

Calculations for Additional lift

We already showed that the total lift coefficient of an arbitrary circulation distribution is proportional to the coefficient of $\sin \theta$ in the Fourier series describing the distribution.

Examining experimental results for many untwisted planforms and devised the approximate rule that the distribution of additional lift, that is, the lift associated with the chord distribution without twist, is nearly proportional at every point to the ordinate that lies halfway between the elliptical and actual chord distributions for the same total area and span. Thus

$$L_a' = \frac{1}{2} \left[C + C_{SE} \sqrt{1 - \frac{y^2}{(b/2)^2}} \right] \frac{L}{S}$$

C : is actual chord.

C_{SE} : chord at plane of symmetry for the elliptical planform

$$S = \int_{-b/2}^{b/2} C dy = \frac{\pi}{4} b \cdot C_{SE}$$

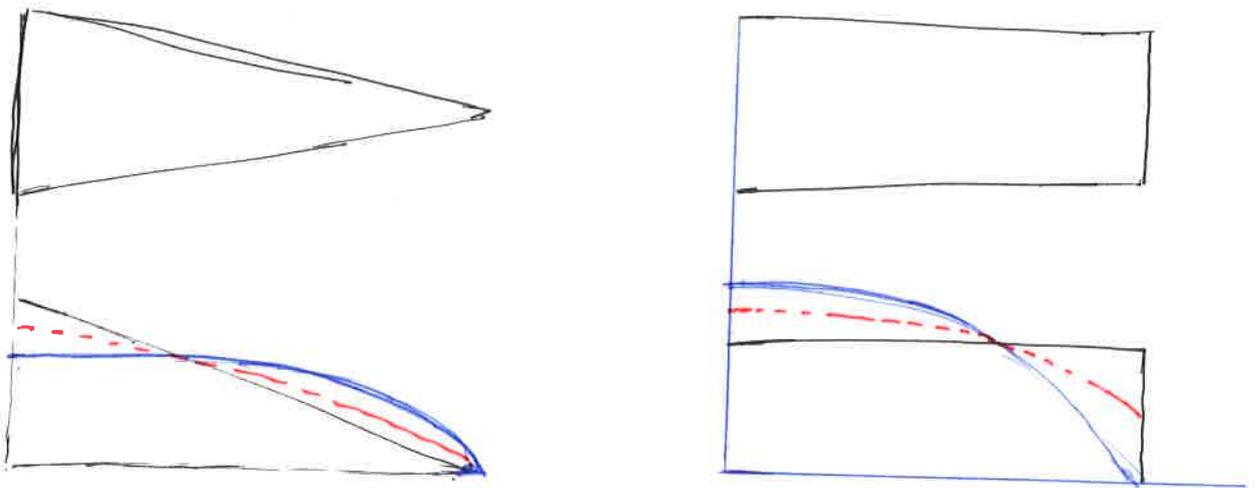
By the use of the relations

$$C_{La}' = \frac{L_a'}{\frac{\rho}{2} C \cdot C_L} \quad \text{and} \quad \bar{C} = \frac{S}{b}$$

We find Schrenck's approximate relation

$$C_{La}' = \frac{1}{2} \left[1 + \frac{4}{\pi} \frac{\bar{C}}{C} \sqrt{1 - \left(\frac{y}{b/2}\right)^2} \right]$$

This relation shows clearly the effect of taper on the spanwise lift distribution



The red lines are drawn halfway between the actual chord and that for an ellipse of the same area and semispan. We can observe that the effect of taper is to increase the load in the outboard portion above that which occur if the additional lift were proportional to the chord.

Characteristic of Finite Wing

(2)

