

Topic 5: Transmission Lines

Telecommunication Systems Fundamentals

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Academic year 2.013-2.014



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Concepts in this Chapter

Mathematical Propagation Model for a guided transmission line

Primary Parameters

Secondary Parameters

Bandwidth and Attenuation

Non Reciprocal Transmission Lines

Coaxial

Microstrip

...

Theory classes: 1.5 sessions (3 hours)

Problems resolution: 0.5 session (1 hour)

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graphy

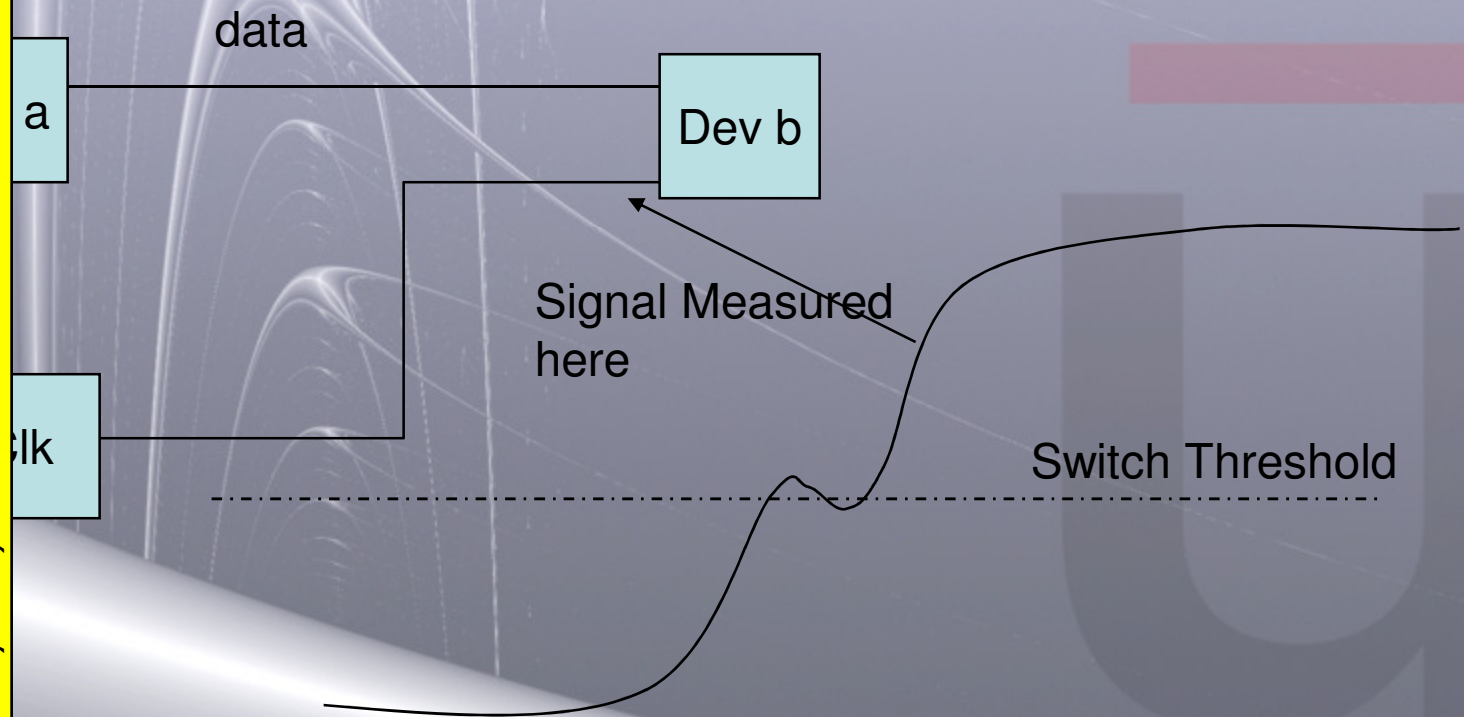
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System Statement: a practical example

... never tells you the measured clock is non-monotonic and because of this the loop internally may double clock the data.



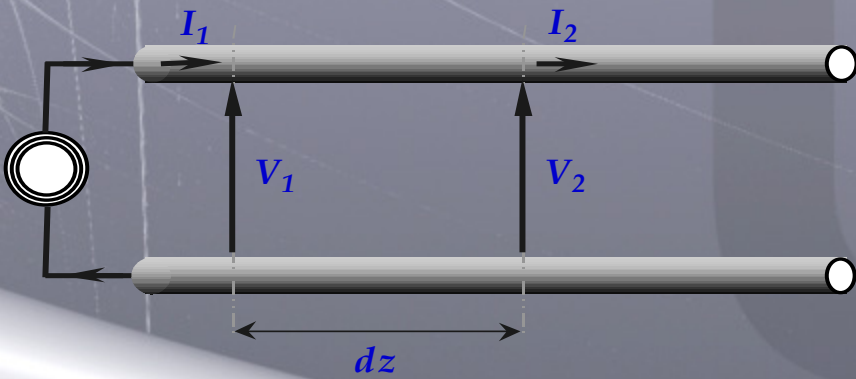
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Transmission Line Concept

Voltage and current on a transmission line is a function of both time and position

Don't just think in terms of position and time to understand transmission line behavior

