AERODINÀMICA I MECÀNICA DE VOL

EXAM PARTIAL **A** **Q1 2016/17**

Formulae

1. INTRODUCTION TO FLIGHT MECHANICS

a. Transformation matrix

$$L_{ba} = \begin{bmatrix} c\delta_2 c\delta_3 & c\delta_2 s\delta_3 & -s\delta_2 \\ s\delta_1 s\delta_2 c\delta_3 - c\delta_1 s\delta_3 & s\delta_1 s\delta_2 s\delta_3 + c\delta_1 c\delta_3 & s\delta_1 c\delta_2 \\ c\delta_1 s\delta_2 c\delta_3 + s\delta_1 s\delta_3 & c\delta_1 s\delta_2 s\delta_3 - s\delta_1 c\delta_3 & c\delta_1 c\delta_2 \end{bmatrix}$$

b. General equations

$$T\cos\varepsilon\cos\nu - D - mg\sin\gamma - m\dot{V} = 0$$

$$T\cos\varepsilon\sin\nu - Q + mg\cos\gamma\sin\mu + mV(\dot{\gamma}\sin\mu - \dot{\chi}\cos\gamma\cos\mu) = 0$$

$$-T\sin\varepsilon - L + mg\cos\gamma\cos\mu + mV(\dot{\gamma}\cos\mu + \dot{\chi}\cos\gamma\sin\mu) = 0$$

$$\dot{x}_e = V\cos\gamma\cos\chi \quad / \quad \dot{y}_e = V\cos\gamma\sin\chi \quad / \quad \dot{z}_e = -V\sin\gamma$$

$$\dot{m} + \varphi = 0$$

2. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Total pitching moment about the center of gravity:

$$C_{M,cg} = C_{M,ac_{wb}} + C_{L_{wb}}\left(x_{cg} - x_{ac_{wb}}\right) - V_H C_{L,t}$$

$$C_{M,cg} = C_{M,ac_{wb}} + a_{wb}\alpha_{wb}\left(x_{cg} - x_{ac_{wb}} - V_H\frac{a_t}{a_{wb}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right) + V_H a_t\left(i_t + \varepsilon_0\right)$$

b. Longitudinal control

$$C_{L,t} = a_t\alpha_t + \frac{dC_{L,t}}{d\delta_e}\delta_e$$

Change in pitching moment acting on the plane

$$\Delta C_M = \frac{dC_{M,cg}}{d\delta_e}\delta_e = -V_H\frac{dC_{L,t}}{d\delta_e}\delta_e$$

Pitching moment equation:

$$C_{M,cg} = C_{M,0} + \frac{dC_{M,cg}}{dC_L}C_L + \frac{dC_{M,cg}}{d\delta_e}\delta_e \qquad \text{with } \frac{dC_{M,cg}}{dC_L} = \left(x_{cg} - x_n\right)$$

c. Directional stability

Contribution of Aircraft components to directional stability: Vertical tail

$$C_N = \frac{N_v}{q_w Sb} = \frac{l_v S_v}{bS}\frac{q_v}{q_w}\frac{dC_L}{d\alpha_v}(\beta + \sigma)$$

$$\frac{dC_{N,v}}{d\beta} = \frac{l_v S_v}{bS}\frac{q_v}{q_w}\frac{dC_L}{d\alpha_v}\left(1 + \frac{d\sigma}{d\beta}\right)$$

d. Directional control

$$C_N = \frac{N}{q_w Sb} = -\frac{q_v}{q_w}\frac{l_v S_v}{Sb}\frac{dC_{L_v}}{d\delta_r}\delta_r$$

$$\frac{dC_N}{d\delta_r} = -\frac{q_v}{q_w}\frac{l_v S_v}{Sb}\frac{dC_{L_v}}{d\delta_r}$$

f. Lateral control

$$\Delta C_l = \frac{\Delta l}{qSb} = \frac{q\,dy\,cC_L y}{qSb} = \frac{dy\,cC_L y}{Sb} \qquad C_l = \frac{2\,\frac{dC_{L,w}}{d\alpha_w}\frac{d\alpha}{d\delta_a}\delta_a}{Sb}\int_{y_1}^{y_2} cy\,dy$$

