LESSON 2: INVISCID FLOW
Part 1: 2D Potential Flow
(Infinite Wings)

José I. Rojas
josep.ignasi.rojas@upc.edu
Building C3, Office 120

CONTENTS

LESSON 7: 2D POTENTIAL FLOW

– Introduction
– Aerodynamic forces on an airfoil in stationary 2D potential flow:
  • D'Alembert's paradox
  • Kutta-Yukovski theorem
– Viscosity effects
– Sharp trailing edge – Streamlining
– Hypothesis of Kutta
– Generation of circulation
– Pressure, lift, drag & pitching moment coef.
2D POTENTIAL FLOW

INTRODUCTION

- **Objective** → to calculate the pressure distribution (forces \( I \) & \( d \)) on 2D airfoils moving horizontally in steady atmosphere

- **Hypotheses:**
  - long or infinite wing → 2D flow
  - \( Re \gg 1 \) & attached Boundary Layer (BL) → viscosity effects neglected
  - \( Fr \gg 1 \) → volume forces neglected
  - Bjerknes-Kelvin Theorem
  - properties far upwind → uniform & stationary
  - \( St \ll 1 \) → stationary problem
  - low speed: \( M < 0.30 \) → compressibility effects neglected: \( \rho = ct \)

2D POTENTIAL FLOW

PROCEDURE TO FULFIL OBJECTIVE

To compute the aerodynamic forces (\( \rho \) distrib.) on the airfoil:

1. compute velocity potential from differential Eq. for velocity potential
2. then compute velocity field from velocity potential
3. then compute pressure field by application of Bernoulli Eq.:

\[
\frac{1}{2} \rho_{\infty} |\nabla \Phi|^2 + p = \frac{1}{2} \rho_{\infty} U_{\infty}^2 + p_{\infty}
\]

2D POTENTIAL FLOW

POTENTIAL FLOW

Uniform approach flow (irrotational)

(a) Curved shock wave introduces rotationality

(b) Viscous regions where Bernoulli's equation fails

Wake flow

Slight separated flow

Laminar boundary layer

Turbulent boundary layer

Wake flow


PRESSURE DISTRIBUTIONS AROUND AIRFOILS (1)

Airfoils with same shape but different AOA

Length of arrows $= C_p$

S denotes $C_p$ at stagnation where $C_p = 1$

Direction of arrows indicates positive or negative $C_p$

(Anderson, J.D., Fundamentals of Aerodynamics, 2001)
2D POTENTIAL FLOW

PRESSURE DISTRIBUTIONS AROUND AIRFOILS (2)

Airfoils with same AOA but different shape

(Arianezhad, M., Numerical study and optimization of a GT car rear-wing aerodynamics, 2015)

2D POTENTIAL FLOW

FORCES ON AIRFOIL IN 2D POTENTIAL FLOW

- relate circulation $\Gamma'$ with Newton's 2nd Principle to calculate lift
- control vol. containing airfoil & with boundaries not too far from it
- Aerodynamic forces on an airfoil in stationary 2D potential flow:
  - **x-axis**: D'Alembert's paradox $\rightarrow$ aerodynamic drag on 2D obstacle in a stationary 2D potential flow is NULL $\rightarrow$ $d = 0$
  - **z-axis**: Kutta-Yukovski theorem $\rightarrow$ $l = \rho U_{\infty} \Gamma'$
- Conclusions:
  - **potential flow**: aerodyn. force perpendicular to incident flow (only lift)
  - airfoils generate lift only if circulation is NOT null
2D POTENTIAL FLOW

VISCOSITY EFFECTS

– Contradiction?
  • topic 1 → Bjerknes-Kelvin + flying in steady atmosphere → potential flow → circulation NULL!
  • previous slide → airfoils generate lift only if circulation is NOT NULL!