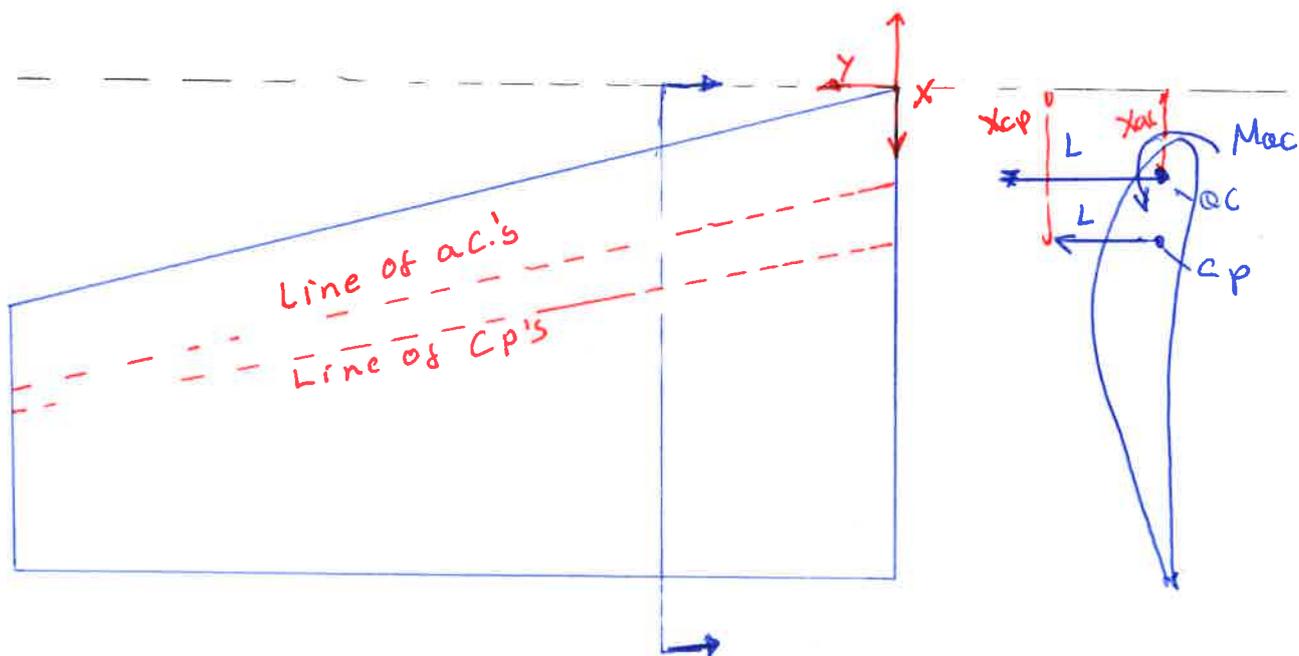
Center of pressure, aerodynamic center and moments about finite wing are calculated as the weighted mean averages of the section characteristics.

The fore and aft location of the center of pressure on an important design parameter, will then be the weighted average of x_{cp} the location for the sections. The distance from a reference line to the center of pressure line for the entire wing x_{cp} for a given angle of attack is

$$x_{cp} = - \frac{M_{RL}}{L}$$

M_{RL} = the moment about the reference line given by

$$M_{RL} = - \int_{-b/2}^{b/2} L' x_{cp} dy = - \int_{-b/2}^{b/2} L' x_{ac} dy + \int_{-b/2}^{b/2} M'_{ac} dy$$



In this equation X_{ac} and M'_{ac} are functions only of Y ; since they are independent of the angle of attack, however, L' is a function of Y , therefore of the sectional absolute angle of attack, α_a . It follows that M_{ac} will be a function of α_a , the absolute angle of attack of the wing and X_{cp} will be also be a function of α_a .

$$X_{cp} = - \frac{\int_{-b/2}^{b/2} [-(C_{L0} + C'_{La} C_L) X_{ac} C + C_{mac} C^2] dy}{\int_{-b/2}^{b/2} C_{La} C_L C dy}$$

$$X_{ac} = - \frac{\int_{-b/2}^{b/2} C'_{La} C_0 X_{ac} dy}{\int_{-b/2}^{b/2} C'_{La} C dy}$$

From C_{L0} and C_{mac} terms must determine M_{ac} ; the moment about the A.C., this follows from the definition of M_{ac} as independent of the angle of attack and, therefore of the additional lift.

