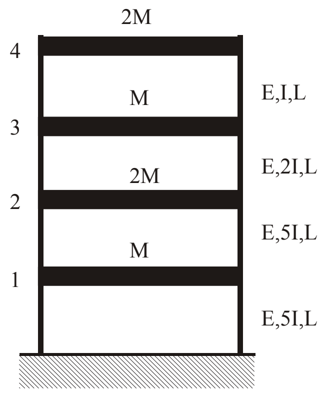
Dynamic Analysis of Structures for the Finite Element Method

Handout 3 (15 points)

Exercise. [Chapters 7 to 10 - Time integration]. Consider the 4 storey building of the figure (the same as that of Handout 2). Use $EI/L^3 = 10^5\,\mathrm{N/m}$ and $M = 10^4\,\mathrm{kg}$. Consider only horizontal degrees of freedom.

1. (1 point) Compute the natural frequencies ω_i in rad/s and f_i in Hz. Compute the periods of the structure T_i. Order them such that T_1 is the largest one and T_4 is the smallest one (that of highest frequency). Which is the maximum time increment that you should use using the central difference method when analyzing the full model of the structure?
2. (1 point) Obtain the $\mathbf{C}$ damping matrix using Rayleigh proportional damping which prescribes a modal damping of $\zeta_1 = \zeta_2 = 2\%$. Assume this damping in the rest of the analysis.
3. (1 point) Write down the equations of motion for a horizontal acceleration at the basement of the structure, see Section 10.2. Note that inertia terms affect all degrees of freedom and obtained displacements are relative to the ground. Obtain the static displacement for a horizontal acceleration of $1g$, being g the value of gravity (since the problem is linear, you can take it as a reference). Compute also the modal participation factors b_i and the mobilized masses b_i^2, see Sections 10.2 and 10.4.
4. (1 point) Download the N-S component of El Centro earthquake accelerogram from the network and divide the accelerations by the maximum absolute registered acceleration in order to normalize it to 1 (see Chapter 10). Create a vector (using MATLAB or similar) which has the loading at each level of the building for each time.
5. (2 point) Obtain the response of the structure using the central differences method for time integration and $\Delta t = T_4/10$. You may adapt and use the code of the notes. Using subplot(4,1,i) plot the response of all degrees of freedom $i = 1, .., 4$.
6. (1 point) Repeat the previous time integration but using $\Delta t = T_4$. Comment the difference in the results.
7. (2 point) Repeat the previous time integration using $\Delta t = T_4$ but employing a Newmark-β method. Comment the difference in the results.
8. (1 point) Repeat the previous time integration using $\Delta t = T_4/10$ but employing a Newmark-β method. Comment the difference in the results.
9. (1 point) Write down the modal decomposition equations and the uncoupled equations of motion for the case of ground acceleration (see again Section 10.2).
10. (1 point) Use modal decomposition to integrate the uncoupled equations of motion with $\Delta t = T_4/10$ and with $\Delta t = T_4$. Verify that the results are almost the same as those using the uncoupled system. Why are them slightly different?
11. (1 point) Use or adapt the code of the notes to compute the response spectra of El Centro earthquake for a 2% of damping.
12. (2 point) Obtain the maximum acceleration response in g using a mode superposition spectral analysis with a SRSS combination method (Section 10.4).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