– Explanation → viscosity effects:
  • if BL thin & not detached, p distribution on airfoil due to inviscid potential flow = p distrib. due to real flow (in BL models, transversal p gradient is NULL)
  • regardless viscosity NEGLECTED, potential flow model allows computing lift

– Yet viscosity plays key role in:
  • flow motion around airfoil
  • generation of circulation on airfoil

2D POTENTIAL FLOW

SHARP TRAILING EDGE (1) – STREAMLINING

potential flow (volunteer for drawing p distribution)  real flow (non-streamlined/blunt/bluff body)

extremely adverse pressure gradient


EETAC – AER – José I. Rojas
**2D POTENTIAL FLOW**

**SHARP TRAILING EDGE (1) – STREAMLINING**

- **potential flow**
  - Beautifully behaved but mythically thin boundary layer and wake
  - $Re_x = 10^3$

- **real flow**
  - (non-streamlined/blunt/bluff body)
  - Outer streams grossly perturbed by broad flow separation and wake
  - $Re_x = 10^5$

---

**2D POTENTIAL FLOW**

**SHARP TRAILING EDGE (2) – STREAMLINING**

Consequences of detachment of BL in blunt/bluff bodies:

- **NO** Trailing Edge (TE) stagnation point $\rightarrow$ D'Alembert's paradox does NOT hold $\rightarrow$ wake (or pressure or form) drag aside from friction drag
- **opposite-sign vortices** detach from upper & lower surfaces $\rightarrow$ yet global vorticity is **NULL**

---


2D POTENTIAL FLOW

SHARP TRAILING EDGE (3) – STREAMLINING

Flow behavior around streamlined bodies: sharp trailing edge (TE):

- POTENTIAL flow → rear stagnation point in upper surf. → NOT realistic
- REAL flow → larger vorticity detached from lower surface
  → circulation appears on the airfoil:
  - opposite sign respect to detached net vorticity
  - approx. equal in absolute value
  - pushes rear stagnation point towards trailing edge

(Anderson, J.D., Fundamentals of Aerodynamics, 2001)

Figure 13. Stagnation point for various angles of attack.
2D POTENTIAL FLOW

HYPOTHESIS OF KUTTA (1)

- Hypothesis of Kutta: “Circulation around airfoil is such that the rear stagnation point is located in the trailing edge (or disappears)”

- Upper & lower flow meet in TE with same pressure → MUST have same velocity (Bernoulli Eq.)

- 2 possibilities:
  - angular TE: 2 tangents → velocities can only be equal if NULL → stagnation point in TE
  - tangential TE: unique tangent → velocities do NOT need to be null → NO stagnation point in TE

**Question:** Is it correct to state that the Kutta hypothesis can only be fulfilled if the velocity of the upper and lower flow in the TE is null?

---

2D POTENTIAL FLOW

HYPOTHESIS OF KUTTA (2)

2 possibilities:

- angular TE: 2 tangents → velocities can only be equal if NULL → stagnation point in TE
- tangential TE: unique tangent → velocities do NOT need to be null → NO stagnation point in TE

![Diagram](attachment:image.png)

At point $a$: $V_1 = V_2 = 0$

At point $a$: $V_1 = V_2 \neq 0$

*Anderson, J.D., Fundamentals of Aerodynamics, 2002*

(Hazen, D.C., Preliminary report on circulation control and the cusp effect, 1953)
2D POTENTIAL FLOW

GENERATION OF CIRCULATION

Show video of starting vortex

Question: What are the differences in behavior of air flow around a cylinder and around an airfoil with very low AOA?

(a) Fluid at rest relative to the airfoil

(Anderson, J.D., Fundamentals of Aerodynamics, 2001)

(b) Picture some moments after the start of the flow

SHARP TRAILING EDGE (4) – STREAMLINING

(Anderson, J.D., Fundamentals of Aerodynamics, 2001)
SHARP TRAILING EDGE (5) – STREAMLINING


2D POTENTIAL FLOW

SUMMARY

- Potential flow model allows computing *p* distribution if BL attached → allows computing *lift* limitation: *drag* is 0! not realistic!
- even if BL attached → *drag* not null (viscosity in BL causes friction)
- streamlining very efficient → *lift* >> *drag* (e.g. *lift/drag* = 30)

(Bienkiewicz, B., A flow visualization technique for low-speed wind-tunnel studies, 1987)
2D POTENTIAL FLOW

PRESSURE COEF. ON AIRFOIL

With \( p \) distrib. on airfoil's surface \( \rightarrow \) distrib. of \( p \) coef. on airfoil:

\[
c_p(x, z_p(x)) = \frac{p(x, z_p(x)) - p_\infty}{\frac{1}{2} \rho \omega U_\infty^2}
\]

Fig. 11. Contour lines of pressure coefficient on the baseline geometry, \( \alpha = 3^\circ \), \( M = 0.85 \).

