Topic 1: Review of Stochastic Processes

Academic Year 2013 - 2014

P1.- Let the stochastic process be defined by the set of all possible realizations $x[n]$, $-5 \leq n \leq 5$, generated by 11 independent releases of a 6-sided die and values $x \epsilon \{1,2,3,4,5,6\}$. Obtain:

a) The PDF of $x[n]$ in the time instant n_1. What is the probability of observing a realization such that $x[0] = 3$?
b) The joint distribution function in the time instants n_1 and n_2. What is the probability of observing a realization such that $x[-1] = 3$ y $x[1] = 3$?
c) The probability of obtaining the value 1 for all $-5 \leq n \leq 5$.
d) The probability of obtaining realizations $x[n] > 3$ for all $-5 \leq n \leq 5$.

P2.- Let the stochastic process be defined by the set of all possible realizations $x[n]$, generated by independent measurements of a process which values are distributed uniformly as $X \sim U(-1,1)$. Obtain:

a) The PDF of the magnitude $x[n]$ at time instant n_1.
b) The joint PDF of the magnitude $x[n]$ at the time instants n_1 and n_2, where $n_1 \neq n_2$.
c) The probability that given a time instant n_1, $0 < x[n_1] < 0.5$.
d) The probability that given two different time instants n_1 and n_2, $0 < x[n_1] < 0.5$ and $-0.5 < x[n_2] < 0$.

P3.- The PDF of the stochastic, stationary and ergodic process $x(t)$, $f(x_1; t_1)$, is a uniform function $X \sim U(1, 2)$. Obtain:

a) The fraction of time in which $x(t) > 1.5$.
b) The fraction of time in which $x(t) < 1.75$.
c) The mean value of $x(t)$.
d) The mean power of $x(t)$.

P4.- What is the autocorrelation $R_{yy}(\tau)$ of an i.i.d. process ("independent and identically distributed" ➔ "independent and stationary") $y[n]$ whose samples are distributed according to $Y \sim N(m_y,\sigma_y^2)$? What is the physical meaning of $R_{yy}(0)$?

P5.- Suppose now that the samples from the process $z[n]$ are distributed according to a random variable Z. If Z is a function of the random variable $Y \sim N(m_y,\sigma_y^2)$, namely $Z = aY + b$, where a and b are constants, what is the autocorrelation $R_{zz}(\tau)$? What is its physical meaning?

P6.- Let $x[n]$ be an uncorrelated process whose samples are distributed according to $X \sim N(0,\sigma_x^2)$. Define the process $z[n]$ as follows: $z[n] = x[n + 1] + x[n] + x[n - 1]$, i.e. a sample of $z[n]$ contains the sum of three consecutive samples of $x[n]$. What is its autocorrelation $R_{zz}(\tau)$? What is its physical meaning?

P7.- Obtain the power or energy spectral density (as appropiate) of the signal defined as:

$$x(t) = \begin{cases} 1, & |t| < T \\ 0, & resto \end{cases}$$

P8.- Obtain the mean value, the energy, the power, the autocorrelation function and the power spectral density of x(t) defined as:

$$x(t) = cos(2\pi ft + \varphi)$$

P9.- Let $n(t)$ be an uncorrelated Gaussian noise, whose magnitude is distributed with mean 0 and variance σ^2. Obtain the PDF (as a function of time, i.e. at each instant t), the mean value, the power, the autocorrelation and power spectral density of the signal:

$$s(t) = n(t) + cos(2\pi ft)$$

P10.- Let $n(t)$ be a process whose power spectral density takes a constant value between $\pm f_c$-B/2 and $\pm f_c$+B/2, with $B << f_c$ and is zero for any frequency outside these two intervals. The total power of the noise is P_n. Consider this signal uncorrelated with any other. Obtain the spectrum of the signal:

$$s(t) = n(t) + cos(2\pi f_c t).$$