AERODINÀMICA I MECÀNICA DE VOL

EXAM PARTIAL **A** **Friday 7th of November 2014**

Formulario T8: Stability and Control

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Total pitching moment about the center of gravity:

$$C_{M,cg} = C_{M,ac_{wb}} + C_{L_{wb}}\left(x_{cg} - x_{ac_{wb}}\right) - V_H C_{L,t}$$

$$C_{M,cg} = C_{M,ac_{wb}} + a_{wb}\alpha_{wb}\left(x_{cg} - x_{ac_{wb}} - V_H \frac{a_t}{a_{wb}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right) + V_H a_t\left(i_t + \varepsilon_0\right)$$

b. Longitudinal control

$$C_{L,t} = a_t\alpha_t + \frac{dC_{L,t}}{d\delta_e}\delta_e$$

Change in pitching moment acting on the plane

$$\Delta C_M = \frac{dC_{M,cg}}{d\delta_e}\delta_e = -V_H \frac{dC_{L,t}}{d\delta_e}\delta_e$$

Pitching moment equation:

$$C_{M,cg} = C_{M,0} + \frac{dC_{M,cg}}{dC_L}C_L + \frac{dC_{M,cg}}{d\delta_e}\delta_e \qquad \text{with } \frac{dC_{M,cg}}{dC_L} = \left(x_{cg} - x_n\right)$$

c. Directional stability

Contribution of Aircraft components to directional stability: Vertical tail

$$C_N = \frac{N_v}{q_w Sb} = \frac{l_v S_v}{bS}\frac{q_v}{q_w}\frac{dC_L}{d\alpha_v}\left(\beta + \sigma\right)$$

$$\frac{dC_{N,v}}{d\beta} = \frac{l_v S_v}{bS}\frac{q_v}{q_w}\frac{dC_L}{d\alpha_v}\left(1 + \frac{d\sigma}{d\beta}\right)$$

d. Directional control

$$C_N = \frac{N}{q_w Sb} = -\frac{q_v}{q_w}\frac{l_v S_v}{Sb}\frac{dC_{L_v}}{d\delta_r}\delta_r$$

$$\frac{dC_N}{d\delta_r} = -\frac{q_v}{q_w}\frac{l_v S_v}{Sb}\frac{dC_{L_v}}{d\delta_r}$$

f. Lateral control

$$\Delta C_l = \frac{\Delta l}{qSb} = \frac{q\,dy\,cC_L y}{qSb} = \frac{dy\,cC_L y}{Sb}$$

$$C_l = \frac{2\,{dC_{L,w}}/{d\alpha_w}\;{d\alpha}/{d\delta_a}\;\delta_a}{Sb}\int_{y_1}^{y_2} cydy$$

2. AIRCRAFT DYNAMICS

$$\tau = \frac{1}{\left|\mathrm{Re}(\lambda_i)\right|} \qquad \zeta = \left|\frac{\mathrm{Re}(\lambda_i)}{\lambda_i}\right| = \frac{|\sigma_i|}{\sqrt{\sigma_i^2 + \omega_i^2}} \qquad T = \frac{2\pi}{\omega_i} \qquad \omega_n = \left|\lambda_i\right|$$

1 Landing performances

There is only one correct option for each question. Each correct answer adds 0.5 points. Each wrong answer subtracts 0.125 points. In the case that the total sum is negative, the total punctuation of this exercise will be zero.

A jet-powered executive aircraft, approximately modeled after the Cessna Citation 3 is considered. For convenience, we will designate our hypothetical jet as the CC3, having the following characteristics:

- wingspan: $b = 16.23$m, wing area: $S = 30\text{m}^2$, Oswald efficiency factor $e = 0.81$
- take-off weight: $m = 8996$kg
- fuel on board at take-off: 1119gal(=3048kg) of aviation gasoline
- power plant: two turbofan engines of 3650lb thrust each at sea level (1lb thrust = $0.454 * 9.81$N)
- drag coefficient during ground roll (spoilers applied): $C_D = 0.2$
- consider $g = 9.81\text{m/s}^2$
- maximum lift coefficient, with flaps fully employed at touchdown:$C_L = 2.5$
- lift coefficient during ground roll (spoilers applied): $C_L = 0.25$
- the runway has a friction coefficient $\mu = 0.02$ with no brakes applied, and $\mu = 0.4$ when the brakes are applied
- we recall that the distance needed to accelerate or decelerate from a velocity V_1 to a velocity V_2 can be written as:

$$d = \int_{V_1}^{V_2} \frac{V}{\frac{dV}{dt}} dV \text{ and we also recall that } \int \frac{f'(x)}{f(x)} dx = \ln(f(x)) \tag{1}$$

We want to calculate the landing ground roll distance at sea level and at Denver for the CC3. At landing we consider that, no thrust reversal is used, and that the spoilers are employed. The fuel tanks are essentially empty, so neglect the weight of any fuel carried by the airplane.