This positional dependence is added when the assumption of the size of the circuit being small compared to the signaling wavelength



$$V = f(z, t)$$

$$I = f(z, t)$$

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Types of Transmission Lines

Coaxial cables and wires

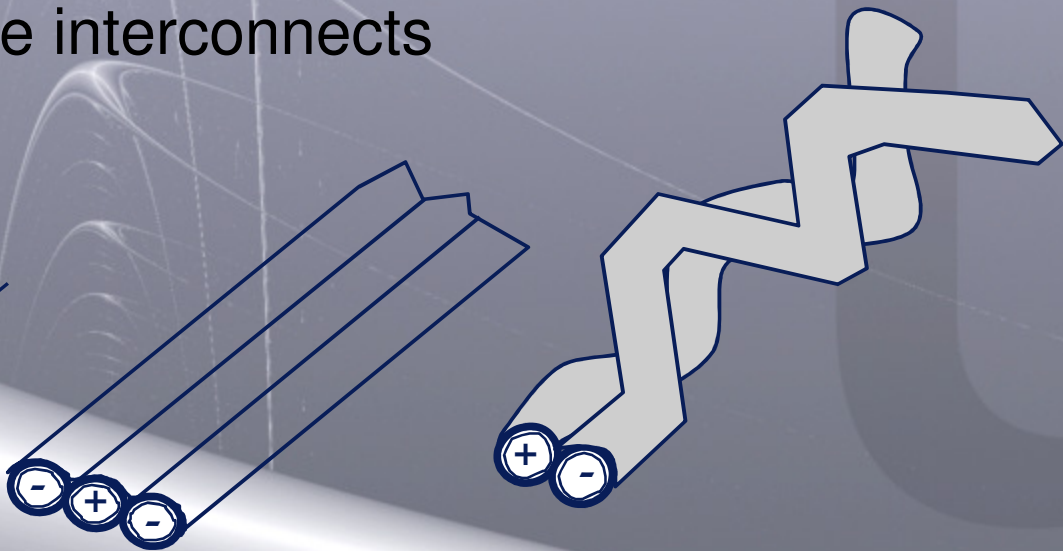
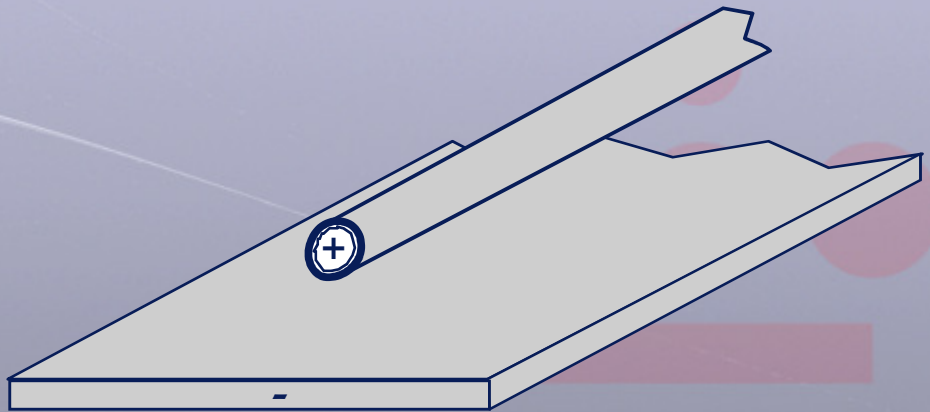
Optical fiber

Power line over ground

Twisted pair

Twisted pair (two-wire line)

Waveguide



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Transmission Line “Definition”

Transmission line: a closed system in which power is transmitted from a source to a destination

In this course we will study only TEM (transversal Electro-magnetic) mode transmission lines

Two conductor wire system with the wires in close proximity, providing relative impedance, velocity and closed current return path to the source

Characteristic impedance is the ratio of the voltage and current waves at any one position on the transmission line

$$Z_0 = \frac{V}{I}$$

Propagation velocity is the speed with which signals are transmitted through the transmission line in its surrounding medium

$$v = \frac{c}{\sqrt{\epsilon_r}}$$

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Transmission Lines as a Complex Electromagnetic Problem

Electric and Magnetic fields are present in the transmission

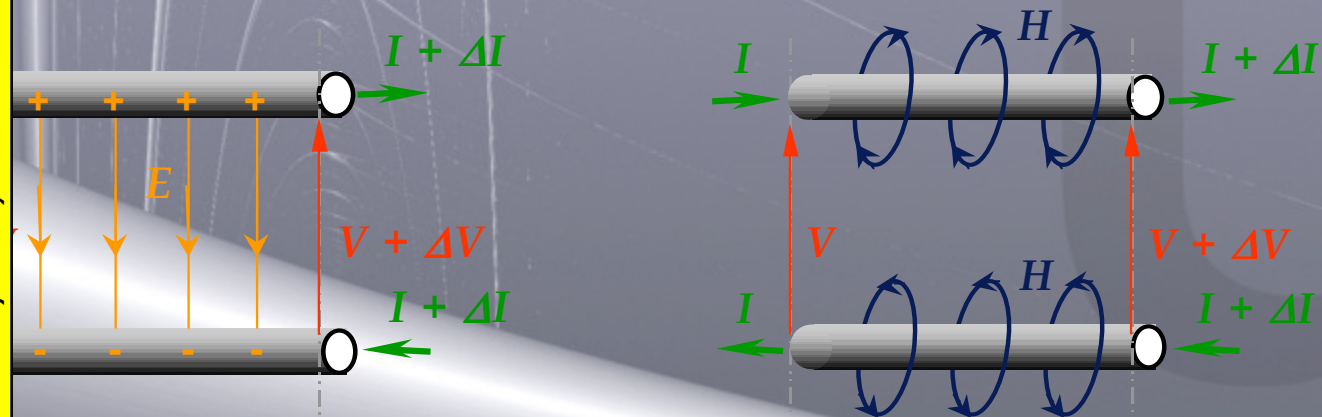
These fields are perpendicular to each other and to the direction of wave propagation for TEM mode waves, which is the simplest mode

Electric field is established by a potential difference between two conductors.