Defining Δx_{ac} as the distance from the A.C. ③
 line to the section aerodynamic center (a.c.)
 and we define $C_{MAC} = \frac{M_{AC}}{\rho_{\infty} \bar{c} S} \Rightarrow \bar{c} = \frac{S}{b}$ mean chord

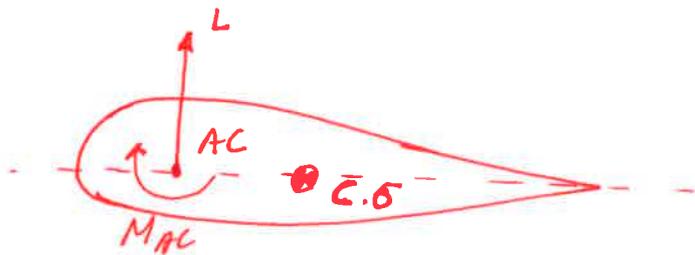
$$C_{MAC} = \int_{-b/2}^{b/2} \left[-C_{Lb} \cdot \frac{\Delta x_{ac}}{\bar{c}^2} \cdot c + C_{mac} \left(\frac{c}{\bar{c}} \right)^2 \right] d\left(\frac{y}{b}\right)$$

The integrand of this equation can also be interpreted as the result of the transfer of moments from the a.c. at a given (y) to the A.C., the centroid of the additional lift of the wing. so

$$M_{AC} = \int_{-b/2}^{b/2} (M'_{ac} - L'_b \Delta x_{ac}) d\left(\frac{y}{b}\right)$$

Stability and trim of wings.

A wing is termed statically stable if, as a result of a small angular disturbance from equilibrium in steady flight, an aerodynamic moment is generated tending to return the wing to equilibrium.



Wing cross section is shown with the load systems acting at aerodynamic center. Consider a wing as a rigid body, any unbalanced moments will cause it to rotate about its center of gravity.

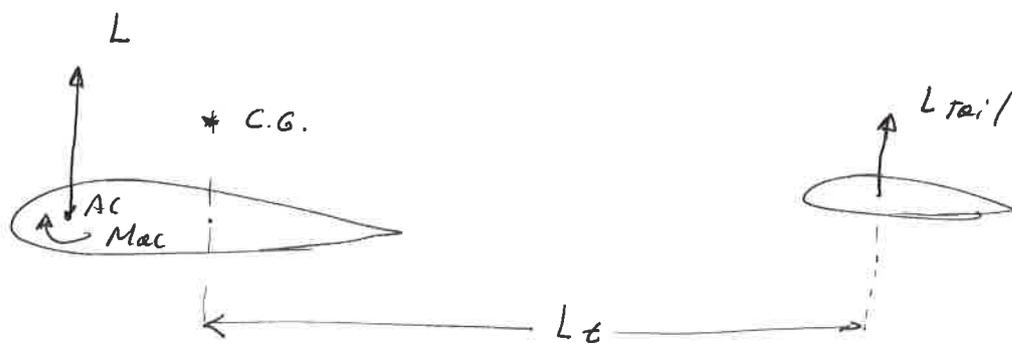
If C.G. is behind the A.C. and M_{CG} is zero or balanced externally, the increment of lift $+\Delta L$, that results from an increment in the angle of attack will cause a moment $+\Delta M_{CG}$.

Conversely, if the angle of attack decreases, the resulting ΔL and M_{CG} will both be negative.

(4)

In either case $dM_{CG}/dL > 0$; so, since the moment generated is in the direction to increase the deviation from equilibrium, this inequality identifies the configuration of the figure unstable. therefore the wing is stable if $X_{AC} > X_{CG}$ so that $dM_{CG}/dL < 0$.

Most configurations, the C.G. lies behind the A.C. and stability is generally achieved by placing horizontal stabilizer behind wing.



It is seen that the tail contributes a stabilizing moment when the wing-tail configuration is disturbed from equilibrium, and by a proper adjustment of the tail area and tail length L_t , this stabilizing moment can be made easily to outweigh the destabilizing effect of wing.

The airplane must be trimmed, meaning that the net moment acting must vanish.

the airplane is stable if $dM_{CG}/dL < 0$; if trim is also to be achieved in steady flight ($L > 0$), it is necessary that $M_{CG} = 0$ at $L = L_{trim}$.

Thus, to satisfy both conditions, it is required that $M_{CG} > 0$ at $L < L_{trim}$. In terms of coefficients the two conditions trim and stability, for steady equilibrium flight are