1. the touchdown velocities, at sea level ($\rho = 1.225\text{kg/m}^3$) and at Denver (mile high city: altitude of 1655m, $\rho = 1.04\text{kg/m}^3$) and considering that $V_{TD} = 1.15V_{stall}$ are

   ☐ $V_{TD,SL} = 13.08$m/s, $V_{TD,Den} = 14.20$m/s ☐ $V_{TD,SL} = 50.41$m/s, $V_{TD,Den} = 54.70$m/s

   ☐ $V_{TD,SL} = 40.99$m/s, $V_{TD,Den} = 44.48$m/s ☐ $V_{TD,SL} = 35.64$m/s, $V_{TD,Den} = 38.68$m/s

   ☐ $V_{TD,SL} = 16.09$m/s, $V_{TD,Den} = 17.47$m/s

2. we can write: $\frac{dV}{dt} = A_0 + A_2V^2$ where

   ☐ $A_0 = -\mu g$, $A_2 = -\frac{1}{2m}\rho C_D S$ ☐ $A_0 = \mu g$, $A_2 = \frac{1}{2m}\rho C_D S$

   ☐ $A_0 = -\mu g - \frac{T}{m}$, $A_2 = \frac{1}{2m}\rho(C_D - \mu C_L)$ ☐ $A_0 = \mu g$, $A_2 = \frac{1}{2m}\rho(C_D - \mu C_L)S$

   ☐ $A_0 = -\mu g$, $A_2 = -\frac{1}{2m}\rho(C_D - \mu C_L)S$

3. the landing ground roll distance at sea level is:

   ☐ 1509m ☐ 304m ☐ 201m ☐ 22m ☐ 1364m

4. the landing ground roll distance at Denver is:

   ☐ 358m ☐ 237m ☐ 26m ☐ 1777m ☐ 1606m

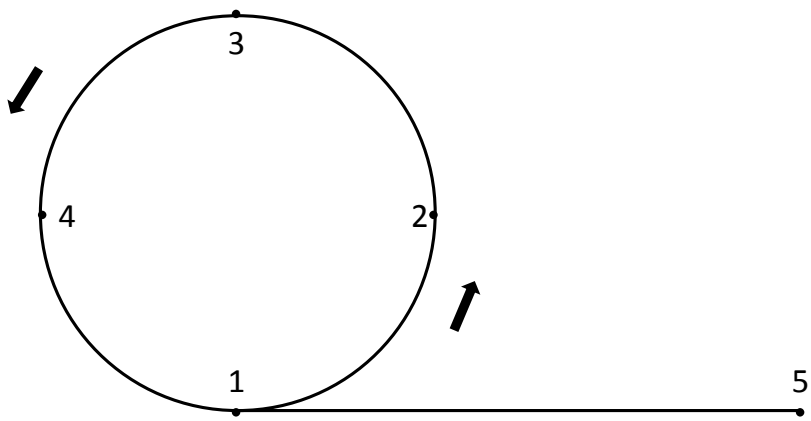
2 Performances

There is only one correct option for each question. Only two significant digits have been considered in the numerical results. Each correct answer adds 0.25 points. Each wrong answer subtracts 0.25/3 points. In the case that the total sum is negative, the total punctuation of this exercise will be zero.