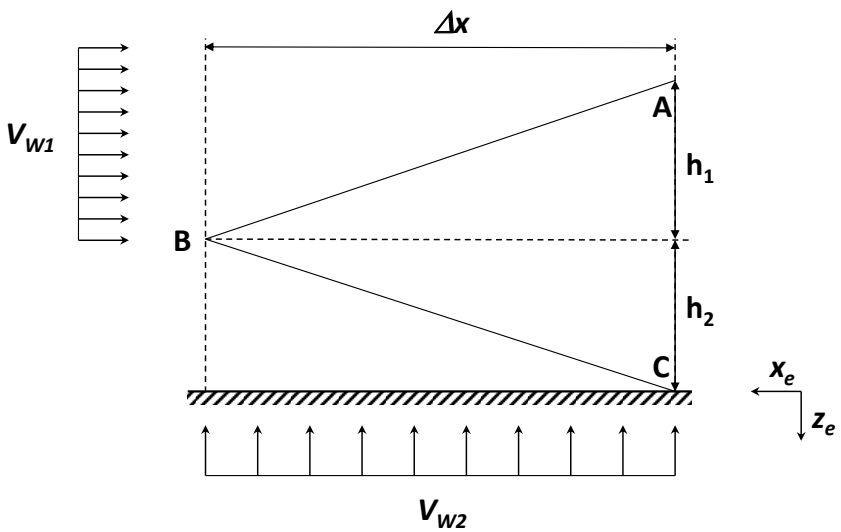
3. AIRCRAFT DYNAMICS

$$\tau = \frac{1}{\left|\mathrm{Re}(\lambda_i)\right|} \qquad \zeta = \left|\frac{\mathrm{Re}(\lambda_i)}{\lambda_i}\right| = \frac{|\sigma_i|}{\sqrt{\sigma_i^2 + \omega_i^2}} \qquad T = \frac{2\pi}{\omega_i} \qquad \omega_n = |\lambda_i|$$

1 Performances

All questions must be answered in the dedicated space below the question, no extra sheet of paper will be accepted. Numerical results with no units or wrong ones will get a 0 grade. Each correct answer adds 0.25 points.

A glider is gliding from a given altitude. The flight is symmetric, stationary, rectilinear, with levelled wings and it is performed in a vertical plane. The glider flights from point **A** to point **B** with a constant and horizontal headwind $\vec{V}_{w1}$. The value of this headwind is $\vec{V}_{w1} = -4 \cdot \vec{i_e}$m/s. During the first part of the flight (from **A** to **B**), the glider's altitude decreases by h_1. At point **B** the glider changes direction. Also, at point **B** $\vec{V}_{w1}$ becomes 0 and a new vertical, ascending wind appears. This second wind $\vec{V}_{w2}$ is equal to $-0.21 \cdot \vec{k_e}$m/s. From point **B** the glider flights towards point **C** (under the only effect of $\vec{V}_{w2}$). At point **C** the glider touches ground. During the second part of the flight (from **B** to **C**), the glider's altitude decreases by h_2. Finally, we know that all along the flight (from point **A** to **C**) the glider's angle of attack is constant and equal to $\alpha = 16°$.



Let's assume the next hypotheses:

- The glider's parabolic drag polar can be described with the following coefficients: $C_{D0} = 1.7 \cdot 10^{-2}$ and $k = 1.2 \cdot 10^{-2}$
- The next expression can be used for the lift coefficient: $C_L = 0.11 \cdot \alpha$ (where α is expressed in degrees). This expression can be used up to a maximum value of α equal to 19°.
- The glider total mass is 450kg.
- The wing area is $S = 18\text{m}^2$.
- The acceleration of gravity must be assumed as a constant, equal to $g = 9.81\text{m/s}^2$.
- We assume here, and through the entire exercise, that the atmospheric density is constant and equal to $\rho = 1.225\text{kg/m}^3$.
- The change in direction at point **B** can be considered as spontaneous (with no time consumption).
- The horizontal distance between point **A** and **B**, Δx, is constant and equal to 1950m.
- All angles γ_d and γ_g involved in this problem **must** be considered as small ones ($<< 1$). Remember: you must work in radians when working with the hypothesis of small angles.
- Remember: from point **A** to point **B** only $\vec{V}_{w1}$ exists ($\vec{V}_{w2} = 0$). From point **B** to point **C** there is only $\vec{V}_{w2}$ ($\vec{V}_{w1} = 0$).

