Problem 1. (4 points)

Given an electric power system with high wind penetration, there is just one thermal unit permanently committed with the following characteristics ($P_T$ measured in MW):

$$F(P_T) = 500 + 30 P_T + 0.02 P_T^2 \ \text{€/h}; \ \ \ \ \ 150 \leq P_T \leq 650$$

Let $D$ (MW) and $P_{wind}$ (MW) be the demand and the available wind generation forecasted for a given hour of the day. In case $P_{wind}$ is very large, it might be necessary to curtail some wind power (this is, to produce less wind generation than the available one) as the thermal unit cannot be shut down in any case as it provides inertia to the system and it is able to react to the unforeseen decreases of wind generation. Let $V_{wind}$ (MW) be the wind curtailment for a given hour of the day. The actual wind power generation would be $P_{wind} - V_{wind}$. Obviously, negative curtailments or curtailments higher than $P_{wind}$ make no sense.

In order to deal with the wind generation uncertainty, the System Operator establishes that the thermal unit has to provide a value of upward reserve higher than 30% of the actual wind power generation:

$$P_T \leq \bar{P}_T - 0.3(P_{wind} - V_{wind})$$

0.8 pt a) Solve the economic dispatch problem for a demand of 1000MW with 600MW of forecasted available wind generation. To do so, formulate the complete optimization problem, identify a reasonable optimality hypothesis, solve it and check the optimality of the obtained solution.

It is necessary to express the system marginal cost as a function of $P_{wind}$ for that hour ($D=1000$MW), for values of $P_{wind}$ ranging from 350 to 1000 MW. In order to achieve this:

0.4 pt b) Using the optimality conditions that correspond to the economic dispatch solution obtained in the previous question, express the value for all primal variables and Lagrange multipliers in terms of $P_{wind}$.

0.3 pt c) Identify the range of values of $P_{wind}$ where such expressions are valid. Explain what happens at both limits of the mentioned range (lower and upper limit)

1 pt d) Discuss whether or not the problem has a solution for values of $P_{wind}$ higher than the upper limit identified previously (it is considered unacceptable to incur in non-served energy, as in that case a second thermal unit would be started-up). In case there is a solution, formulate the optimality hypothesis for such values of $P_{wind}$ and obtain the new expressions of all the variables (primal ones, and Lagrange multipliers) in terms of $P_{wind}$. Identify the range of values of $P_{wind}$ for which those expressions are valid. Repeat the process until a value of 1000 MW for $P_{wind}$ is reached.

1 pt e) Discuss whether or not the problem has a solution for values of $P_{wind}$ lower than the lower limit identified previously (it is considered unacceptable to incur in non-served energy, as in that case a second thermal unit would be started-up). In case there is a solution, formulate the optimality hypothesis for such values of $P_{wind}$ and obtain the new expressions of all the variables (primal ones, and Lagrange multipliers) in terms of $P_{wind}$. Identify the range of values of $P_{wind}$ for which those expressions are valid. Repeat the process until a value of 350 MW for $P_{wind}$ is reached.

0.5 pt f) Express the system marginal cost in terms of the available $P_{wind}$ forecasted for that hour ($D=1000$MW), for a range of values of $P_{wind}$ between 350 and 1000 MW.
Problem 2 (4 points)

The electrical system of the figure consists of three identical lines with an admittance $y = 4$ p.u. where only line 1-3 has a maximum capacity of 55 MW in both directions. There are 2 thermal units permanently committed with the following cost functions and power limits:

$$F(P_1) = 300 + 30 \cdot P_1 + 0.01 \cdot P_1^2 \quad [\text{€/h}] \quad 100 \leq P_1 \leq 400 \quad [\text{MW}]$$

$$F(P_2) = 150 + 40 \cdot P_2 + 0.02 \cdot P_2^2 \quad [\text{€/h}] \quad 100 \leq P_2 \leq 400 \quad [\text{MW}]$$

The demand at each node is $D_1$, $D_2$ and $D_3$ respectively. The System Operator has installed a BESS (Battery Energy Storage System) at node 3. This battery allows to storage energy by consuming power from the grid (as if is was another additional load), and also to produce energy injecting electric power into the grid (as any other kind of generator). The round trip efficiency of the battery is 90%.

The mathematical model of the battery can be done by defining a variable that represents the useful energy stored at the end of every hour $E_h$, the power consumed from the grid during each hour $CON_h$ and the power generated each hour $GEN_h$. Variables $CON_h$ and $GEN_h$ cannot take negative values. Thus, the energy balance at the battery can be expressed as:

$$E_h = E_{(h-1)} - GEN_h + 0.9 \cdot CON_h.$$

The time horizon is 2 consecutive hourly periods (A and B), where the demand at each node is shown in the following table.

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hour A</td>
<td>300</td>
<td>150</td>
<td>100</td>
</tr>
<tr>
<td>Hour B</td>
<td>250</td>
<td>100</td>
<td>50</td>
</tr>
</tbody>
</table>

The initial storage level of the battery is $E_0 = 50$ MWh that has to be kept at the end of hour B. Maximum and minimum energy storage limits can be ignored. However, the limits of the power consumed/generated by the battery (25 in both cases) have to be taken into account.

Please answer the following questions providing an adequate justification:

a) Let assume that the system marginal cost at node 3 in hour A is $SMC_{3A} = 60$ Eur/MWh, and that in hour B is $SMC_{3B} = 35$ Eur/MWh. In case the battery owner sells/purchases the generation/consumption of the battery at the system marginal cost of node 3, and neglecting in this question the impact of the battery operation on that marginal cost, what would be the optimal operation of the battery in their hours A and B in order to maximize its market profit? You can answer this question by common sense, without being necessary to formulate an optimization problem. What is the value of such market profit?
b) Taking node 1 as the slack bus, build matrix $\tilde{Q}$ corresponding to the compact formulation. Consider the power flows vector as $[f_{12} f_{13} f_{23}]^T$.

Assuming there is NO battery installed:

c) Obtain the economic dispatch solution for both hours taking into account the following information: in the optimum, line 1-3 in hour A is congested in its positive direction, while in hour B, the system marginal cost is the same for the 3 nodes.

d) Compute the system marginal cost at node 3 for both hours.

Assuming the battery IS installed at node 3:

e) Solve the transmission constrained economic dispatch for both hours taking into account the operation of the battery. Formulate the complete optimization problem, write down the optimality conditions of the problem, identify a reasonable optimality hypothesis, solve the corresponding equations, and check clearly the validity of the obtained solution until you find the optimum.

Note: in order to elaborate the optimality hypothesis, take into account the operation of the battery obtained in question a)

f) What would be the economic impact for the system if the maximum power generated by the battery is increased 1 MW? And if such increase is for the maximum power consumed when charging?

g) Assuming that the generators are remunerated with the corresponding nodal system marginal cost, what is the operational profit of both generators (consider only variable costs)?