Let's consider the following information on a given acrobatic single-engine turbojet:

- Its parabolic drag polar can be described with the following coefficients: $C_{D0} = 2.2 \cdot 10^{-2}$ and $k = 1.8 \cdot 10^{-2}$
- Due to the fact of being an acrobatic aeroplane, the aerofoil is symmetric, and the next expression can be used for C_L: $C_L = 7.5 \cdot 10^{-2} \cdot \alpha$ (where α is expressed in degrees). This expression can be used up to a maximum value of α equal to 20°
- The aeroplane total mass is 900 kg. This mass can be considered constant along the exercise
- The wing area is S=12 m^2
- The gravity must be assumed as a constant, equal to $g_0 = 10\ m/s^2$
- The typical flight altitudes along the exercise are not so high. We assume here, and through the entire exercise, that the atmospheric density is constant and equal to $\rho_0 = 1.21\ kg/m^3$
- The flight can be assumed as a symmetric one along the exercise. The Thrust's angle of attack can be also neglected

Firstly, the aeroplane performs a looping manoeuvre (consider $t = 0$ corresponding to the lower point of the trajectory, point 1) reaching a load factor equal to 0 at the higher point (point 3). The aeroplane's centre of gravity describes a circumference with a constant velocity. When the aeroplane completes the looping, the pilot changes appropriately the angle of attack, and the aeroplane continues flying with an horizontal trajectory. The aerodynamic velocity, V, is constant and equal to $50\ m/s$ all along the looping as well as along the horizontal trajectory. You are asked to compute:



a. The value of C_{Lopt}:

☐ 0.79 ☐ 0.90 ☐ 1.11 ☐ 1.26

b. The value of α_{opt}:

☐ 15.81° ☐ 14.74° ☐ 12.06° ☐ 9.88°

c. The value of V_B:

☐ 33.49 m/s ☐ 10.59 m/s ☐ 26.40 m/s ☐ 8.35 m/s

d. The value of V_S:

☐ 6.27 m/s ☐ 9.09 m/s ☐ 28.75 m/s ☐ 19.84 m/s

e. The value of the dimensionless drag, $\hat{D}$, at point 2:

☐ 1.74 ☐ 1.34 ☐ 2.68 ☐ 1.00

f. The radius R of the looping:

☐ 336.47 m ☐ 500.00 m ☐ 250.00 m ☐ 220.90 m

g. The value of the aerodynamic efficiency, E, at point 4:

☐ 19.76 ☐ 11.34 ☐ 9.38 ☐ 18.76

h. Time required to perform the looping:

☐ 22.24 s ☐ 72.00 s ☐ 29.53 s ☐ 31.42 s

i. The maximum load factor, n_{max}, during the entire manoeuvre:

☐ 2.83 ☐ 1.66 ☐ 2 ☐ 3

j. The thrust, T, at point 5:

☐ 705.34 N ☐ 176.34 N ☐ 119.91 N ☐ 479.63 N

k. The angle of attack, α, at point 5:

☐ 2.23° ☐ 2.57° ☐ 6.61° ☐ 4.99°

l. The control parameter/s:

☐ π and α ☐ α ☐ π ☐ 0

3 Test

Note that there is only one correct option for each question. Each correct answer adds 0.25 points. Each wrong answer subtracts 0.25/4 points

1. A glider is gliding with constant angle of attack along its path. We can also assume the hypothesis of "small angles". Under these conditions, the following has to be fulfilled:

   a. none of the other answers is correct
   b. V must remain constant
   c. $C_L = C_{Lopt}$
   d. $\hat{D} = cte$
   e. $\alpha = \alpha_{max}$

2. Consider a glider performing a symmetric, rectilinear and stationary flight with $\dot{x}_e$=80 m/s, V_d =2 m/s. There is a constant, tail, atmospheric wind of 80 m/s ($\vec{V}_w = 80\vec{i}_e$). Comparing this scenario with the stagnant condition in which $V_W = 0$, the glider will reach:

   a. double range, double endurance
   b. same range, double endurance
   c. half the range, half endurance
   d. none of the other answers is correct
   e. double range, same endurance

3. Consider a glider performing a symmetric, rectilinear and stationary flight. There is a constant wind with both up and head components: $\vec{V}_w = -60\vec{i}_e - 2\vec{k}_e$. By knowing that $\vec{V}_g = 0$, we can deduce that the aerodynamic efficiency at which the glider is flying is equal to:

   a. 30
   b. 15
   c. 60
   d. none of the other answers is correct
   e. 20

4. Consider a turbojet, performing a symmetrical, horizontal, rectilinear and stationary flight, which is flying with $T > T_B$. For that T, $\hat{V}_S = \hat{V}_2$. Additionally, we also know $C_{Lmax}/C_{Lopt} = 4$. Therefore, we can assume:

   a. $\hat{V}_1 = \hat{V}_2$
   b. $\hat{V}_1 = 0.5$
   c. $\hat{V}_1 = 2$
   d. $\hat{V}_1 = 4$
   e. $\hat{V}_1 = 16$