You are asked to calculate:

1. The value of the aerodynamic velocity V at which the glider is flying from point **A** to point **B**:

2. The value of $\hat{D}$ at point **A**:

3. The value of the lift force, L, at point **A**:

4. The value of γ_d at which the glider is flying from point **A** to point **B**:

5. The value of γ_g at which the glider is flying from point **A** to point **B**:

6. The value of h_1:

7. The time required to decrease in h_1 the altitude:

8. The value of γ_g at which the glider is flying from point **B** to point **C**:

9. The value of h_2:

10. The time required to decrease in h_2 the altitude:

11. In the case of **no wind** ($\vec{V}_{w1}$=$\vec{V}_{w2}$=0), what would be the value $h_1 + h_2$?:

12. In the case of **no wind** ($\vec{V}_{w1}$=$\vec{V}_{w2}$=0) as in the previous question, what would be the total time required to go from point **A** to point **C**?:

2 A Boeing C17 Globemaster III study

This problem considers a Boeing C17 Globemaster III. All three parts of the problem are completely independent. **All questions must be answered in the dedicated space below the question, no extra sheet of paper will be accepted. Numerical results with no units or wrong ones will get a 0 grade.**
General data:

- Maximum Take-off weight $MTOW = 265350$kg, Cruise mass (longitudinal study) $m_{cruise} = 188000$kg
- Power-Plant: 4 Pratt and Whitney F117-PW-100 turbofans, of 180000N each
- wing surface: $S = 353$m^2, wingspan: $b = 50$m.
- Sea-level air density $\rho_{SL} = 1.225$kg/m^3, acceleration of gravity: $g = 9.81$m/s^2
- When needed, you can make the following assumption for small angles: $\cos(\alpha) = 1$ and $\sin(\alpha) = \alpha$ [rad]

2.1 Take-off (1 point)

In this part we consider that the Boeing C17 Globemaster III takes-off at sea-level from a runway that has a positive inclination (it goes up) of $\gamma = 2^o$, see Figure 1.

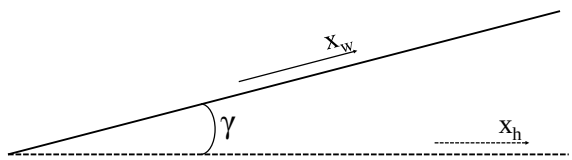


Figure 1: Runway for take-off with a positive angle.

1. Calculate the lift-off velocity, considering a maximum take-off weight $MTOW = 265350$kg, that $V_{LOF} = 1.1V_{stall}$ and that $C_{L,MAX,Take-Off} = 1.5$.

2. Considering that all wheels are on the ground and that the thrust is constant with velocity, draw the force diagram on Figure 1, calculate the expression of the force that drives the acceleration of the aircraft, that is the force F in $m\frac{dV}{dt} = F$, and prove that the acceleration can be written as:

$$\frac{dV}{dt} = A_0 + A_2V^2$$

Give the expressions of A_0 and A_2.

Write here the end of question 2 (if needed).

3. We recall that the distance D needed to reach (from zero velocity) the airspeed V can be written as:

$$D = \int_0^V \frac{V}{\frac{dV}{dt}} dV \text{ and we also recall that } \int \frac{f'(x)}{f(x)} dx = \ln(f(x))$$

Assuming that during ground roll C_L is constant and equal to $C_{L,Max,Take-Off} = 1.5$ and that $C_D = 0.13$, calculate the sea-level lift-off distance on a paved runway ($\mu = 0.02$).