Electric field implies equivalent circuit model must contain capacitor

Magnetic field induced by current flowing on the line

Magnetic field implies equivalent circuit model must contain inductor



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Transmission Line Equivalent Circuit

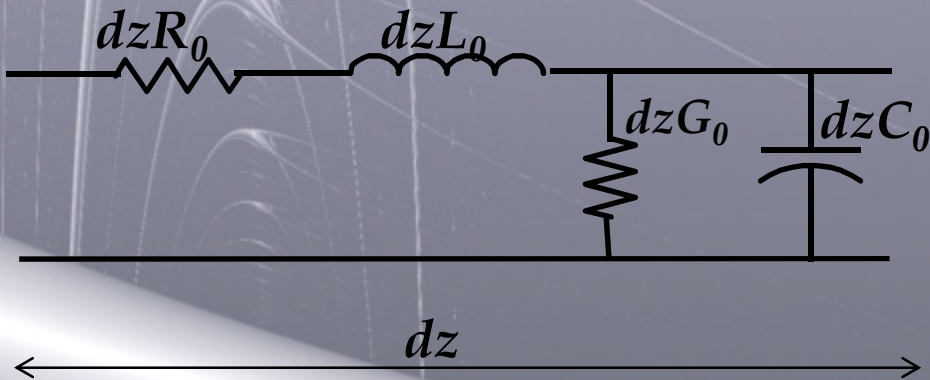
General Characteristics (Primary Parameters)

per-unit-length Capacitance (C_0) [pf/m]

per-unit-length Inductance (L_0) [nf/m]

per-unit-length (Series) Resistance (R_0) [Ω /m]

per-unit-length (Parallel) Conductance (G_0) [$1/\Omega$ m]



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Transmission Line

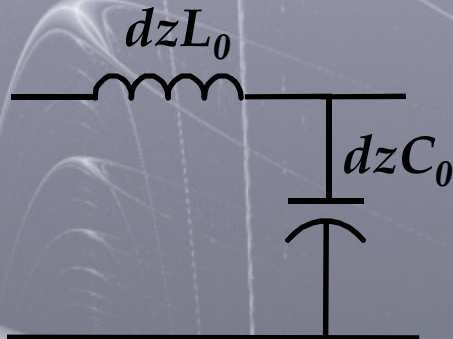
I (lossless) Characteristics of Transmission

Real TL assumes:

Uniform line

Perfect (lossless) conductor ($R_0 \rightarrow 0$)

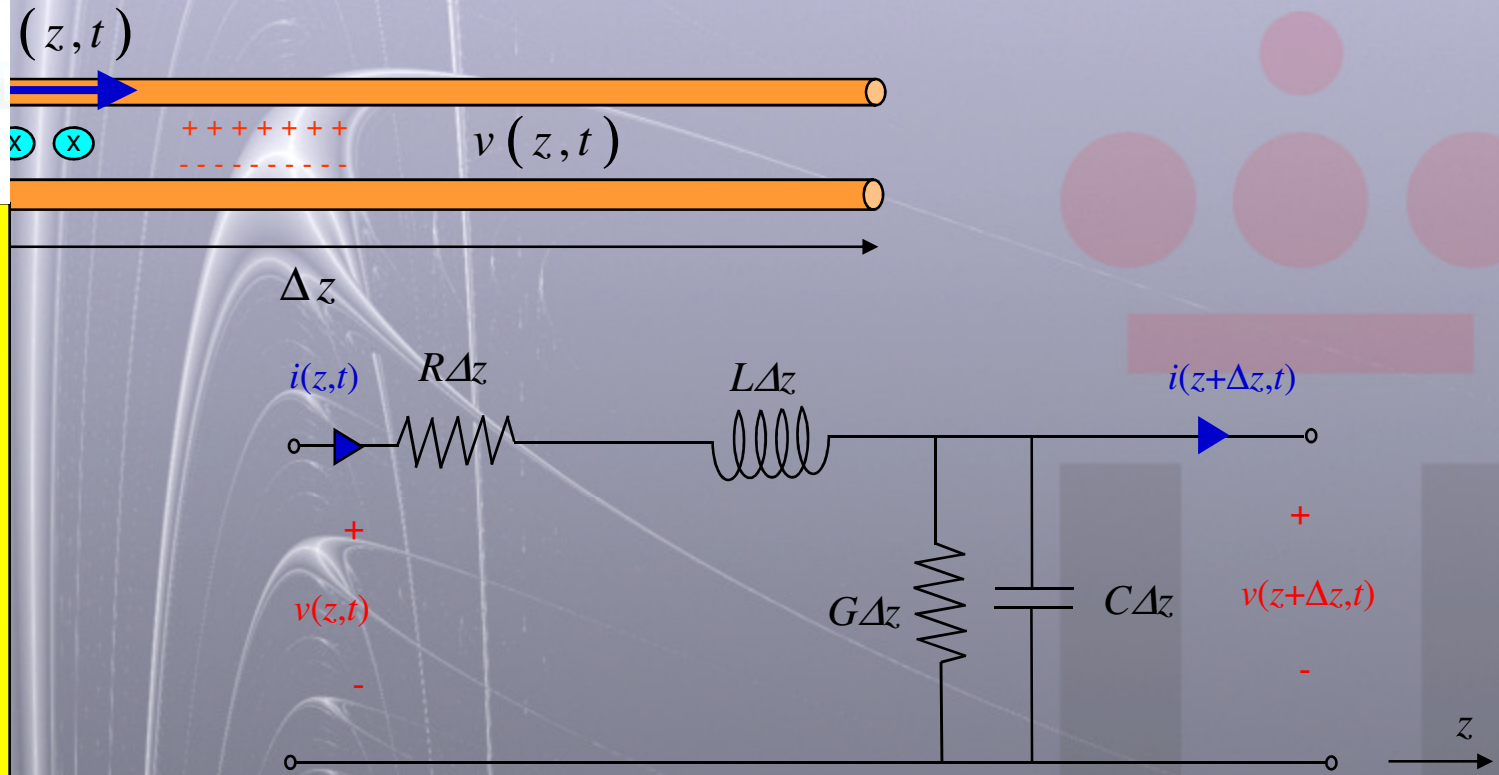
Perfect (lossless) dielectric ($G_0 \rightarrow 0$)