5. Consider a simple turbojet (x=0.7), performing a symmetrical, horizontal, rectilinear and stationary flight, and additionally fulfilling the following relationship: $\hat{T}_{11max} = 1$. Therefore:

   a. the theoretical ceiling is at the tropopause and reaches its maximum possible velocity at the tropopause.
   b. the theoretical ceiling is at the troposphere and reaches its maximum possible velocity at the troposphere.
   c. the theoretical ceiling is at the stratosphere and reaches its maximum possible velocity at the troposphere.
   d. the theoretical ceiling is at the tropopause and reaches its maximum possible velocity at the troposphere.
   e. none of the other answers is correct

6. ν is the angle between:

   a. T and the plane $x_h - y_h$
   b. T and the plane $x_b - z_b$
   c. T and the axis x_w
   d. T and the plane $x_e - z_e$
   e. none of the other answers is correct

7. Consider a glider performing a symmetric, rectilinear and stationary flight. Due to the small flight level, we can assume that $\rho = cte$. If the glider is flying with $V = V_B = cte$, then:

   a. the glider will reach maximum endurance
   b. the glider will reach maximum range
   c. the glider is flying with constant aerodynamics velocity (not with constant angle of attack), and therefore neither the maximum range nor the maximum endurance can be reached
   d. none of the other answers is correct
   e. the dimensionless velocity, $\hat{V}$, will change along the flight

8. An airplane flights with a zero angle of attack as well as non-symmetric conditions ($\beta \neq 0$) all along its trajectory. We can assume:

   a. $L_{wb,11} = 1$
   b. none of the other answers is correct
   c. $L_{wb,33} = 1$
   d. $L_{wb,22} = 1$
   e. $L_{wb,11} = 0$

9. The following eigenvalues are obtained for a longitudinal mode of a given aircraft: $\lambda_{i,j} = -0.002 \pm 0.05j$

   a. this mode is the short-period oscillation mode, with $\tau = 0.002$s and $\zeta = 0.4$
   b. this mode is the phugoid mode, with $\tau = 500$s and $\zeta = 0.04$
   c. this mode is the short-period oscillation mode, with $\tau = 500$s and $\zeta = 0.04$
   d. this mode is the phugoid mode, with $\tau = 0.002$s and $\zeta = 0.4$
   e. none of the other answers is correct

The $C_{M,CG}$ versus C_L curve for a general aviation airplane with the landing gear and flaps in their retracted position can be seen in Figure 1. Use Figure 1 and the following information to answer the following 4 questions (10 to 13)

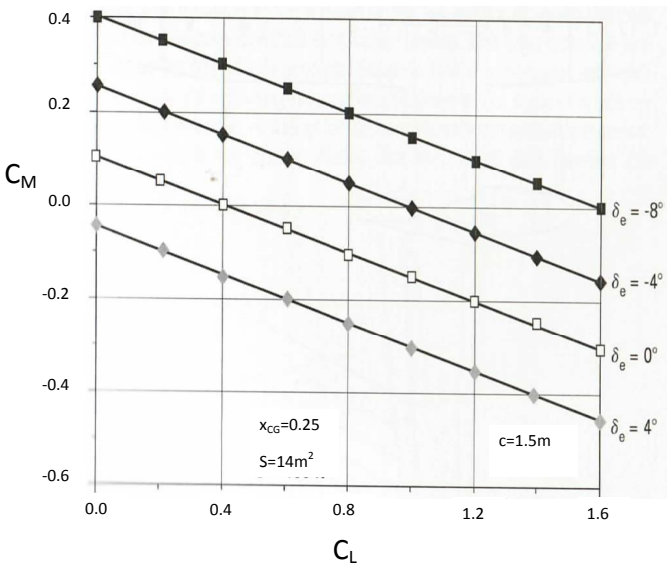


Figure 1: $C_{M,CG}$ versus C_L curve for $x_{CG} = 0.25$ (expressed as a fraction of the wing chord)

10. Where is the stick fixed neutral point located?
    a. $x_N = -0.5$
    b. $x_N = 0.5$
    c. $x_N = 0$
    d. $x_N = 0.25$
    e. $x_N = -0.25$