2.2 Longitudinal study (1.25 point)

The $C_{M,CG}$ versus C_L curve for the Boeing C17 Globemaster III with the landing gear and flaps in their retracted position can be seen in Figure 2. Use Figure 2 and the following information to answer the following questions.

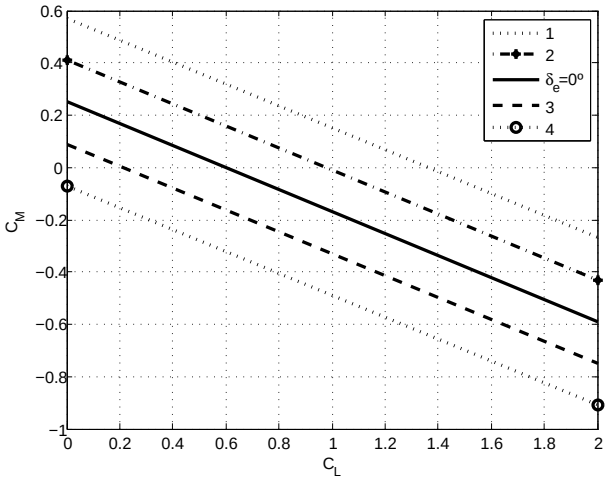


Figure 2: $C_{M,CG}$ versus C_L curve for $x_{CG} = 0.25$ (expressed as a fraction of the wing chord). The middle graph is for $\delta_e = 0^o$, the other graphs correspond to $\delta_e = 5^o$, $\delta_e = -5^o$, $\delta_e = 10^o$, $\delta_e = -10^o$ (order to determine).

1. Calculate the location of the stick fixed neutral point (x_n).

2. Complete Figure 2, by placing the values of δ_e on their corresponding lines, and calculate the approximate value of the elevator control power $\frac{dC_{M,CG}}{d\delta_e}$.

3. If the airplane is flying at 830km/h at 8500m ($\rho_{8500m} = 0.5$kg/m^3), with a cruise mass $m_{cruise} = 188000$kg, calculate the C_L required for trim.

4. If the airplane is flying at 830km/h at 8500m ($\rho_{8500m} = 0.5$kg/m^3), calculate the elevator angle required for trim.

5. We now consider that the airplane is loaded so that the center of gravity comes forward during landing. Assuming that $\frac{dC_{M,CG}}{d\delta_e}$, and C_{M0} are unaffected by the center of gravity travel, that $\delta_{e,max} = \pm 20^o$ and that $C_{L_{landing}} = 1.5$, calculate the most forward position of the center of gravity so that the airplane can be trimmed during landing.

2.3 Assymetric thrust (0.75 point)

The exterior right engine of the Boeing C17 Globemaster III is now inoperative during cruise, see Figure 3. Consider that each of the 3 remaining turbofans provides a thrust of 180000N. The distances between the engines and the aircraft longitudinal axis are given on Figure 3. The rudder is located at a distance $l_v = 28$m behind the center of gravity. The rudder maximum deflection is $\delta_r = \pm 30^o$, $\frac{dC_{Lv}}{d\delta_r} = 2.32/\text{rad}$, $\frac{q_v}{q_w} = 0.95$.

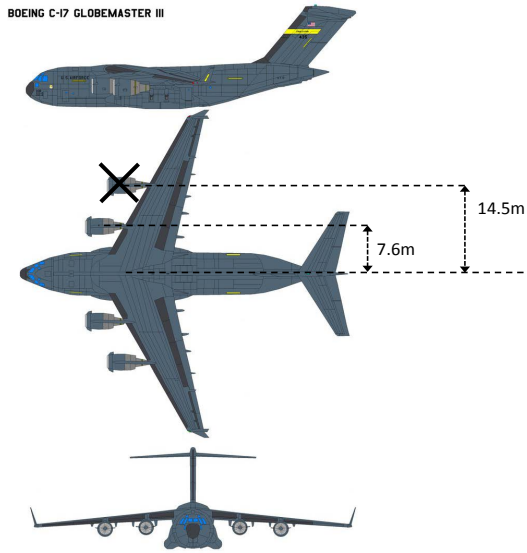


Figure 3: 3D sketch of a Boeing C17 Globemaster III

Calculate the minimum value of the rudder surface S_v needed to maintain directional trim in an asymmetric thrust flight condition at sea level. Consider that the aircraft minimum controllable speed is 80% of stall speed (use $C_{L,Max} = 1.5$, and $m_{cruise} = 188000$kg).