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Transmission Line Equivalent Circuit



$$v(z + \Delta z, t) + i(z, t)R\Delta z + L\Delta z \frac{\partial i(z, t)}{\partial t}$$

$$i(z + \Delta z, t) + v(z + \Delta z, t)G\Delta z + C\Delta z \frac{\partial v(z + \Delta z, t)}{\partial t}$$

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Transmission Line Signal Model

$$\frac{v(z + \Delta z, t) - v(z, t)}{\Delta z} = -Ri(z, t) - L \frac{\partial i(z, t)}{\partial t}$$

$$\frac{i(z + \Delta z, t) - i(z, t)}{\Delta z} = -Gv(z + \Delta z, t) - C \frac{\partial v(z + \Delta z, t)}{\partial t}$$

$\Delta z \rightarrow 0$

$$\frac{\partial v}{\partial z} = -Ri - L \frac{\partial i}{\partial t}$$

$$\frac{\partial i}{\partial z} = -Gv - C \frac{\partial v}{\partial t}$$

known as “Telegrapher’s Equations”

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Transmission Line Signal Model

Combine these, take the derivative of the first one respect to z :

$$\frac{\partial^2 v}{\partial z^2} = -R \frac{\partial i}{\partial z} - L \frac{\partial}{\partial z} \left(\frac{\partial i}{\partial t} \right)$$

$$= -R \frac{\partial i}{\partial z} - L \frac{\partial}{\partial t} \left(\frac{\partial i}{\partial z} \right)$$

$$= -R \left[-Gv - C \frac{\partial v}{\partial t} \right]$$

$$- L \left[-G \frac{\partial v}{\partial t} - C \frac{\partial^2 v}{\partial t^2} \right]$$

Switch the order of the derivatives

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Transmission Line Signal Model

$$\frac{\partial^2 v}{\partial z^2} = -R \left[-Gv - C \frac{\partial v}{\partial t} \right] - L \left[-G \frac{\partial v}{\partial t} - C \frac{\partial^2 v}{\partial t^2} \right]$$

Therefore, we have

$$-(RG)v - (RC + LG) \frac{\partial v}{\partial t} - LC \left(\frac{\partial^2 v}{\partial t^2} \right) = 0$$

The same equation also holds for i



$$-(RG)V - (RC + LG)j\omega V - LC(-\omega^2)V = 0$$

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Transmission Line Signal Model

$$\frac{\partial V}{\partial z} = (RG)V + j\omega(RC + LG)V - (\omega^2 LC)V$$

$$j\omega(RC + LG) - \omega^2 LC = (R + j\omega L)(G + j\omega C)$$

$$Z = R + j\omega L = \text{series impedance/length}$$

$$Y = G + j\omega C = \text{parallel admittance/length}$$

can re-write

$$\frac{d^2V}{dz^2} = (ZY)V$$

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Transmission Line Signal Model

Defining $\gamma^2 = ZY$

The differential equation governing the signal is

$$\frac{d^2V}{dz^2} = (\gamma^2)V$$

The general solution is

$$V(z) = Ae^{-\gamma z} + Be^{+\gamma z}$$

where

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\sqrt{z} = \sqrt{|z|} e^{j\theta/2}$$

$$-\pi < \theta < \pi$$

$$\gamma = \alpha + j\beta$$

$$\alpha \geq 0, \beta \geq 0$$

α = attenuation constant

β = phase constant

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Transmission Line Signal Model

Forward travelling wave (a wave traveling in the positive z direction):

$$V^+(z) = V_0^+ e^{-\gamma z} = V_0^+ e^{-\alpha z} e^{-j\beta z}$$

$$v^+(z, t) = \text{Re} \left\{ \left(V_0^+ e^{-\alpha z} e^{-j\beta z} \right) e^{j\omega t} \right\}$$

$$= \text{Re} \left\{ \left(|V_0^+| e^{j\phi} e^{-\alpha z} e^{-j\beta z} \right) e^{j\omega t} \right\}$$

