Operation of Electric Power Systems
Chapter 1: Simplified centralized operation and investment planning exercise

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The basic centralized context

Optimal long-term planning

• Optimal mix with a simplified representation
  • Demand by means of the load-duration curve
  • Generation: annualized investment and variable costs

Determine:
- Optimal mix
- Total cost

<table>
<thead>
<tr>
<th>Technology</th>
<th>N</th>
<th>C</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fixed cost</strong></td>
<td></td>
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<tr>
<td>(annualised investment cost)</td>
<td>210 €/kW</td>
<td>114 €/kW</td>
<td>72 €/kW</td>
</tr>
<tr>
<td><strong>Variable cost</strong></td>
<td></td>
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<td></td>
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<tr>
<td>(operation cost)</td>
<td>6 €/MWh</td>
<td>18 €/MWh</td>
<td>30 €/MWh</td>
</tr>
</tbody>
</table>
Content

• Stylized representation of gen. and dem.
• Thermal mix
  • Centralized short-term operation
  • Centralized long-term investment planning
• Hydrothermal mix
  • Centralized short-term operation
  • Centralized long-term investment planning
• Mathematic formulation of the stylized problems
Comparing costs

• Comparing total costs of each technology for covering each slice
  
  • Slice 1: 2500 MW, 8760 h.
    
    • N: 656,4 M€
      • FC = 210 * 2 500 000 = 525 M€
      • VC = 6 * 2 500 * 8760 = 131,4 M€
    
    • C: 679,2 M€
      • FC = 114 * 2 500 000 = 285 M€
      • VC = 18 * 2 500 * 8760 = 394,2 M€
    
    • F: 837 M€
      • FC = 72 * 2 500 000 = 180 M€
      • VC = 30 * 2 500 * 8760 = 657 M€
Comparing costs

• Therefore:
  • Slice 1: 2500 MW, 8760 h.
    • N: 656,4 M€ ⇒ 29,97 €/MWh
  • Slice 2: 1500 MW, 6000 h
    • C: 333 M€ ⇒ 37 €/MWh
  • Slice 3: 1000 MW, 4500 h
    • C: 195 M€ ⇒ 43,33 €/MWh
  • Slice 4: 2000 MW, 1500 h
    • F: 234 M€ ⇒ 78 €/MWh
  • Slice 5: 2000 MW, 500 h
    • F: 174 M€ ⇒ 174 €/MWh

• Total: 39 400 GWh at 1 592,4 M€: 40,4 €/MWh
Results

<table>
<thead>
<tr>
<th>MW</th>
<th>500 h</th>
<th>1500 h</th>
<th>4500 h</th>
<th>6000 h</th>
<th>8760 h</th>
</tr>
</thead>
<tbody>
<tr>
<td>2500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>5000</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>7000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9000</td>
<td></td>
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</tr>
</tbody>
</table>

- F: 2500 MW
- C: 4000 MW
- N: 8760 MW

Chapter 1: Simplified centralized operation and investment planning exercise

Course 2019-2020
Analytical comparison

• $T_1$ substitutes $T_2$ if:

$$FC_1 + VC_1 \times h \leq FC_2 + VC_2 \times h$$

$$\iff h \geq \frac{(FC_1 - FC_2)}{(VC_2 - VC_1)} = h^*$$

Then, $T_1$ substitutes $T_2$ if the running hours are greater than $h^*$.

• $N$ vs. $C$:

• $h^* = \frac{(210000 - 114000)}{(18 - 6)} = 8000$ h

• $C$ vs. $F$:

• $h^* = \frac{(114000 - 72000)}{(30 - 18)} = 3500$ h
Total annualized unitary (per MW) costs curves as a function of the running hours (screening curves)
Production Structure

• **N**: produces 21 900 GWh:
  - produces at **2500 MW** for 8760h.

• **C**: produces 13 500 GWh:
  - produces at **2500 MW** for 4500 h
  - and at 1500 MW for another 1500 h.

• **F**: produces 4 000 GWh:
  - produces at **4000 MW** for 500 h,
  - and at 2000 MW for another 1000 h.
The basic centralized context

Optimal long-term planning

Energy produced:
- 56% of energy produced
- 34% of energy produced

Installed capacity:
- 28% of installed capacity
- 28% of installed capacity
- 44% of installed capacity

Optimal long-term planning

- N: 500 h, 1500 h, 4500 h, 6000 h, 8760 h
- C: 2500 MW, 4000 MW, 5000 MW, 7000 MW, 9000 MW
- F: 9000 MW, 7000 MW, 5000 MW, 4000 MW, 2500 MW

Installation capacity for 2019-2020
The basic centralized context

Optimal long-term planning

• How to include in the study the NSE (non-served energy) cost?
  • The maximum price the demand is willing to pay for the electricity
  • Consider a non-served energy cost of 80 €/MWh