2D POTENTIAL FLOW

PRESSURE COEF. ALONG CHORD (1)

- instead of $p$ coef. on airfoil, work with $p$ coef. along chord: $c_p(x)$
- $c_p(x) \rightarrow$ projection along $z$ axis of force per unit length on each point of airfoil
- it can be demonstrated that $c_p(x) = c_p(x, z_p(x)) = \frac{p(x, z_p(x)) - p_{in}}{\frac{1}{2} \rho_0 U_0^2}$
- Graphically, we usually represent $-c_p(x)$
- When the flow velocity increases, so does $-c_p(x)$

Graphically, we usually represent $-c_p(x)$

(Anderson, J.D., Fundamentals of Aerodynamics, 2001)
2D POTENTIAL FLOW

PRESSURE DISTRIBUTIONS AROUND AIRFOILS (1)

Airfoils with same shape but different AOA

(Anderson, J.D., Fundamentals of Aerodynamics, 2001)

Question:
How much is the global airfoil’s lift coefficient?
### 2D POTENTIAL FLOW

**LIFT COEF. ALONG CHORD & GLOBAL LIFT COEF. (1)**

Distribution of lift coef. along chord:

$$c_l(x) = c_{p, lower}(x) - c_{p, upper}(x)$$

Global airfoil’s lift coef.:

- for $|\alpha| << 1 \rightarrow c = x_{TE} - x_{LE}$
- for $|\alpha| << 1 \rightarrow$ lift coef. linear with $\alpha$: $c_l \propto \alpha$ (slope is constant)
- theory fails if $|\alpha|$ large:
  - as $\alpha$ grows/decreases $\rightarrow$ BL eventually detaches, slope starts to decrease & reaches $0 \rightarrow$ max./min. lift coef.
  - if $\alpha$ continues to grow/decrease $\rightarrow$ slope becomes increasingly negative
    - we say that the airfoil is in STALL
- typically, $c_{l, max} \approx 1.5$ for airfoils with no hyper-lift devices

---

**Figure 2.7:** Performance comparison in different Reynolds numbers computed by XFOIL


2D POTENTIAL FLOW

LIFT COEF. ALONG CHORD & GLOBAL LIFT COEF. (2)

Detachment of BL:

(Anderson, J.D., Fundamentals of Aerodynamics, 2001)


2D POTENTIAL FLOW

LIFT COEF. ALONG CHORD & GLOBAL LIFT COEF. (3)

Detachment of BL:

(Rojas, J.I., 2006)

2D POTENTIAL FLOW

LIFT COEF. ALONG CHORD & GLOBAL LIFT COEF. (4)

Global airfoil’s lift coef.:

\[ c_l = \frac{1}{c} \int_{x_{LE}}^{x_{TE}} c_l(x) dx \]

- for \(|\alpha| \ll 1 \rightarrow \text{lift coef. linear with } \alpha\):
  \[ c_l \propto \alpha \]
  - E.g.: Yukovsky transformation
  \[ c_l = 2\pi \frac{\delta + (1 + y)\alpha}{1 + \frac{y}{1 + 2y}} \]
  - E.g.: thin airfoil
  \[ c_l = 2\pi (\delta + (1 + y)\alpha) \]
  - E.g.: flat plate
  \[ c_l = 2\pi \alpha \]

**Question:** Compute the global lift coefficient for a flat plate with angle of attack (AOA) 10º. Compute the global lift coefficient for a symmetric thin airfoil.

2D POTENTIAL FLOW

FORCES ON AIRFOIL

In 2D, we compute forces per unit span [N/m]:

- lift \( \rightarrow \)
  \[ l = \frac{1}{2} \rho U_{\infty}^2 c c_l \]

- drag \( \rightarrow \)
  \[ d = \frac{1}{2} \rho U_{\infty}^2 c c_d \]

**Question:** Compute lift for previous plate at SL in ISA at 60 km/h, if chord is 30 cm.

Global airfoil’s aerodynamic drag coef.: \( c_d \)

- typically, \( c_{d,\text{min}} \approx 0.004 \) for a laminar airfoil
2D POTENTIAL FLOW

AERODYNAMIC DRAG COEF. – POLAR CURVE

- Polar curve: relationship between lift & drag coef. \( c_d = f(c_l) \)
- E.g.: laminar airfoil \( \rightarrow \) laminar BL in a small range of \( \alpha \)/lift coef.
- Catalogues listing airfoil data:
  e.g.: UIUC:
  [http://www.ae.uiuc.edu/m-selig/ads.html](http://www.ae.uiuc.edu/m-selig/ads.html)


![Graph](image1.png)

Fig. 8. Lift-drag polar for baseline geometry \( M = 0.85 \).