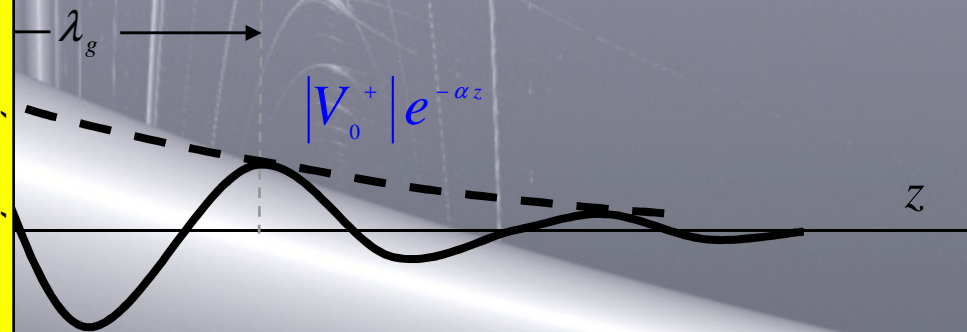
$$= |V_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \phi)$$

The wave “repeats” when

$$\beta \lambda_g = 2\pi$$

Hence:

$$\beta = \frac{2\pi}{\lambda_g}$$



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Transmission Line Secondary Parameters

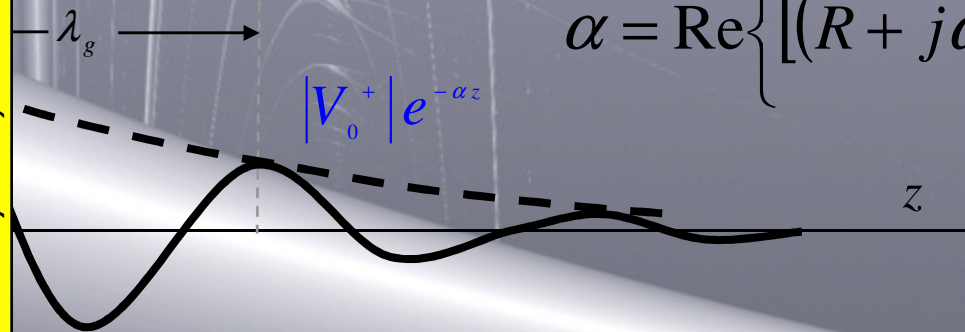
Equation

$$V^+(z) = V_0^+ e^{-\gamma z} = V_0^+ e^{-\alpha z} e^{-j\beta z}$$

$$|V^+(z)| = |V_0^+ e^{-\gamma z}| = |V_0^+| |e^{-\alpha z}| |e^{-j\beta z}| = |V_0^+| |e^{-\alpha z}|$$

$$|e^{-\alpha z}| = \text{attenuation}$$

$$\alpha = \text{Re} \left\{ \left[(R + j\omega L)(G + j\omega C) \right]^{1/2} \right\}$$



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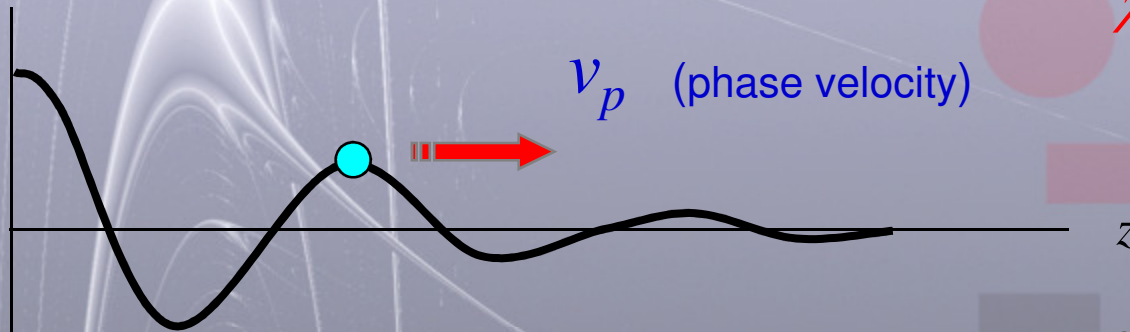
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Transmission Line Secondary Parameters

Phase Velocity

guided wavelength $\equiv \lambda_g$

$$\lambda_g = \frac{2\pi}{\beta} \text{ [m]}$$



Set $\omega t - \beta z = \text{constant}$

$$\omega - \beta \frac{dz}{dt} = 0$$

$$\frac{dz}{dt} = \frac{\omega}{\beta}$$

$$V(z, t) = |V_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \phi)$$

In expanded form:

$$v_p = \frac{\omega}{\text{Im}\{[(R + j\omega L)(G + j\omega C)]^{1/2}\}}$$

$$v_p = \frac{\omega}{\beta}$$

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Transmission Line Secondary Parameters

Characteristic Impedance Z_0

A wave is traveling in the positive z direction.

$$Z_0 \equiv \frac{V^+(z)}{I^+(z)}$$

$$Z_0 = \frac{V_0^+}{I_0^+}$$

(Z_0 is a number, not a function of position)

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{Z}{\gamma} = \left(\frac{Z}{Y} \right)^{1/2}$$

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

$$-\gamma W_0^+ e^{-\gamma z} = -Z I_0^+ e^{-\gamma z}$$

$$Z_0 = \left(\frac{R + j\omega L}{G + j\omega C} \right)^{1/2}$$

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-wise

Power is proportional to the voltage squared

$$P(d) \propto e^{-2\alpha d}$$

Attenuation dependence with distance is

$$At = K e^{2\alpha d}$$

When expressing it on dBs

$$At(dB) = K_1 + K_2 x$$

(constants vary with frequency)

