

**Degree in Biomedical Engineering**  
**MIDTERM EXAM OF ALGEBRA - Chapter 4**

**INSTRUCTIONS:**

1. Write your name and surnames **now** using **capitals**.
2. Every paper on your table, **written or not**, must have your name and surname on it.
3. Use a pen. Anything written in pencil will not be marked.
4. You will be asked to hand in your exam **at once** if it comes into other student's sight.
5. The use of notes, books, calculators or mobile phones is **not** allowed at any point.
6. You must hand in **all** papers: this one, your problem answers and your draft calculations.
7. **THEORY:**
  - a) Read the questions **carefully**. Don't make any assumptions. Choose the most appropriate answer.
  - b) Write your answers in the boxes below. Only these will be marked. Don't give reasons to justify your answer.
  - c) MARKS: Correct answer = 0.6, Incorrect answer = -0.2, Empty = 0. (If negative, theory mark = 0)  
If you are not sure about the answer, you might well leave the question unanswered.

**THEORY (3 marks)**

1.- Given  $A (m \times n)$ ,  $B (p \times n)$  and  $C (m \times p)$ , how many of these expressions make sense?

$AB^T C$

$C^* AB^T$

$BA^* C$

$A^T C B$

- A) Just one  
B) Only two
- C) Three of them  
D) All four

2.- If the linear system  $Ax = b$  has a unique solution for all  $b$ , then

- A)  $A$  is square  
B)  $A$  is invertible
- C) A) and B) are true  
D) None of the above

3.- If  $A$  and  $B$  are  $(n \times n)$  invertible matrices, which of the following expressions is incorrect?

- A)  $(AB)^T = B^T A^T$   
B)  $(AB)^{-1} = B^{-1} A^{-1}$
- C)  $(AB)^* = B^* A^*$   
D) None of the above

4.- If  $A$  is an  $(n \times n)$  non-invertible matrix, then:

- A)  $\text{Col } A = \mathbb{K}^n$   
B)  $\dim \text{Nul } A = n$
- C) A) and B) are true  
D) None of the above

5.- If  $A$  and  $B$  are  $(n \times n)$  matrices, then,

- A)  $|AB| = |A| |B|$   
B)  $|A + B| = |A| + |B|$
- C) A) and B) are true  
D) None of the above

**THEORY ANSWERS**

1	2	3	4	5
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<b>FIRST NAME:</b>	<b>GROUP:</b>
<b>SURNAMES:</b>	

## PROBLEM (7 marks)

We want to determine under what conditions this block matrix is invertible:

$$M = \begin{bmatrix} \mathbb{I} & A \\ A & \mathbb{I} \end{bmatrix}$$

- a) Find a matrix  $S$  so that  $M$  can be factorized this way: (1 mark)

$$M = \begin{bmatrix} \mathbb{I} & 0 \\ A & \mathbb{I} \end{bmatrix} \begin{bmatrix} \mathbb{I} & A \\ 0 & S \end{bmatrix}$$

- b) Find the inverse of the matrix (1 mark)

$$\begin{bmatrix} \mathbb{I} & 0 \\ A & \mathbb{I} \end{bmatrix}$$

- c) Determine when this matrix is invertible (1 mark) and, when possible, find its inverse (1 mark)

$$\begin{bmatrix} \mathbb{I} & A \\ 0 & S \end{bmatrix}$$

- d) Find the determinant of the matrix  $M$  in terms of the matrix  $A$  (1 mark)

- e) Using the factorization of section a), determine the inverse of the matrix  $M$  (2 marks)