 \theta d\theta = B_0 \left( \frac{3\pi^2}{4} \right) \]
\[ D = 4\pi \frac{i_{\text{max}}}{P_r} = 1.697 \]
\[ G = \eta \cdot D = \left( 1 - |r|^2 \right) D = \left( 1 - \left| \frac{73 - 50}{73 + 50} \right|^2 \right) 1.697 = 0.965 \cdot 1.697 = 1.638 = 2.14\ dB \]

Radiation Diagram

What parameter are useful?
Radiation Diagram

- Parameters to characterize the lobe structure
  - Beamwidth
    - Null to Null Beamwidth
    - Half Power Beamwidth (HPBW) – 3dBs
    - 10 dB Beamwidth
  - Lobes
    - Main lobe
    - Side lobes
      - First lobe
    - Backlobe
  - Forward-backward ratio

Radiation Diagram: Classification

- Isotropic
- Omnidirectional
- Directive
- Multi-beam
Polarization

- Of a Plane-wave, it refers to the spatial orientation of the time-variation of the electric field
- Of an antenna, it refers to the polarization of the radiated field
  - Generally speaking polarization is defined according to the propagation direction

Co-Polar and Cross-Polar components

[Graphs and diagrams showing polarization patterns]
Antenna Bandwidth

- Frequency margin where the defined parameters for the antenna remain valid (impedance, beamwidth, sidelobes ratio, etc.)
  - Narrowband Antennas (<10% central frequency)
  - Broadband Antennas (>10% central frequency)

Antenna Radiation on Free-Space Condition

- What is Free-Space condition: no obstacles or material to influence the radiation pattern – not even the ground
Radiation Zones

- When the distance is much greater than the wavelength, $R \gg \lambda$, the observed wave behaves as a Plane-Wave.
- When can we consider we are “far enough”?

Far-Field zone: $R \approx r$

Radiation Zones

- Simplifying but useful approach: three zones are defined:
  - Near-Field: Rayleigh (spheric propagation)
  - Intermediate-Field: Fresnel (interferences)
  - Far-Field: Fraunhofer

\[
\frac{D^2}{2\lambda} \quad \frac{2D^2}{\lambda}
\]

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Radiation Zones: Far-Field

• Conclusions:
  – Power decreases as square of the distance
  – Satisfy condition of Plane-Wave
    • E and H fields are perpendicular
    • Amplitude of E and H fields are related to each other through the transmission mean impedance
  – The type of the transmitting antenna affects only on the angular variation of the transmitted power flux (Radiation Diagram)
  – Transmission direction of the wave propagation coincides with line of sight of the transmitting antenna

EIRP - Equivalent Isotropic Radiated Power

• The product of the antenna Gain by the available Power

\[
PIRE = G \cdot P_{in}
\]

\[
i_{iso} = \frac{P_{in}}{4\pi} \quad i_{ant} = \frac{G \cdot P_{in}}{4\pi} = \frac{PIRE}{4\pi}
\]
**EIRP**

- Its units are dBW (or dBm)
- We will see later, this parameter (expressed on dBW) is quite useful when computing the “availability” of the radio-link
- Example: if the Gain of an antenna is 2dBi and it gets a power of 10W, How much is its EIRP?

\[
G = 2 \text{ dB} \\
P_{in} = 10 \cdot \log(10) = 10 \text{ dB} \\
PIRE = 12 \text{ dB}
\]

**Lineal Antennas**

- Antennas built with thin electrically conductive wires (very small diameter compared to \( \lambda \)).
- They are used extensively in the MF, HF, VHF and UHF bands, and mobile communications.
- Among others:
  - Dipole
  - Monopole
  - Yagi antenna
  - Loops
  - Helixes
Lineal Antennas: Infinitesimal Dipole

- Formed by two short conductive wires symmetrically fed at its center, being \( l \ll \lambda \).

\[
E_\theta = jZ_0 \frac{l \cdot I_0}{2\pi \cdot \lambda \cdot r} \cdot \frac{\sin(\theta)}{r} \cdot e^{-j2\pi r/\lambda}
\]

With Linear Polarization
And radiation diagram

Lineal Antennas: Infinitesimal Dipole

- Computing the magnetic field form the electrical, calculating the power flux and integrating for all \( \theta \) we get

\[
Pr = Z_0 \left( \frac{2\pi}{3} \right) \left( \frac{l \cdot I_0}{\lambda} \right)^2 = I_0^2 R_i
\]

\[
R_i = 80\pi^2 \left( \frac{l}{\lambda} \right)^2
\]

- and consequently \( D = \frac{3}{2} \)

- What is the Gain of this antenna?
Lineal Antennas: Half-Wavelength Dipole

- Very common antenna, with a “convenient” radiation impedance of 73Ω

\[
l = \frac{\lambda}{2}
\]

\[
E_\theta = jZ_0 \frac{I_0 e^{-j\frac{\omega l}{\lambda}}}{2\pi r} \left[ \cos \left( \frac{\pi}{2} \cos \theta \right) \right]
\]

Lineal Antennas: Half-Wavelength Dipole

- Radiation parameters

Directividad: \( D_\theta = 1.64 = 2.15 \text{ dB} \)

\( R_{\text{radiación}} = 73 \text{ Ω} \)
Radiated Field over a Perfect Conductor

- Up to this point we have assumed the antennas are in the free-space environment. However they usually are close to the ground.
- When the distance to ground is comparable to the wavelength, and the beamwidth is large, antenna radiation is heavily affected by the presence of the ground.
- For these scenarios, we will assume the ground is a perfect conductor, infinite and plane.