3 Test

Note that there is only one correct option for each question. Each correct answer adds 0.25 points. Each wrong answer subtracts 0.25/4 points

1. Consider a glider with a weight of 3500N and a maximum aerodynamic efficiency of $E_m = 35$. What is the value of the minimum drag force of this glider when it is performing a symmetric, rectilinear and stationary flight? (In this question you cannot assume the hypothesis of small angles):

   a. 3500 N
   b. 350 N
   c. 0 N
   d. 99.95 N
   e. 3498.57 N

2. An airplane is performing a regular looping manoeuvre (the aeroplane's centre of gravity describes a circumference in a vertical plane with a constant velocity in symmetrical conditions). We know that $V =$ 250km/h, $R = 1200$m, $\rho = 1.21$Kg/m^3, $W = 8700$N, $S = 16$m^2 and $\varepsilon = 0$. This flight has: (d.f.: degrees of freedom; p.c.: parameters of control)

   a. 0 d.f. and 2 p.c.
   b. 0 d.f. and 1 p.c.
   c. 1 d.f. and 2 p.c.
   d. 1 d.f. and 0 p.c.
   e. 1 d.f. and 1 p.c.

3. Consider a turbojet with a parabolic drag polar performing a symmetrical, horizontal, rectilinear and stationary flight. Regarding this turbojet we know that W=40000 N, E_m=20, $C_{D0} = 0.1$. Additionally, we know that this turbojet has its theoretical ceiling at the stratosphere. We can deduce that:

   a. $T_{11,max} < 1000N$
   b. $T_{11,max} = 1000N$
   c. none of the other answers is correct
   d. $T_{11,max} > 2000N$
   e. $T_{11,max} = 2000N$

4. An airplane flights with $\beta = \alpha = 45°$. Therefore: (note: $L_{bw,ij}$, where i defines the row and j the column)

   a. $L_{bw,12} = 0$
   b. none of the other answers is correct
   c. $L_{bw,21} = 1$
   d. $L_{bw,32} = -0.5$
   e. $L_{bw,33} = 0.5$

5. In a conventional wing-tail combination (wing is forward the horizontal stabilizer):

   a. none of the other answers is correct
   b. $C_{M,0}$ is increased by adding negative lift in the horizontal stabilizer
   c. $C_{M,0}$ is decreased by adding positive lift in the horizontal stabilizer
   d. $C_{M,0}$ is decreased by adding negative lift in the horizontal stabilizer
   e. $C_{M,0}$ is increased by adding positive lift in the horizontal stabilizer

6. A glider is gliding in symmetrical, rectilinear, stationary conditions. Given to the low temperatures of that day, some ice has deposited over the glider's wing, increasing its value of C_{D0}. As a consequence:

   a. x_{max} (maximum range) does not change
   b. none of the other answers is correct
   c. t_{max} (maximum endurance) does not change
   d. t_{max} (maximum endurance) decreases
   e. t_{max} (maximum endurance) increases

7. A glider is gliding in symmetrical, rectilinear, stationary conditions. We can assume the air density to be constant. If the glider wants to reach the maximum range, it is supposed to glide with:

   a. $V \neq$constant and $\alpha =$constant
   b. $V =$constant and $\alpha \neq$constant
   c. $V \neq$constant and $\alpha \neq$constant
   d. $V =$constant and $\alpha =$constant
   e. none of the other answers is correct