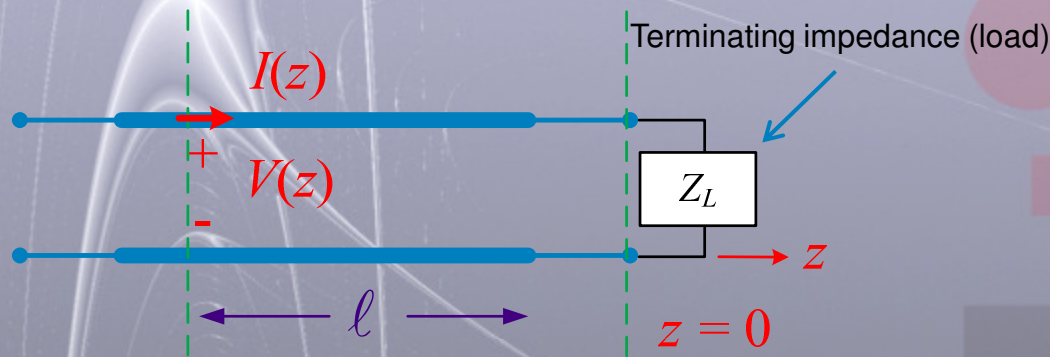
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Transmission Line Model

with forward and backward traveling waves



$$V_0^- e^{-\gamma z} + V_0^+ e^{+\gamma z} = V_0^+ e^{-\gamma z} \left(1 + \frac{V_0^-}{V_0^+} e^{2\gamma z} \right) = V_0^+ e^{-\gamma z} (1 + \Gamma_L e^{2\gamma z})$$

$$\left[V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z} \right]$$

$\Gamma_L \equiv$ Load reflection coefficient

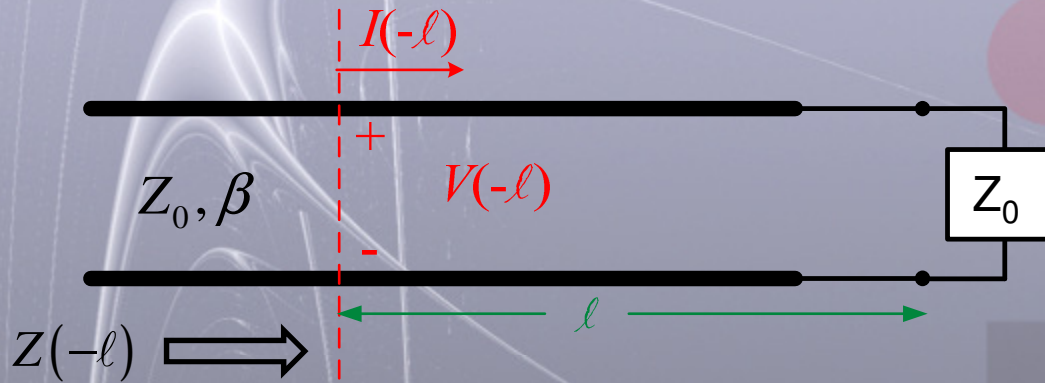
$$\Rightarrow \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

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Impedance Matching in Transmission Lines

1: Matched load: ($Z_L = Z_0$)



$$\frac{Z_L - Z_0}{Z_L + Z_0} = 0$$

No reflection from the load

$$V(-l) = V_0 e^{+j\beta l}$$

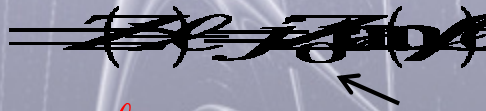
$$Z(-l) = Z_0 \quad \text{For any } l$$

$$I(-l) = \frac{V_0}{Z_0} e^{+j\beta l}$$

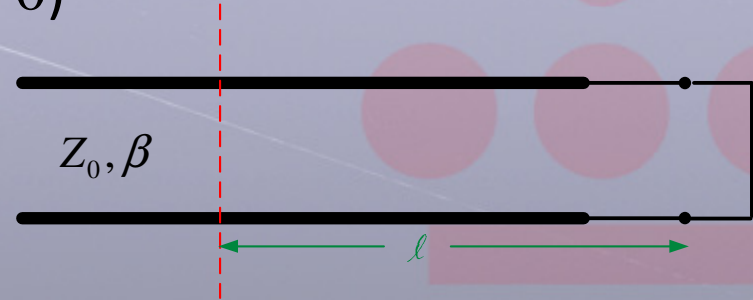
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Impedance Matching in Transmission Lines

2: Short circuit load: ($Z_L = 0$)



$$\beta l = 2\pi \frac{l}{\lambda_g}$$

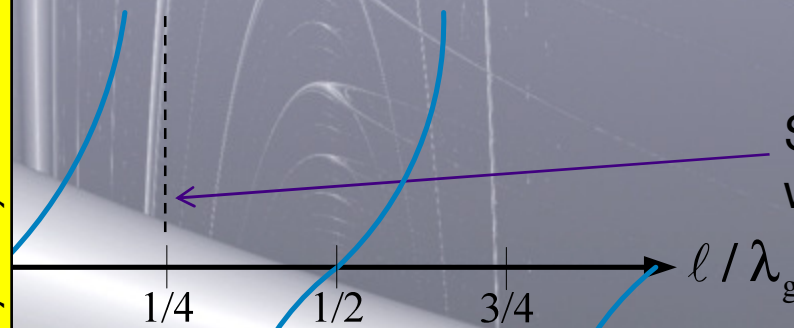


Always imaginary!

$$\Rightarrow Z_{in} = jX_c$$

$$X_c = Z_0 \tan(\beta l)$$

S.C. can become an O.C. with a $\lambda_g/4$ trans. line



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Impedance Matching in Transmission Lines