11. If the airplane weighs 1134kg and is flying at 45.7m/s at sea level ($\rho = 1.225$kg/m^3, $g = 9.81$m/s^{-2}), what is the C_L required for trim?
    a. $C_{L,trim} = 0.06$
    b. $C_{L,trim} = 0.08$
    c. $C_{L,trim} = 0.8$
    d. $C_{L,trim} = 0.6$
    e. $C_{L,trim} = 0.3$

12. If the airplane weighs 1134kg and is flying at 45.7m/s at sea level ($\rho = 1.225$kg/m^3, $g = 9.81$m/s^{-2})), what is the elevator angle required for trim?
    a. $\delta e, trim \approx -1.3\,\text{deg}$
    b. $\delta e, trim \approx -2.6\,\text{deg}$
    c. $\delta e, trim \approx 1.5\,\text{deg}$
    d. $\delta e, trim \approx 2.1\,\text{deg}$
    e. $\delta e, trim \approx 0\,\text{deg}$

13. Estimate the elevator control power $\frac{dC_{M,CG}}{d\delta_e}$.
    a. $\frac{dC_{M,CG}}{d\delta_e} \approx 0.395/\,\text{deg}$
    b. $\frac{dC_{M,CG}}{d\delta_e} \approx -0.355/\,\text{deg}$
    c. $\frac{dC_{M,CG}}{d\delta_e} \approx 0.0410/\,\text{deg}$
    d. $\frac{dC_{M,CG}}{d\delta_e} \approx -0.0375/\,\text{deg}$
    e. $\frac{dC_{M,CG}}{d\delta_e} \approx -0.250/\,\text{deg}$

14. The term $R_{1,33}$ (term 3-3 of the matrix R_1) is:
    a. $-sin(\delta_3)$
    b. $sin(\delta_3)$
    c. none of the other answers is correct
    d. $cos(\delta_3)$
    e. 0

15. Consider an airplane with the following non-parabolic drag polar: $C_D = C_{D0} + k{C_L}^4$, being $C_{D0} = 10^{-2}$ and $k = 3 \cdot 10^{-2}$. Which one is the value of C_{Lopt}:
    a. 3
    b. 1
    c. $1/\sqrt{3}$
    d. $\sqrt{3}$
    e. none of the other answers is correct

16. In order to change the value of α_e without an elevator, the best way is:
    a. to modify $\frac{dC_{M,CG}}{d\alpha_a}$ by changing the aircraft velocity
    b. to modify $\frac{dC_{M,CG}}{d\alpha_a}$ by shifting the position of the aircraft center of gravity
    c. to modify $C_{M,0}$ by shifting the position of the aircraft center of gravity
    d. to modify $C_{M,0}$ by changing the aircraft velocity
    e. none of the other answers is correct

17. Which one of the following statements is correct?
    a. inherent instability can never be tolerated for an aircraft with no flight control system
    b. a statically unstable system will be for sure dynamically unstable as well
    c. none of the other answers is correct
    d. a statically stable system will be for sure dynamically stable as well
    e. mild instability in closed loop systems can be tolerated

18. For a given aircraft, $C_{M,0} = 0.05$ and $\frac{dC_{M,CG}}{d\delta_e} = -0.01/deg$. During landing, the center of gravity of the airplane is located at 0.1 and $\frac{dC_{M,CG}}{dC_L} = -0.25$. Can the airplane be trimmed during landing (with $C_L = 1$)? Consider that $-22^o \leq \delta_e \leq 22^o$

    a. none of the other answers is correct
    b. it can be trimmed and $\delta_{e,trim} = -15^o$
    c. it can be trimmed and $\delta_{e,trim} = -20^o$
    d. it can be trimmed and $\delta_{e,trim} = 20^o$
    e. it can be trimmed and $\delta_{e,trim} = 15^o$

19. Consider a turbojet, performing a symmetrical, horizontal, rectilinear and stationary flight, which is flying with $\hat{D} = 3$. Therefore, the Thrust value is:

    a. T=3
    b. none of the other answers is correct
    c. $T = \sqrt{3}E_m/W$
    d. $T = 3W/E_m$
    e. $T = \sqrt{3}$

20. While decreasing it flight altitude, a glider is also increasing its:

    a. V_d
    b. V_S
    c. minimum γ_d
    d. none of the other answers is correct
    e. V_B

	A	B	C	D	E
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