8. An aircraft is landing at Barcelona-El Prat airport:

   a. if it is a very hot day, the landing ground distance will be longer because the density of the air is smaller
   b. if an aircraft carries 20% more weight it will need 40% more landing ground distance
   c. the pilot can shift the position of the center of gravity (by moving the fuel in different tanks) in order to reduce the landing ground distance
   d. after touchdown, the use of the flaps will increase the lift and thus decrease the landing ground distance
   e. none of the other answers is correct

9. Consider a turbojet with a parabolic drag polar performing a symmetrical, horizontal, rectilinear and stationary flight. For this turbojet we know that $T_{11,max} =$ 2000N, W=8000N and E_m=11. What is the minimum $\hat{V}$ that this turbojet can reach at the tropopause?:

   a. 0.18
   b. 0.43
   c. 1
   d. 0.83
   e. 2.75

10. An airplane is performing a regular looping manoeuvre ($V = cte$, $R = cte$, $\rho = cte$, $W = cte$, $\varepsilon = 0$) in a vertical plane in symmetrical conditions. If the value of $V^2/(g \cdot R)$ is equal to 2, we can assure that the value of the load factor at the highest point of the looping is equal to:

    a. 3
    b. 2
    c. 1
    d. 0
    e. none of the other answers is correct

11. The following eigenvalues are obtained for a lateral mode of a given aircraft: $\lambda_{i,j} = -0.25 \pm 0.5j$

    a. this mode is the phugoid mode
    b. this mode is the short-period oscillation mode
    c. this mode is the Dutch roll mode
    d. none of the other answers is correct
    e. this mode is the roll mode

12. Consider a single-engine turbojet with a parabolic drag polar performing a symmetrical, horizontal, rectilinear and stationary flight. This engine has a x=0.6 for the troposphere and x=1 for the stratosphere (remember the condition of maximum: $\hat{T}_{max} = 1/\sqrt{1-x^2}$). We know that $\hat{T}_{11,max}$=1.3 for this engine. At which altitude would this turbojet reach its $V_{max,max}$?: (we assume here than the tropopause is placed exactly at 11000m)

    a. $h_{V_{max,max}} = 11000\text{m}$
    b. $h_{V_{max,max}} > 11000\text{m}$
    c. $h_{V_{max,max}} \leq 11000\text{m}$
    d. $h_{V_{max,max}} < 11000\text{m}$
    e. $h_{V_{max,max}} \geq 11000\text{m}$

13. A glider is gliding in symmetrical, rectilinear, stationary conditions with an aerodynamic velocity V and the corresponding γ_d (you can assume $\gamma_d \ll 1$). The atmosphere is not in quiescence conditions, in fact, there is a given and constant wind: $\vec{V}_w = -2 \cdot V \cdot \gamma_d \vec{k}_e$. As a consequence:

    a. $\gamma_g = 2\gamma_d$
    b. $\gamma_g = \gamma_d$
    c. none of the other answers is correct
    d. $2\gamma_g = \gamma_d$
    e. $\gamma_g = -\gamma_d$

14. A single-engine airplane with the engine fixed to the structure is flying in symmetric conditions. We also know that at a given instant of time $\varepsilon = 8°$ and $\alpha = 3°$. What is the value of the angle between the engine's Thrust and the axis X_b at that given instant of time?:

    a. $8°$
    b. $5°$
    c. none of the other answers is correct
    d. $11°$
    e. $-3°$

15. To improve the lateral stability of an aircraft

    a. both wing forward sweep and low-wing placement would help
    b. both positive wing dihedral angle and low-wing placement would help
    c. both wing backward sweep and high-wing placement would help
    d. both wing forward sweep and positive wing dihedral angle would help
    e. none of the other answers is correct

16. A glider is gliding in symmetrical, rectilinear, stationary conditions. We cannot assume the air density to be constant. If the glider wants to reach the maximum range, it is supposed to glide with:

    a. α =constant and $C_L = C_{Lopt}$
    b. V =constant and $C_L = C_{Lopt}$
    c. α =constant and $C_L = \sqrt{3}C_{Lopt}$
    d. V =constant and $C_L = \sqrt{3}C_{Lopt}$
    e. none of the other answers is correct

	A	B	C	D	E
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