3: Open Circuit ($Z_L = \infty$)

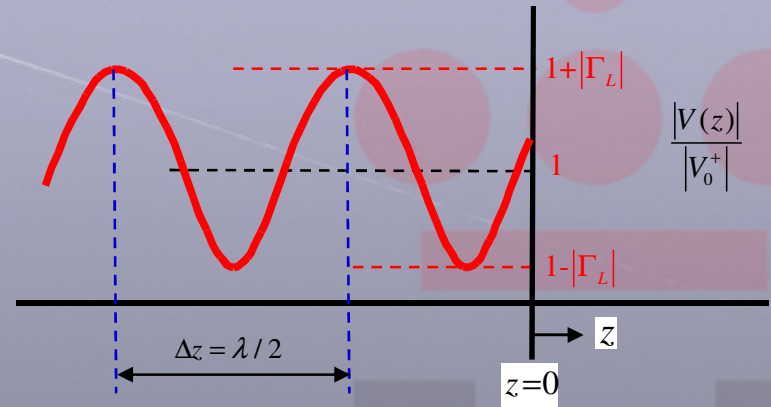
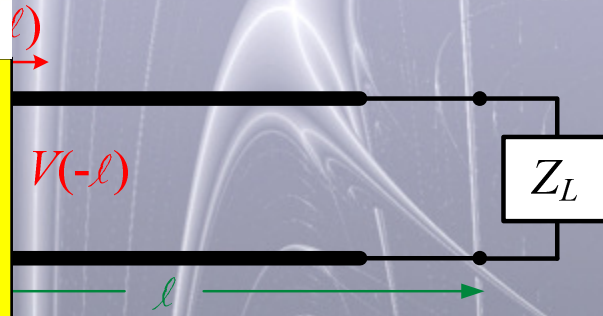
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Impedance Matching in Transmission Lines

Large Standing Wave Ratio



$$V_{\max} = |V_0^+| (1 + |\Gamma_L|)$$

$$V_{\min} = |V_0^+| (1 - |\Gamma_L|)$$

$$\text{Standing Wave Ratio (VSWR)} = \frac{V_{\max}}{V_{\min}}$$

$$\text{VSWR} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

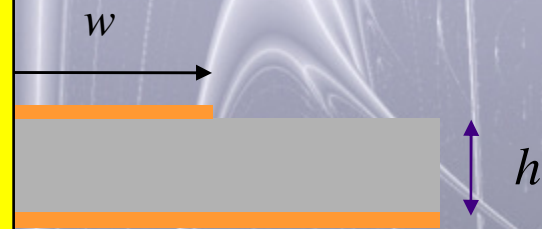
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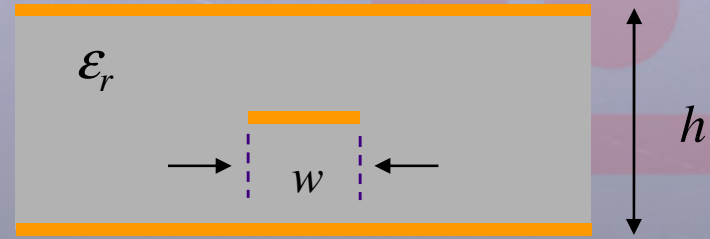


Transmission Lines

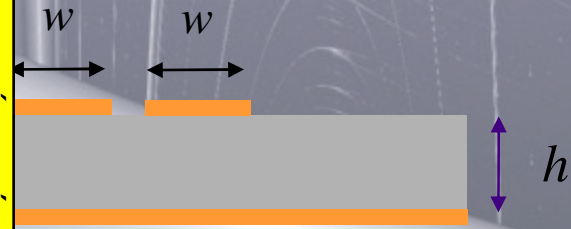
Transmission lines commonly met on printed-circuit boards



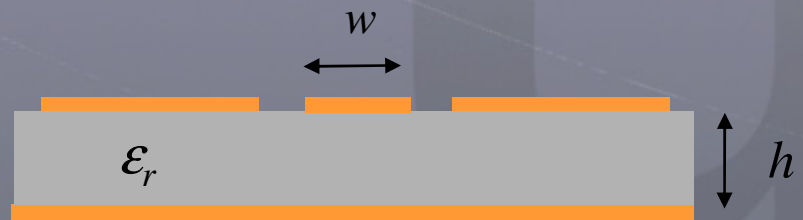
Microstrip



Stripline



Coplanar strips



Coplanar waveguide (CPW)



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Transmission Lines

Electric permittivity

$8.85 \times 10^{-12} \text{ F/m}$ (free space)

Relative dielectric constant

ϵ_r : 3.29 paper; 1 air;

polyethylene and 4 to 6 PVC

Magnetic permeability

$4\pi \times 10^{-7} \text{ H/m}$ (free space)

Relative permeability

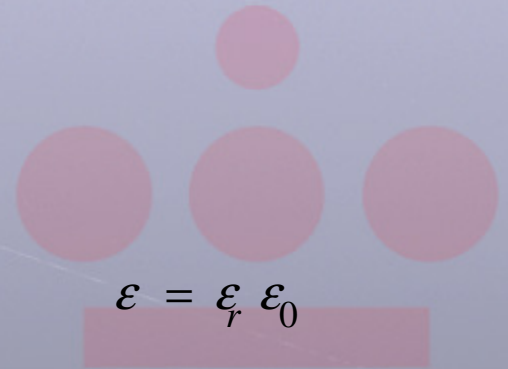
Most of dielectric materials, μ_r is close to one