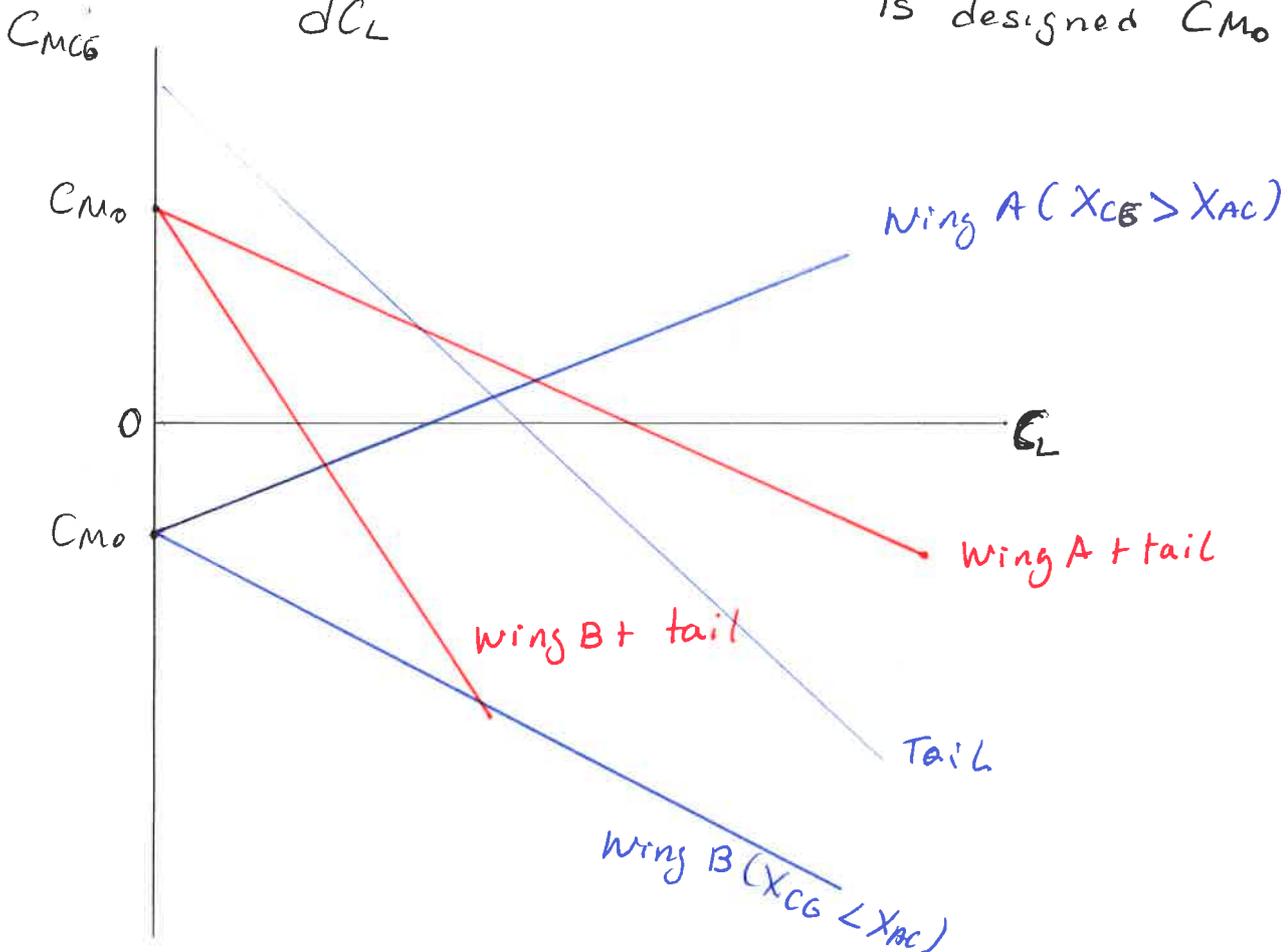
$$C_{MEG} > 0 \quad C_L = 0$$

$$\frac{dC_{MEG}}{dC_L} < 0$$

These conditions are shown in below figure.