• How would the solution change if...
  • ...we change the rate of return of the project
  • ...we introduce a price for CO2 (provided technology specific emission rates)
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• Hydrothermal mix
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  • Centralized long-term investment planning

• Mathematic formulation of the stylized problems
The basic centralized context

Basic hydrothermal coordination

• Economic hydro thermal dispatch based on variable costs

• Data

• Consider the same demand and thermal units

• Consider the following stylized representation of hydro units:
  • First case: Pmax \(\rightarrow\) 4 GW, Energy that can produce \(\rightarrow\) 4 GWh
  • Second case: Pmax \(\rightarrow\) 2 GW, Energy that can produce \(\rightarrow\) 5 GWh

<table>
<thead>
<tr>
<th>CV (€/MWh)</th>
<th>N</th>
<th>C</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>18</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Pmax (MW)</td>
<td>3000</td>
<td>4000</td>
<td>2000</td>
</tr>
</tbody>
</table>
The basic centralized context

Basic hydrothermal coordination

• Economic hydro thermal dispatch based on variable costs
• When to use the hydro resources
The basic centralized context

Optimal thermal mix (with pre-existing hydro)

• Let us reconsider the previous problem of determining the optimal mix

• But now we have in the system (before installing the thermal generation) a hydro plant (the investment has already been done)
  • Maximum power output: 500 MW
  • The size of the reservoir is 1500 GWh

• Which is the optimal mix now?
  • What is it more economical when considering the investment costs?
    • Dispatching the hydro in the peak or as a base load?
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Optimal thermal mix (with pre-existing hydro)

• Baseload dispatch

Hydro: 500 MW and 1500 GWh
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Optimal thermal mix (with pre-existing hydro)

- Baseload dispatch

\[
P = \frac{1500000}{8760} = 171.233 \text{ MW}
\]

- Savings (nuclear):
  - CF: \(210 \times 1000 \times 171.233 = 35,96 \text{ M€}\)
  - CV: \(6 \times 8760 \times 171.233 = 9 \text{ M€}\)
  - Savings = 44,96 M€
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Optimal thermal mix (with pre-existing hydro)

- Peak-load dispatch

Hydro: 500 MW and 1500 GWh
The basic centralized context

**Optimal thermal mix (with pre-existing hydro)**

- Peak-load dispatch

\[ \text{Slice 5: } P_5 = 500 \text{ MW} ; \ 500 \text{ h} \Rightarrow E_5 = 250 \text{ GWh} \]

\[ \text{Slice 4: } P_4 = 500 \text{ MW} ; \ 1000 \text{ h} \Rightarrow E_4 = 500 \text{ GWh} \]

\[ \text{Slice 3: } E_3 = 750 \text{ GWh} ; \ 3000 \text{ h} \Rightarrow P_3 = 250 \text{ MW} \]

- What are the savings?
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Optimal thermal mix (with pre-existing hydro)

Curvas de coste total

Coste total (pts/kW)

Horas

Coste total (pts/kW)

Horas

Coste total (pts/kW)

Horas

Coste total (pts/kW)

Horas

N

C

F

9000 MW
8500 MW
7000 MW
6500 MW
5000 MW
4750 MW
4000 MW
2500 MW

Hydro

500 h 1500 h 4500 h 6000 h 8760 h
The basic centralized context

Optimal thermal mix (with pre-existing hydro)
The basic centralized context
Optimal thermal mix (with pre-existing hydro)

• Peak-load dispatch

• Investment cost savings
  • 500 MW of fuel (slices 5 and 4)
    • $500,000 \times 72 = 36 \text{ M€}$
  • 250 MW of C (slice 3)
    • that are substituted with hydro during 3000 h
    • But that need to be supplied with F during 1500 h: $CF_3 = 250,000 \times (114-72) = 10,5 \text{ M€}$

• Variable cost savings
  • VC of F: $(500\times1500 - 250\times1500) \times 30 = 11,25 \text{ M€}$
  • VC de C: $250 \times 4500 \times 18 = 20,25 \text{ M€}$

• Total savings: $78 \text{ M€} (> 44,96 \text{ M€})$
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The basic centralized context
Mathematical formulation

• The simplified setting:
  • Two periods with different demand values and duration
  • Three technologies (N, C, F)
    • Variable costs (€/MWh) \( v_c^N, v_c^C, v_c^F \)
    • Installed capacity (MW) \( \bar{P}^N, \bar{P}^C, \bar{P}^F \)
    • Investment costs (€/MW) \( i_c^N, i_c^C, i_c^F \)

• Formulate
  • Minimum cost dispatch
    • Capacity is already installed
  • Minimum cost mix
    • No capacity is installed
    • There is a certain amount of capacity installed

