Consider the two-dimensional wave equation
\[ \partial_{tt} u - \Delta u = f(x, y, t) \text{ if } t > 0, \quad u(x, y, 0) = \partial_t u(x, y, 0) = 0, \]
and heat equation
\[ \partial_t u - \Delta u = f(x, y, t) \text{ if } t > 0, \quad u(x, y, 0), \]
where the forcing function \( f \) is given by
\[ f(x, y, t) = \cos 2t \exp[-(2x^2 + 3y^2)/4]. \]

1. Solve the wave equation in the whole plane, with \( u \rightarrow 0 \) as \( x^2 + y^2 \rightarrow \infty \), using the Green function.

2. Solve the wave equation in the domain \( \Omega : -1 < x < 1, -1 < y < 1 \), with \( u = 0 \) at \( \partial \Omega \), using a spectral representation.

3. For both the unbounded and bounded domain:
   3.1 Elucidate whether the solutions are in phase with the forcing.
   3.2 Compare the CPU time that is required to construct a snapshot of the solution calculated in questions 1 and 2 in the domain \( \Omega \) in a \( 100 \times 100 \) equispaced grid at \( t = \pi/2 \).
   3.3 Construct the appropriate graphical representations of the solution calculated in questions 1 and 2 to illustrate the solution in the domain \( \Omega \) as time proceeds.

4. Repeat questions 2, 3, 3.1, and 3.3 for the damped wave equation
\[ \partial_{tt} u + \varepsilon \partial_t u - \Delta u = f(x, y, t) \text{ if } t > 0, \quad u(x, y, 0) = \partial_t u(x, y, 0) = 0, \]
with \( \varepsilon = 0.01 \).

5. (Extra credit) Repeat questions 1, 2, and 3 (including 3.1, 3.2, and 3.3) for the heat equation.