$C_L = 0$, the $C_{MCG} (= C_{Mac})$ is designed C_{M0}

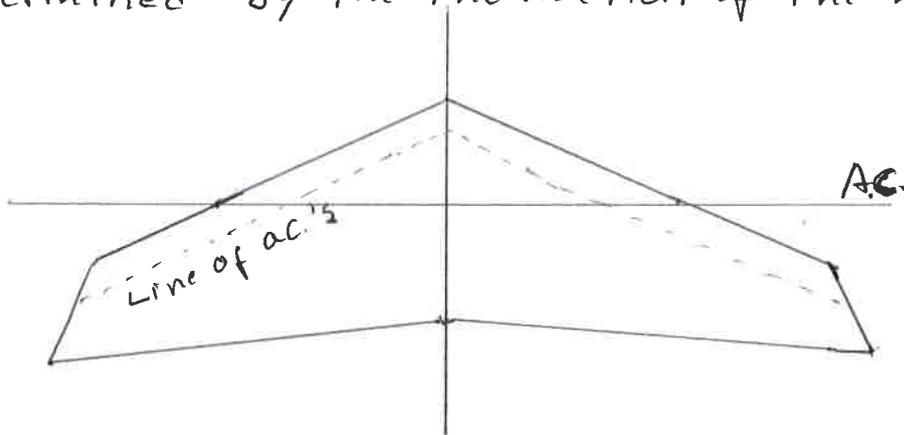
$C_{M0} (C_{MCG} = C_{Mac})$



The above figure shows schematically the dependence of $C_{m\alpha}$ on C_L for the wings with the C.G. ahead of and behind the A.C., for tail (behind A.C.) and for the wing-tail combination the wing A alone trims but is unstable if the C.G. is behind the A.C. and the wing B alone is stable but does not trim if the C.G. is ahead of A.C.

For both of these wings, stability and trim are achieved by adding the tail. For a conventional wing (Wing B) with positive camber, $C_{m0} < 0$ so that the tail is required for steady equilibrium flight.

The contribution of the basic lift to C_{m0} may be either positive or negative, depending on the twist and sweep of the wing. Consider the sweepback wing. The direction of sweep of a wing is determined by the inclination of the line of a.c.



Presume that the wing is set at $C_L = 0$ so that the lift acting at any section is basic lift, then if the wing is washed out at the tips, the lift acting on the outboard sections will be down, whereas the lift acting on the inboard section will be up. This consequence of the fact that the integral of the basic lift is zero.

the basic lift thus distributed will cause a positive moment about the aerodynamic center.

By the same argument, it can be shown that a combination of sweepforward and washin at tips will also result in a positive contribution to the $C_{m_{ac}}$ by the basic lift.

In conclusion, the C_{m_0} of a wing may be made positive by reflexing the trailing edges of the wing sections or providing the proper amount of twist and sweep.

The combination of sweepback and washout is useful in flying wing design. The sweepback moves the aerodynamic center of the wing rearward, thereby facilitating the stability condition that requires the C.G. lie in front of the aerodynamic center. The sweepback and washout obtains a positive C_{m_0} which is a necessary condition for trim.

Aerodynamics for Airplane Lift and drag

We know that the aerodynamic force on any body moving through the air is due only to basic sources, pressure and shear stress. distribution exerted over the body surface. Lift is created by the pressure distribution, shear stress has only a minor effect on lift. Assumption of inviscid flow is given a good approximation to calculate the lift over an object.

Drag, on the other hand, is created by both pressure and shear stress distribution, analysis just for inviscid flow is not sufficient for prediction of drag.

Lift is produced by the fuselage of an airplane as well as the wing. Wing-body combination is referred as wing plus fuselage. The analysis of the flow field over the body modifies the flow field over the wing and vice-versa. These configurations and interaction must be tested in wind tunnel or computational fluid dynamic calculation must be made.

Other components of the airplane such as a horizontal tail, canard surfaces, and wing strakes can contribute to lift, either in a positive or negative sense.

For subsonic speeds, however, data obtained using different fuselage thicknesses, d , mounted on wings with different spans, b , show that the total lift for wing-body combination is constant for $(\frac{d}{b})$ varying from 0 (wing only) to 6 (fat fuselage).

Lift of the wing-body combination can be treated as simply the lift on the complete wing by itself, including that portion of the wing that is masked by the fuselage.

For drag on airplane cannot be obtained as the simple sum of drag on each component. For example for wing-body combination, the drag usually is higher than the sum of the separate drag forces on the wing and the body, giving rise to an extra drag component called interference drag.

We will discuss only for a simple extension of the equation for finite wing.

$$C_D = C_{D_i} + \frac{C_L^2}{\pi e A R}$$

For the whole airplane, the equation is rewritten as (2)

$$C_D = C_{D_e} + \frac{C_L^2}{\pi e A R}$$

C_D : total drag coefficient

C_{D_e} : parasite drag coefficient

These contain not only pressure drag and skin drag but also the friction and pressure drag of the tail fuselage and any other component of the airplane.