• Reformulate including the non-served energy
The basic centralized context
Mathematical formulation

• Short-term operation
• Variables: $p_1^N, p_2^N, p_1^C, p_2^C, p_1^F, p_2^F$

Objective function:

$$OF = vc^N \cdot (p_1^N \cdot t1 + p_2^N \cdot t2) +
vc^C \cdot (p_1^C \cdot t1 + p_2^C \cdot t2) +
vc^F \cdot (p_1^F \cdot t1 + p_2^F \cdot t2)$$

Constraints:

$$d_1 = p_1^N + p_1^C + p_1^F$$
$$d_2 = p_2^N + p_2^C + p_2^F$$

$$0 \leq p_1^N \leq \bar{P}^N, 0 \leq p_2^N \leq \bar{P}^N$$
$$0 \leq p_1^C \leq \bar{P}^C, 0 \leq p_2^C \leq \bar{P}^C$$
$$0 \leq p_1^F \leq \bar{P}^F, 0 \leq p_2^F \leq \bar{P}^F$$
The basic centralized context
Mathematical formulation

• Now including non-served energy value  $nsev$
• Variables:  $p_1^N, p_2^N, p_1^C, p_2^C, p_1^F, p_2^F, p_1^{NSE}, p_2^{NSE}$

Objective function:

$$OF = vc^N \cdot (p_1^N \cdot t1 + p_2^N \cdot t2) +$$
$$vc^C \cdot (p_1^C \cdot t1 + p_2^C \cdot t2) + vc^F \cdot (p_1^F \cdot t1 + p_2^F \cdot t2) +$$
$$nsev \cdot (p_1^{NSE} \cdot t1 + p_2^{NSE} \cdot t2)$$

Constraints:

$$d_1 = p_1^N + p_1^C + p_1^F + p_1^{NSE}$$
$$d_2 = p_2^N + p_2^C + p_2^F + p_2^{NSE}$$
$$0 \leq p_1^N \leq \bar{P}^N, 0 \leq p_2^N \leq \bar{P}^N$$
$$0 \leq p_1^C \leq \bar{P}^C, 0 \leq p_2^C \leq \bar{P}^C$$
$$0 \leq p_1^F \leq \bar{P}^F, 0 \leq p_2^F \leq \bar{P}^F$$
$$0 \leq p_1^{NSE}, 0 \leq p_2^{NSE}$$
The basic centralized context
Mathematical formulation

- Optimal mix
- Variables: $p_1^N, p_2^N, p_1^C, p_2^C, p_1^F, p_2^F, p_1^{NSE}, p_2^{NSE}, \bar{p}_N, \bar{p}_C, \bar{p}_F$

Objective function:
$$OF = ic^N \cdot \bar{p}_N + ic^C \cdot \bar{p}_C + ic^F \cdot \bar{p}_F +$$
$$vc^N \cdot (p_1^N \cdot t1 + p_2^N \cdot t2) +$$
$$vc^C \cdot (p_1^C \cdot t1 + p_2^C \cdot t2) +$$
$$vc^F \cdot (p_1^F \cdot t1 + p_2^F \cdot t2) + nsev \cdot (p_1^{NSE} \cdot t1 + p_2^{NSE} \cdot t2)$$

Constraints:
$$d_1 = p_1^N + p_1^C + p_1^F$$
$$d_2 = p_2^N + p_2^C + p_2^F$$
$$0 \leq p_1^N \leq \bar{p}_N, 0 \leq p_2^N \leq \bar{p}_N$$
$$0 \leq p_1^C \leq \bar{p}_C, 0 \leq p_2^C \leq \bar{p}_C$$
$$0 \leq p_1^F \leq \bar{p}_F, 0 \leq p_2^F \leq \bar{p}_F$$
The basic centralized context

Mathematical formulation

• Investment under uncertainty: two potential scenarios (with associated probabilities)
  • Consider the non-served energy cost

![Diagram](image)
The basic centralized context
Mathematical formulation

• Variables:
  \[
p_{a1}^N, p_{a2}^N, p_{a1}^C, p_{a2}^C, p_{a1}^F, p_{a2}^F, p_{b1}^N, p_{b2}^N, p_{b1}^C, p_{b2}^C, p_{b1}^F, p_{b2}^F
  \]
  \[
  \tilde{p}^N, \tilde{p}^C, \tilde{p}^F
  \]