Image Theory

- Intuitively, the field is reflected on the ground
  - Perfect conductor: the transmitted wave is reflected
  - The field $P$ is the result of the sum of the direct and reflected waves
  - Fiend $P$ is the result of the primary and image waves in the equivalent free-space scenario
    - Which is valid only for the upper half semi-space.
**Image Theory**

- **Example:** field produced by a Infinitesimal dipole

\[
E_\phi = jZ_0 I_0 e^{-\frac{2\pi}{\lambda}} \sin \theta \left[ 2 \cos \left( \frac{2\pi \cdot h}{\lambda} \cos \theta \right) \right]
\]

- **Isolated Dipole Field**
- **Joint Contribution of two Antennas**

Radiation Properties change as function of the ratio 
\[ \frac{h}{\lambda} \]

---

**Image Theory**

- **Example (cont.)**
  - If \( \lambda \gg h \) then, directivity increases by 3dB! \( \rightarrow \) decrease the size
  - Otherwise….
Monopole

- Vertical Dipole divided to its half, that is fed between wire end and a conductive plane

  - Applying Image theory, it can be proven that a monopole above a conductive plane exhibit the same behavior than a dipole with a length twice the height of the monopole

  \[ \alpha = \infty \]

  \[ l = 2h \]

  Consider only \( z > 0 \) and \( \lambda >> h \)

  \[
  \begin{align*}
  P_{r_{\text{mono}}} &= \frac{1}{2} P_{r_{\text{dipolo}}} \\
  R_{r_{\text{mono}}} &= \frac{1}{2} R_{r_{\text{dipolo}}} \\
  i_{\text{mono}} &= i_{\text{dipolo}} \\
  D_{\text{mono}} &= 2D_{\text{dipolo}}
  \end{align*}
  \]

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Monopole

- Example: \( \lambda/4 \) monopole

  - As shown before, the monopole exhibit the same performances than a \( \lambda/2 \) dipole and therefore its directivity is

  \[ D = 5.15 \text{ dBi} \]

- At low frequencies, this antenna has quite large physical dimensions
  - Example: the standard AM transmitter for frequency carrier around 1MHz, corresponds to a wavelength of 300 m, and therefore this \( \lambda/4 \) monopole has a heigh of 75 m.
Reception of an Electromagnetic Field

• If an electromagnetic wave, with a plane-wave like propagation, runs over a conductor (antenna) it generates a current distribution over it
Equivalent Circuit Model for a Receiving Antenna

- An antenna at reception is designed to optimize the power handed out at its terminal

\[ Z_a = R_a + jX_a \]

- Available power at antenna

\[ P_a = \frac{|V_a|^2}{8R_a} \]

- Power handed out

\[ P_L = \frac{1}{2} |I|^2 R_L = P_0 (1 - |\Gamma|^2) \]

\[ \Gamma = \frac{Z_a - Z_L}{Z_a + Z_L} \]

Reciprocity Theorem

- It allows to relate the properties of an antenna when receiving and transmitting

- “The relationship between an oscillating current and the resulting electric field is unchanged if one interchanges the points where the current is placed and where the field is measured”
  
  – For the specific case of an electrical network, it is sometimes phrased as the statement that voltages and currents at different points in the network can be interchanged"
Reciprocity Theorem

• Suppose an anechoic chamber (no echo) in which are placed two antennas, both can transmit and receive, and operating at the same frequency.

• The roles of sending and receiving can exchanged. Thus, the radiation patterns of transmitting and receiving are the same.

\[ \text{If} \ I_1 \rightarrow V_{2,ca} \Rightarrow V_{1,ca} = V_{2,ca} \text{ when } I_2 \rightarrow V_{1,ca} \]

Effective Aperture

• Effective Aperture of an antenna characterizes the electromagnetic energy that it is able to capture.
• Intuitively a large antenna captures more power, as it has more area.
Effective Aperture

- Equivalent aperture is defined as
  \[ A_{\text{eff}} = \frac{\text{Potencia entregada}}{\text{Densidad de potencia incidente}} = \frac{P_L}{\phi_i} = \frac{|I_1|^2 R_L}{2} \]

- The value does not have to match the dimensions (physical) of the antenna.
- When the antenna is flat, the physical relationship between the opening \( A_f \) and the effective aperture \( A_{\text{eff}} \) is known as aperture efficiency, verifying that:
  \[ A_{\text{eff}} = \varepsilon_{\text{ap}} A_f \quad \text{con} \ 0 \leq \varepsilon_{\text{ap}} \leq 1 \]