As the lift changes with the angle of attack, we can say that C_{D_e} is, also, a function of C_L . A reasonable function is

$$C_{D_e} = C_{D_0} + \gamma C_L^2$$

γ : is an empirically constant, for $C_L = 0$ then $C_{D_e} = C_{D_0}$

C_{D_0} is defined as the parasite drag coefficient at zero lift or zero-lift drag coefficient.

$$C_D = C_{D_0} + \left(\gamma + \frac{1}{\pi e A R} \right) C_L^2$$

Consider e , that includes the effect of the variation of parasite drag with lift, so the above equation is defined

$$\left[C_D = C_{D_0} + \frac{1}{\pi e A R} C_L^2 \right]$$

where C_{D0} is the parasite drag coefficient at zero lift and the term $\frac{C_L^2}{\pi e AR}$ is the drag coefficient due to lift including both induced drag and the contribution to parasite drag due to lift.

In the equation, the redefined e is called the Oswald efficiency factor. The Oswald factor for different airplanes typically varies between 0.7 and 0.85 whereas the span efficiency factor varies from 0.9 to 1.0.

The empirical expression for the Oswald factor for straight-wing aircraft is.

$$[e = 1.78(1 - 0.045 AR^{0.68}) - 0.64]$$

This equation works for not very large aspect ratio.

This equation $[C_D = C_{D0} + \frac{C_L^2}{\pi e AR}]$ is called the drag polar for airplane, representing the variation of C_D with C_L . It is the cornerstone for conceptual design of airplane and for predictions of the performance of a given aircraft.

Ex. For the airplane Seversky P-35, this airplane has a wing planform area of 20.4 m^2 and a wingspan (10.8 m) the break down for drag is 18% for complete airplane configuration. For 18% the total drag is given as $C_D = 0.0275$. when the airplane is at particular angle of attack $C_L = 0.15$.

Solution.

$$AR = \frac{b^2}{S} = \frac{(10.8)^2}{20.4} = 5.72$$

$$\text{Oswald } e = 1.78 (1 - 0.045 (5.72)^{0.68}) = 0.64$$

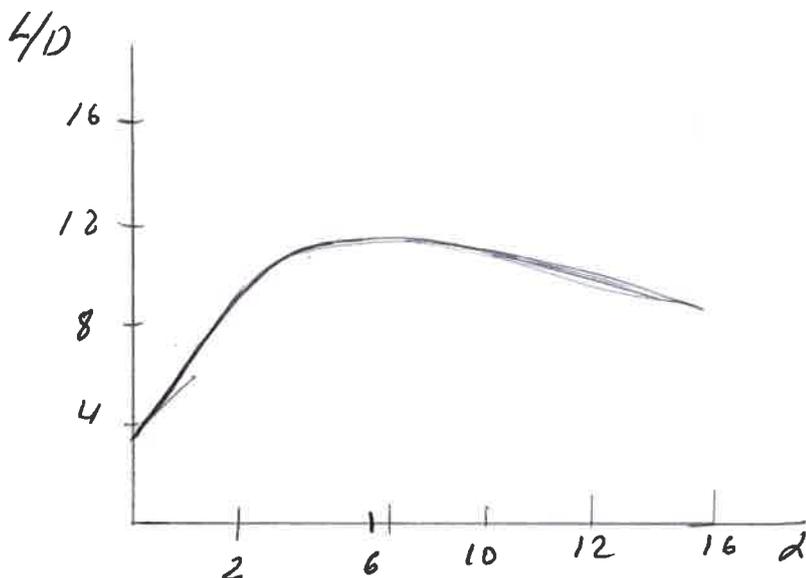
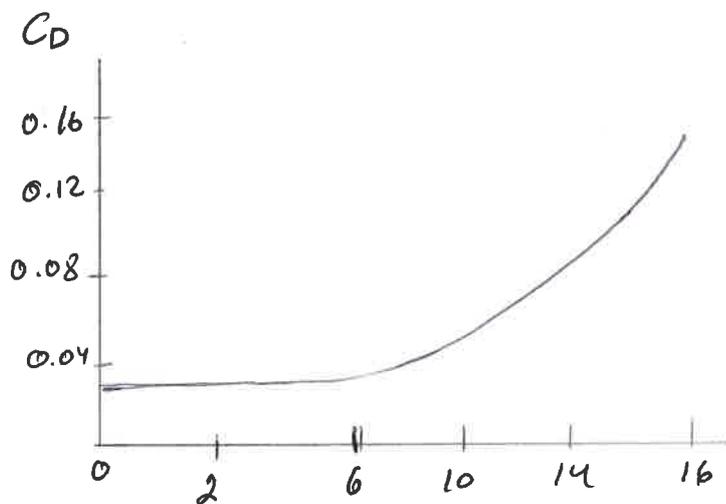
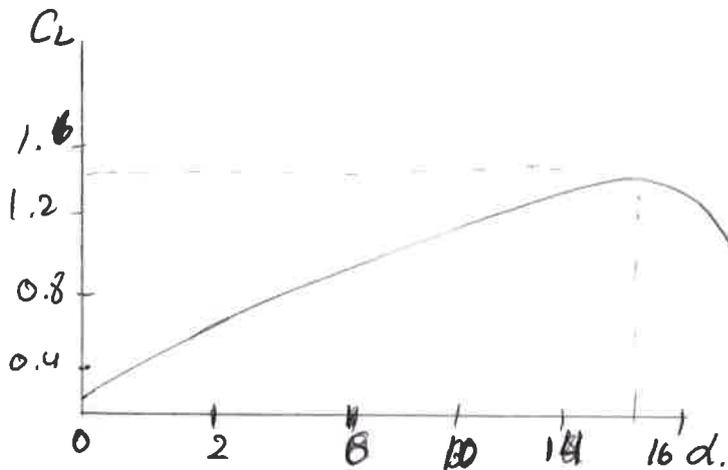
$$e = 0.878$$