Conductivity:

Aluminum: 37.7; Bronze 19; Iron 120 ($\times 10^{-8} \Omega/m$)

$$\mu = \mu_r \mu_0;$$

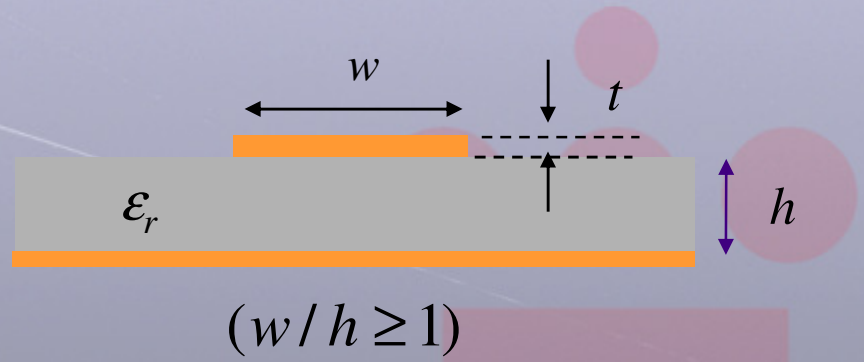
$$\epsilon = \epsilon_r \epsilon_0$$



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Microstrip Transmission Line



$$\frac{120\pi}{\epsilon_r(0) \left[(w'/h) + 1.393 + 0.667 \ln \left((w'/h) + 1.444 \right) \right]} \left(\frac{\epsilon_r^{eff}(f) - 1}{\epsilon_r^{eff}(0) - 1} \right) \sqrt{\frac{\epsilon_r^{eff}(0)}{\epsilon_r^{eff}(f)}}$$

$$\frac{1}{\epsilon_r(0) + \frac{\sqrt{\epsilon_r} - \sqrt{\epsilon_r^{eff}(0)}}{1 + 4F^{-1.5}}}$$

$$\frac{\epsilon_r - 1}{2} \left(\frac{1}{\sqrt{1 + 12(h/w)}} \right) - \left(\frac{\epsilon_r - 1}{4.6} \right) \left(\frac{t/h}{\sqrt{w/h}} \right)$$

$$w' = w + \frac{t}{\pi} \left(1 + \ln \left(\frac{2h}{t} \right) \right)$$

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Copper Pair / Twisted Pair

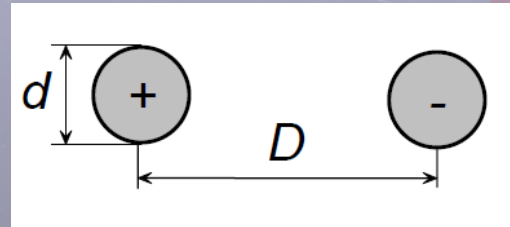
Primary parameters of a Copper Pair

$$\frac{\mu}{\pi} \log\left(2 \frac{D}{d}\right)$$

$$d^2$$

$$\frac{\pi \epsilon}{\log\left(\frac{2D}{d}\right)}$$

Inductance depends on the dielectric, usually negligible but depending on the frequency



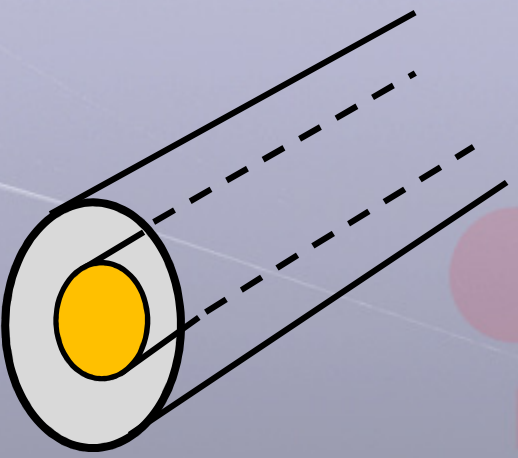
- D: Distance between cables
- d: diameter of each cable

$$Z_0 = 120 \cosh^{-1}\left(\frac{D}{d}\right)$$

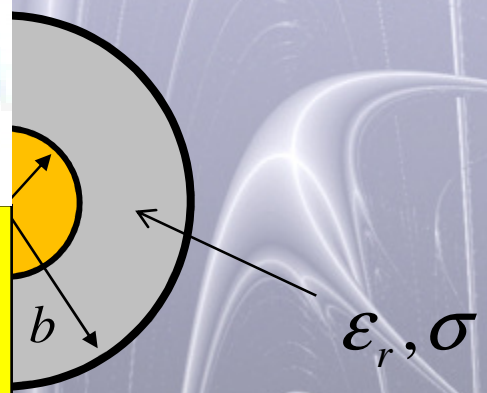
$$\approx 276 \log\left(\frac{2D}{d}\right)$$

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Coaxial Cable



$$R = \rho \left(\frac{1}{2\pi a} + \frac{1}{2\pi b} \right)$$

$$G = \frac{2\pi\sigma}{\ln\left(\frac{b}{a}\right)} \quad [\text{S/m}]$$

$$C = \frac{2\pi\epsilon_r}{\ln\left(\frac{b}{a}\right)} \quad [\text{F/m}]$$

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \quad [\text{H/m}]$$



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Summary of Concepts in this Chapter

are the primary parameters of the transmission

calculation of attenuation, and propagation velocity for transmission lines

calculate above parameters for common transmission

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