Objective function:
\[
OF = ic^N \cdot \tilde{p}^N + ic^C \cdot \tilde{p}^C + ic^F \cdot \tilde{p}^F +
pa \cdot [vc^N \cdot (p_{a1}^N \cdot ta1 + p_{a2}^N \cdot ta2) + vc^C \cdot (p_{a1}^C \cdot ta1 + p_{a2}^C \cdot ta2) +
vc^F \cdot (p_{a1}^F \cdot ta1 + p_{a2}^F \cdot ta2)]
+ pb \cdot [vc^N \cdot (p_{b1}^N \cdot tb1 + p_{b2}^N \cdot tb2) + vc^C \cdot (p_{b1}^C \cdot tb1 + p_{b2}^C \cdot tb2) +
vc^F \cdot (p_{b1}^F \cdot tb1 + p_{b2}^F \cdot tb2)]
\]

Constraints:
\[
d_{a1} = p_{a1}^N + p_{a1}^C + p_{a1}^F
\]
\[
d_{a2} = p_{a2}^N + p_{a2}^C + p_{a2}^F
\]
\[
d_{b1} = p_{b1}^N + p_{b1}^C + p_{b1}^F
\]
\[
d_{b2} = p_{b2}^N + p_{b2}^C + p_{b2}^F
\]
\[
0 \leq p_{a1}^N \leq \tilde{p}^N, 0 \leq p_{a2}^N \leq \tilde{p}^N
\]
\[
0 \leq p_{a1}^C \leq \tilde{p}^C, 0 \leq p_{a2}^C \leq \tilde{p}^C
\]
\[
0 \leq p_{a1}^F \leq \tilde{p}^F, 0 \leq p_{a2}^F \leq \tilde{p}^F
\]
The basic centralized context
Mathematical formulation

- **Variables:**

\[ p_{a1}, p_{a2}, p_{a1}^N, p_{a2}^N, p_{a1}^C, p_{a2}^C, p_{a1}^F, p_{a2}^F, p_{a1}^{NSE}, p_{a2}^{NSE} \]

\[ p_{b1}, p_{b2}, p_{b1}^N, p_{b2}^N, p_{b1}^C, p_{b2}^C, p_{b1}^F, p_{b2}^F, p_{b1}^{NSE}, p_{b2}^{NSE} \]

- **Objective function:**

\[ OF = ic^N \cdot \bar{p}^N + ic^C \cdot \bar{p}^C + ic^F \cdot \bar{p}^F + \]

\[ pa \cdot [vc^N \cdot (p_{a1}^N \cdot ta1 + p_{a2}^N \cdot ta2) + vc^C \cdot (p_{a1}^C \cdot ta1 + p_{a2}^C \cdot ta2) + vc^F \cdot (p_{a1}^F \cdot ta1 + p_{a2}^F \cdot ta2) + nsev \cdot (p_{a1}^{NSE} \cdot t1 + p_{a2}^{NSE} \cdot t2)] + pb \cdot [vc^N \cdot (p_{b1}^N \cdot tb1 + p_{b2}^N \cdot tb2) + vc^C \cdot (p_{b1}^C \cdot tb1 + p_{b2}^C \cdot tb2) + vc^F \cdot (p_{b1}^F \cdot tb1 + p_{b2}^F \cdot tb2) + nsev \cdot (p_{b1}^{NSE} \cdot t1 + p_{b2}^{NSE} \cdot t2)] \]

- **Constraints:**

\[ d_{a1} = p_{a1}^N + p_{a1}^C + p_{a1}^F + p_{a1}^{NSE} \]

\[ d_{a2} = p_{a2}^N + p_{a2}^C + p_{a2}^F + p_{a2}^{NSE} \]

\[ 0 \leq p_{a1}^N \leq \bar{p}^N, 0 \leq p_{a2}^N \leq \bar{p}^N \]

\[ 0 \leq p_{a1}^C \leq \bar{p}^C, 0 \leq p_{a2}^C \leq \bar{p}^C \]

\[ 0 \leq p_{a1}^F \leq \bar{p}^F, 0 \leq p_{a2}^F \leq \bar{p}^F \]

\[ 0 \leq p_{a1}^{NSE}, 0 \leq p_{a2}^{NSE} \]

\[ d_{b1} = p_{b1}^N + p_{b1}^C + p_{b1}^F + p_{b1}^{NSE} \]

\[ d_{b2} = p_{b2}^N + p_{b2}^C + p_{b2}^F + p_{b2}^{NSE} \]

\[ 0 \leq p_{b1}^N \leq \bar{p}^N, 0 \leq p_{b2}^N \leq \bar{p}^N \]

\[ 0 \leq p_{b1}^C \leq \bar{p}^C, 0 \leq p_{b2}^C \leq \bar{p}^C \]

\[ 0 \leq p_{b1}^F \leq \bar{p}^F, 0 \leq p_{b2}^F \leq \bar{p}^F \]

\[ 0 \leq p_{b1}^{NSE}, 0 \leq p_{b2}^{NSE} \]