$$C_{D0} = C_D - \frac{C_L^2}{\pi e AR} = 0.0275 - \frac{(0.15)^2}{\pi (0.878) (5.72)}$$

$$[C_{D0} = 0.026]$$

Air plane Lift-to-Drag ratio.

The following graph show the variation of C_L vs α , C_D vs α and L/D vs α for an airplane.



The $\frac{C_L}{C_D}$ increases with angle of attack, reach a certain maximum value for an α , and then decreases

the maximum lift-to-drag ratio $(L/D)_{\max} = \left(\frac{C_L}{C_D}\right)_{\max}$ (4)

is a direct measure of the aerodynamic efficiency of the airplane. and therefore its value is of great importance in airplane design and in prediction of airplane performance.

$$\frac{C_L}{C_D} = \frac{C_L}{C_{D0} + \frac{C_L^2}{\pi e AR}} \quad \text{for maximum differentiate with respect } C_L$$

$$\frac{d(C_L/C_D)}{d(C_L)} = 0 = \frac{C_{D0} + \frac{C_L^2}{\pi e AR} - C_L [2C_L / (\pi e AR)]}{[C_{D0} + C_L^2 / (\pi e AR)]^2}$$

$$C_{D0} + \frac{C_L^2}{\pi e AR} - \frac{C_L^2 \cdot 2}{\pi e AR} = 0$$

$$\left[C_{D0} = \frac{2C_L^2}{\pi e AR} \right]$$

When the airplane is flying at the specific angle of attack where lift-drag-ratio is maximum the zero-lift drag and the drag due to lift are precisely equal.

$$\left[C_L = \sqrt{\pi e AR C_{D0}} \right]$$

then
$$\left(\frac{C_L}{C_D}\right)_{\max} = \frac{(\pi e AR C_{D,0})^{1/2}}{C_{D,0} + \frac{\pi e AR C_{D,0}}{\pi e AR}}$$

$$\left[\left(\frac{C_L}{C_D}\right)_{\max} = \frac{(\pi e AR C_{D,0})^{1/2}}{2 C_{D,0}}\right]$$
 only depends of zero-lift drag coefficient $C_{D,0}$, e , AR .

Ex. From the above example, calculate the maximum lift-to-drag ratio for the airplane mentioned above.

We calculated $C_{D,0} = 0.026$

$e = 0.878$

$AR = 5.72$

$$\left(\frac{C_L}{C_D}\right)_{\max} = \frac{(\pi e C_{D,0})^{1/2}}{2 C_{D,0}} = \frac{(\pi \cdot 0.878 \cdot 0.026)^{1/2}}{2 \cdot 0.026}$$

$$\left(\frac{C_L}{C_D}\right)_{\max} = 12.32$$
 the value tabulated for the airplane is 11.8. Our calculation is about 5%.