EXAM 2012-13Q2

Part 1. Consider an obstacle in a 2D wind tunnel with parallel floor and ceiling as shown in Fig. 1. The tests show that far upwind (in the test chamber inlet) the pressure and velocity profiles are uniform and known, and that far downwind (in the test chamber outlet) the pressure profile is also uniform (and equal to that far upwind). We can assume that the problem is stationary, \( Re >> 1 \) and \( M << 1 \), and that we also know the density \( \rho \) far upwind. In this case:

1. Deduce an expression for the lift of the obstacle, \( l \), as a function of known variables as shown in Fig. 1.

\[
\begin{align*}
U(z) & = \frac{U_\infty}{2} \left( 1 - 9 \left( \frac{z}{a} \right)^2 \right) \quad \text{for } |z| \leq a \\
p_c(x) & = p_\infty \left[ 1 - \delta \left( 1 + \cos \frac{\pi x}{l} \right) \right] \quad \text{for } |x| < l \\
p_f(x) & = p_\infty \left[ 1 + \delta \left( 1 + \cos \frac{\pi x}{l} \right) \right] \quad \text{for } |x| < l \\
p_c(x) & = p_f(x) = p_\infty \quad \text{for } |x| \geq l
\end{align*}
\]

2. Compute the lift of the obstacle, \( l \), given these experimental data.