

z-TRANSFORM PAIRS

The index-domain signal is $x[n]$ for $-\infty < n < \infty$; and the z -transform is:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \quad \Longleftrightarrow \quad x[n] = \frac{1}{2\pi j} \oint X(z) z^n \frac{dz}{z}$$

The ROC is the set of complex numbers z where the z -transform sum converges.

Signal: $x[n]$ $-\infty < n < \infty$	z -Transform: $X(z)$	Region of Convergence
$\delta[n]$	1	All z
$\delta[n - n_0]$	z^{-n_0}	$ z > 0$, if $n_0 > 0$ $ z < \infty$, if $n_0 < 0$
$u[n]$	$\frac{1}{1 - z^{-1}}$	$ z > 1$
$-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$ z < 1$
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$ z > a $
$-a^n u[-n - 1]$	$\frac{1}{1 - az^{-1}}$	$ z < a $
$n a^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z > a $
$-n a^n u[-n - 1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z < a $
$(n + 1) a^n u[n]$	$\frac{1}{(1 - az^{-1})^2}$	$ z > a $
$[\cos \omega_0 n] u[n]$	$\frac{1 - [\cos \omega_0] z^{-1}}{1 - 2[\cos \omega_0] z^{-1} + z^{-2}}$	$ z > 1$
$[\sin \omega_0 n] u[n]$	$\frac{[\sin \omega_0] z^{-1}}{1 - 2[\cos \omega_0] z^{-1} + z^{-2}}$	$ z > 1$
$[r^n \cos \omega_0 n] u[n]$	$\frac{1 - [r \cos \omega_0] z^{-1}}{1 - 2r[\cos \omega_0] z^{-1} + r^2 z^{-2}}$	$ z > r $
$[r^n \sin \omega_0 n] u[n]$	$\frac{[r \sin \omega_0] z^{-1}}{1 - 2r[\cos \omega_0] z^{-1} + r^2 z^{-2}}$	$ z > r $
$x[n] = \begin{cases} a^n, & 0 \leq n < L \\ 0, & \text{otherwise} \end{cases}$	$\frac{1 - a^L z^{-L}}{1 - az^{-1}}$	$ z > 0$