1) Let \( \{ \vec{e}_1, \vec{e}_2 \} \) be a basis of a Euclidean vector space \( E \) satisfying:

\[
\begin{align*}
\vec{e}_1, (3\vec{e}_1 + \vec{e}_2) &= 2 \\
\vec{e}_2, \vec{e}_2 &= 2 \\
\vec{e}_2, (-2\vec{e}_1 + \vec{e}_2) &= 4
\end{align*}
\]

a) Find the scalar product matrix with respect to \( B \) and the angle between the vector of the basis.

b) If \( W \) is a subspace with equations (on \( B \)) \( x - 2y = 0 \), find the equations of \( W^\perp \) (on \( B \)).

c) Find an orthonormal basis of \( E \).

d) Find the equations (on \( B \)) of the orthogonal reflection with respect to \( W \).

2) Let \( \{ \vec{e}_1, \vec{e}_2 \} \) be a basis of a Euclidean vector space \( E \)

\[
\left\| \vec{e}_2 \right\| = \sqrt{2}, \quad \vec{e}_1 - \vec{e}_2 \text{ is a unit vector,} \quad (\vec{e}_1 - \vec{e}_2) \cdot \vec{e}_2 = -1
\]

a) Find the scalar product matrix with respect to \( B \) and the angle that \( \vec{e}_1 \) has with \( \vec{e}_2 \).

b) Find the equations of the orthogonal projection onto the subspace \( W \) spanned by \( \vec{e}_2 \) and the equations of the orthogonal reflection with respect to \( W \).

3) Let \( \{ \vec{e}_1, \vec{e}_2 \} \) be a basis for a Euclidean vector space \( E \) such that the scalar (inner) product matrix with respect to \( B \) is

\[
\frac{2}{\sqrt{5}} \quad \frac{-1}{\sqrt{5}} \\
\frac{-1}{\sqrt{5}} \quad \frac{2}{\sqrt{5}}
\]

a) Find the angle that \( \vec{e}_1 \) has with \( \vec{e}_2 \).

b) If \( f \) is a linear map such that \( f(\vec{e}_1) = \frac{7}{5}\vec{e}_1 + \frac{8}{5}\vec{e}_2 \) and \( f(\vec{e}_2) = -\frac{4}{5}\vec{e}_1 - \frac{1}{5}\vec{e}_2 \), is \( f \) a symmetric tensor? If possible, find a spectral basis for \( f \) of \( E \).

4) Consider a basis \( \{ \vec{e}_1, \vec{e}_2 \} \) for a Euclidean vector plane verifying \( \left\| \vec{e}_1 \right\| = 1 \) \( y \vec{e}_1 \cdot \vec{e}_2 = 2 \). If

\[
\frac{2}{\sqrt{5}} \quad \frac{-1}{\sqrt{5}} \\
\frac{-1}{\sqrt{5}} \quad \frac{2}{\sqrt{5}}
\]

is the matrix of an orthogonal tensor with respect to \( B \), find the inner product matrix with respect to \( B \) and an orthonormal basis.

5) Let \( \{ \vec{e}_1, \vec{e}_2 \} \) be a basis for a Euclidean vector space satisfying:

\[
\begin{align*}
\vec{e}_1 \cdot \vec{e}_2 &= -1 \\
(\vec{e}_2 - \vec{e}_1)(\vec{e}_1 + \vec{e}_2) &= -3 \\
\left\| \vec{e}_2 \right\| &= 1
\end{align*}
\]

a) Find the scalar product matrix with respect to \( B \) and the angle between the vectors of the basis.

b) If \( W \) is a subspace with equations on \( B \) : \( x_1 - x_2 = 0 \), find the equations of the orthogonal complement of \( W, W^\perp \) (on \( B \)).

c) If \( f \) is a linear map such that \( f(\vec{e}_1) = 3\vec{e}_1 + \vec{e}_2 \) and \( f(\vec{e}_2) = 2\vec{e}_2 \), is \( f \) an orthogonal vector reflection? Justify your answer.

d) Find the image under the vector rotation of angle \( \frac{\pi}{2} \) of the vector \( \vec{e}_2 \).