[Anderson, J.D., Introduction to Flight, 2002]

![Graph](image2.png)

2D POTENTIAL FLOW

PITCHING MOMENT & PITCHING MOMENT COEF. (1)

- aerodynamic loading NOT fully defined by simply stating *lift & drag*

- it is necessary to provide also:
  - point of application of *lift & drag*; OR
  - moment produced by *lift & drag* in a reference point

- typical reference points:
  - pressure center (cp): point of application of *lift & drag* (moment is NULL)
  - aerodynamic center (ca): pitching moment coef. independent of $\alpha$, for $|\alpha| \ll 1$
    - subsonic regime: ca located approx. in 25% of chord
    - supersonic regime: ca located approx. in 50% of chord

Pitching moment coef. respect to a generic point:

$$c_{m,point} = \frac{-1}{c^2} \int_{x_{LE}}^{x_{TE}} c_l(x)(x - x_{point})dx$$

$$c_{m,2} = c_{m,1} - c_l \left( \frac{x_1}{c} - \frac{x_2}{c} \right)$$

Pitching moment respect to a generic point:

$$m_{point} = \frac{1}{2} \rho U^2 c^2 c_{m,point}$$
2D POTENTIAL FLOW

PITCHING MOMENT & PITCHING MOMENT COEF. (3)

Resultant force at leading edge

Resultant force at quarter-chord point


2D POTENTIAL FLOW

PITCHING MOMENT & PITCHING MOMENT COEF. (4)


(e) $M = 0.72$
Lesson 2: Inviscid Flow

Part 1: 2D Potential Flow

(Infinitive Wings)

Thanks for your attention

Any question?
**CYLINDER**

Figure 3.49  Pressure distribution over a circular cylinder in low-speed flow. Comparison of the theoretical pressure distribution with two experimental pressure distributions—one for a subcritical Re and the other for a supercritical Re.

**MAGNUS EFFECT**

Figure 3.32  The synthesis of lifting flow over a circular cylinder.

Figure 3.33  Stagnation points for the lifting flow over a circular cylinder.
BOUNDARY LAYER

Streamline just outside the shear-layer region

Boundary layer where shear stress is significant


(Anderson, J.D., Fundamentals of Aerodynamics, 2001)
FLOW VELOCITY IN BOUNDARY LAYER


(a) Favorable gradient: \( \frac{dU}{dx} > 0 \)
- No separation.
- PI inside wall.

(b) Zero gradient: \( \frac{dU}{dx} = 0 \)
- No separation.
- PI at wall.

(c) Weak adverse gradient: \( \frac{dU}{dx} < 0 \)
- No separation.
- PI in the flow.

(d) Critical adverse gradient: \( \frac{dU}{dx} > 0 \)
- Zero slope at the wall.
- Separation.

(e) Excessive adverse gradient:
- Backflow at the wall.
- Separated flow region.

BOUNDARY LAYER

LAMINAR & TURBULENT BOUNDARY LAYER (1)

Surface of golf balls:

- characteristic length (diameter) & velocity not high → small Re
- plain surface → laminar BL → detaches earlier → thicker wake → \( d \uparrow \)
- carved surface → forces transition to turbulent BL → detaches later → thinner wake → \( d \downarrow \)

BOUNDARY LAYER

LAMINAR & TURBULENT BOUNDARY LAYER (2)

Surface of golf balls:

- characteristic length (diameter) & velocity not high → small $Re$
- plain surface → laminar BL → detaches earlier → thicker wake → $d \uparrow$
- carved surface → forces transition to turbulent BL → detaches later → thinner wake → $d \downarrow$

[Anderson, J.D., Fundamentals of Aerodynamics, 2001]