Gradiente, Divergencia, Rotacional, Laplaciano

Coordenadas Cartesianas $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$

$$f = f(x, y, z) \qquad ; \qquad \overrightarrow{G} = G_x \hat{\imath} + G_y \hat{\jmath} + G_z \hat{k}$$

$$\begin{aligned}
\overrightarrow{\nabla} f &= \frac{\partial f}{\partial x}\,\hat{\imath} + \frac{\partial f}{\partial y}\,\hat{\jmath} + \frac{\partial f}{\partial z}\,\hat{k} \\
\overrightarrow{\nabla} \cdot \overrightarrow{G} &= \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y} + \frac{\partial G_z}{\partial z} \\
\overrightarrow{\nabla} \times \overrightarrow{G} &= \left(\frac{\partial G_z}{\partial y} - \frac{\partial G_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial G_x}{\partial z} - \frac{\partial G_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial G_y}{\partial x} - \frac{\partial G_x}{\partial y}\right)\hat{k} \\
\nabla^2 f &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}
\end{aligned}$$

Coordenadas Cilíndricas $(\boldsymbol{\rho}, \boldsymbol{\phi}, \boldsymbol{z})$

$$f = f(\rho, \phi, z) \qquad ; \qquad \overrightarrow{G} = G_\rho \hat{\rho} + G_\phi \hat{\phi} + G_z \hat{z}$$

$$\begin{aligned}
\overrightarrow{\nabla} f &= \frac{\partial f}{\partial \rho}\,\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\,\hat{\phi} + \frac{\partial f}{\partial z}\,\hat{z} \\
\overrightarrow{\nabla} \cdot \overrightarrow{G} &= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho\, G_\rho) + \frac{1}{\rho}\frac{\partial G_\phi}{\partial \phi} + \frac{\partial G_z}{\partial z}
\end{aligned}$$

Coordenadas Esféricas $(\boldsymbol{r}, \boldsymbol{\theta}, \boldsymbol{\phi})$

$$f = f(r,\theta,\phi) \qquad ; \qquad \overrightarrow{G} = G_r\hat{r} + G_\theta\hat{\theta} + G_\phi\hat{\phi}$$

$$\overrightarrow{\nabla} f = \frac{\partial f}{\partial r}\,\hat{r} + \frac{1}{r}\,\frac{\partial f}{\partial \theta}\,\hat{\theta} + \frac{1}{r\,\mathrm{sen}\,\theta}\,\frac{\partial f}{\partial \phi}\,\hat{\phi}$$

$$\overrightarrow{\nabla}\cdot\overrightarrow{G} = \frac{1}{r^2}\,\frac{\partial}{\partial r}\left(r^2\,G_r\right) + \frac{1}{r\,\mathrm{sen}\,\theta}\,\frac{\partial}{\partial \theta}\left(\mathrm{sen}\,\theta\,G_\theta\right) + \frac{1}{r\,\mathrm{sen}\,\theta}\,\frac{\partial G_\phi}{\partial \phi}$$

$$\begin{aligned}\overrightarrow{\nabla}\times\overrightarrow{G} &= \frac{1}{r\,\mathrm{sen}\,\theta}\left[\frac{\partial}{\partial \theta}\left(\mathrm{sen}\,\theta\,G_\phi\right) - \frac{\partial G_\theta}{\partial \phi}\right]\hat{r} \\ &+ \left[\frac{1}{r\,\mathrm{sen}\,\theta}\,\frac{\partial G_r}{\partial \phi} - \frac{1}{r}\,\frac{\partial}{\partial r}\left(r\,G_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\,G_\theta\right) - \frac{\partial G_r}{\partial \theta}\right]\hat{\phi}\end{aligned}$$

$$\nabla^2 f = \frac{1}{r^2}\,\frac{\partial}{\partial r}\left(r^2\,\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\,\mathrm{sen}\,\theta}\,\frac{\partial}{\partial \theta}\left(\mathrm{sen}\,\theta\,\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\,\mathrm{sen}^2\theta}\,\frac{\partial^2 f}{\partial \phi^2}$$