Directivity vs Maximum Effective Aperture

\[ \phi_{tx} = \frac{P_{tx}}{4\pi \cdot r^2} D_{tx} \]

\[ P_{rx} = \phi_{tx} A_{rx} = \frac{P_{tx} D_{tx} A_{tx}}{4\pi \cdot r^2} \quad \rightarrow \quad D_{rx} A_{rx} = \frac{P_{rx}}{P_{tx}} \left(4\pi r^2\right) \]

\[ D_{rx} A_{rx} = \frac{P_{tx}}{P_{rx}} \left(4\pi r^2\right) \]
Directivity vs Maximum Effective Aperture

\[
\frac{D_{tx}}{A_{rx}} = \frac{D_{rx}}{A_{tx,m}} \Rightarrow \frac{D_{tx}}{A_{rx}} = \frac{D_{rx}}{A_{tx,m}}
\]

- The above solution is valid for any antenna. For an infinitesimal dipole it can be proof that

\[
A_{ef} = \frac{3\lambda^2}{8\pi} = \frac{\lambda^2}{4\pi} D
\]

Directivity vs Maximum Effective Aperture

- In case of losses associated to the antenna, the maximum effective aperture is

\[
A_{ef} = \eta_{cd} \left(1 - |\Gamma|^2 \right) \frac{\lambda^2}{4\pi} D = \frac{\lambda^2}{4\pi} G
\]

\[
G = \frac{4\pi}{\lambda^2} A_{ef}
\]
Polarization Mismatch

- The difference of polarization between transmitting and receiving antennas, it is known as **Polarization Loss Factor**

\[
L_{\text{polarization}} = \left| \mathbf{e}_{\text{tx}} \cdot \mathbf{e}_{\text{rx}}^* \right|^2
\]

Polarization Vector
For Tx antenna

Polarization Vector
For Rx antenna

\[z\]
\[x\]
\[r\]

No-Loss
Max. Losses

Gains in the Radio Link: Tx and Rx Gains

\[
\phi_{\text{tx}} = \frac{P_{\text{tx}}G_{\text{tx}}}{4\pi r^2} = \text{PIRE}
\]

\[
P_{\text{rx}} = \phi_{\text{tx}}A_{\text{rx}} = \phi_{\text{tx}} \frac{\lambda^2}{4\pi} G_{\text{rx}} = \frac{P_{\text{tx}}G_{\text{tx}}G_{\text{rx}}}{(4\pi)^2}
\]

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Friis Transmission Equation

- For Isotropic antennas and free-space propagation
  - The basic losses are
  \[
  p_{\text{Rx}} = \phi_{\text{ISO}} \cdot s_{\text{eq,ISO}} = \phi_{\text{ISO}} \cdot \frac{\lambda^2}{4\pi} \Rightarrow l_{\text{bf}} = \frac{p_{\text{TX}}}{p_{\text{Rx}}} = \left(\frac{4\pi d}{\lambda}\right)^2
  \]

  \[L_{\text{bf}} (dB) = 32.45 + 20\log f (\text{MHz}) + 20\log d (\text{km})\]

  \[L_{\text{bf}} (dB) = 92.45 + 20\log f (\text{GHz}) + 20\log d (\text{km})\]
Friis Transmission Equation

- Sumarizing, for any antenna

\[ l_t = \frac{P_{tx}}{P_{rx}} = a_t \left( \frac{4\pi f^2}{\lambda} \right) \frac{1}{G_{ts} G_{rs}} \]

\[ L_t = L_{bf} + A_s - G_t = L_{bf} - G_t - G_r - G_f \]

<table>
<thead>
<tr>
<th>Usual Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{bf} )</td>
<td>Free-Space Basic Loss</td>
</tr>
<tr>
<td>( L_f )</td>
<td>Isotripic Basic Loss</td>
</tr>
<tr>
<td>( L_{gf} )</td>
<td>Free-Space Transmission Loss</td>
</tr>
<tr>
<td>( L_t )</td>
<td>Any Transmission Loss</td>
</tr>
</tbody>
</table>

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Friis Transmission Equation

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Link Budget

- Link Budget = expression for available power at the receiver as a function of
  - Transmitted Power
  - Rx and Tx Antenna Gains
  - All the losses in the link

\[ P_{Rx} = P_{Tx} - L_{bf} + G_{Tx} + G_{Rx} - A_c \]

Other factors affecting the link
- Normalized Noise Power
  - SNR
  - Power-limited Systems
    - Minimum Received Power (Sensibility) + Fading Margin
    - The maximum distance between Tx and Rx is calculated by the Link Budget

\[
\begin{align*}
    p_n &= k T_0 \cdot b \cdot f_{sis} \\
    P_n(dBm) &= F_{sis}(dB) + 10 \log b(Hz) - 174
\end{align*}
\]

- Interference
  - C/I; SINR
  - Interference-limited Systems
  - Performances: BER, PER, \( P_{out} \ldots \)
Important Concepts in this Topic

- Poynting Vector
- Radiated Power Flux Density
- Antenna Directivity
- Antenna Gain
- Antenna Efficiency
- Antenna Effective Aperture
- Polarization
- Reciprocity Theorem
- Most common simple antennas
- Friis Equation and Link Budget
- Free-Space Basic Propagation Loss