

Flutter Phenomenon

PART 1: AEROELASTIC EQUATIONS WITHOUT EXPLICIT FORMULATION OF AERODYNAMIC TERMS

Next weeks...

PART 2: FORMULATION OF THE AERODYNAMIC FORCES

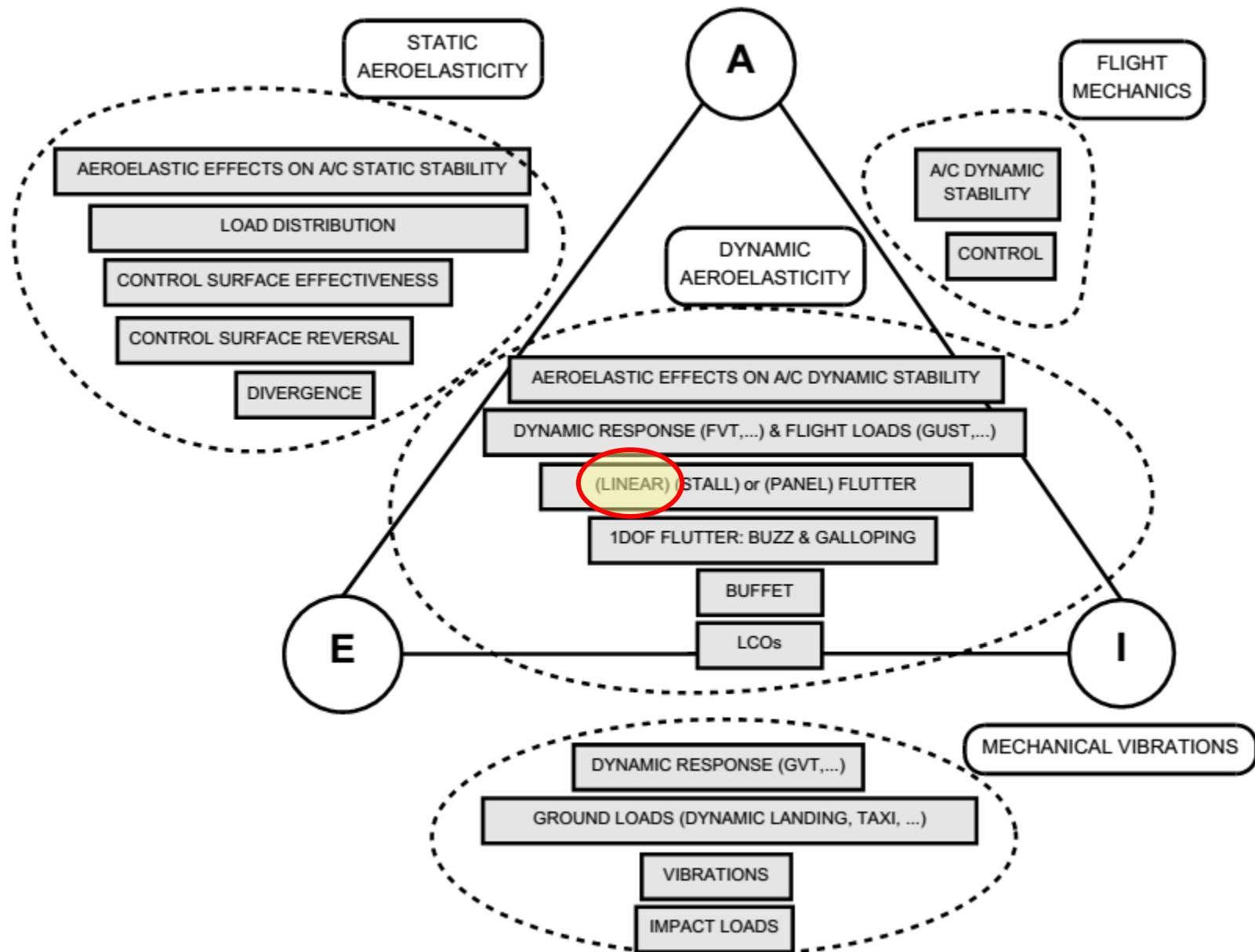
Vibraciones y Aeroelasticidad

Dpto. de Vehículos Aeroespaciales

P. García-Fogeda Núñez & F. Arévalo Lozano

CLASSICAL LINEAR FLUTTER

WHERE WE ARE IN THE COLLAR'S DIAGRAM ?



☐ General →



Video

☐ V-Tail Flutter →



Video

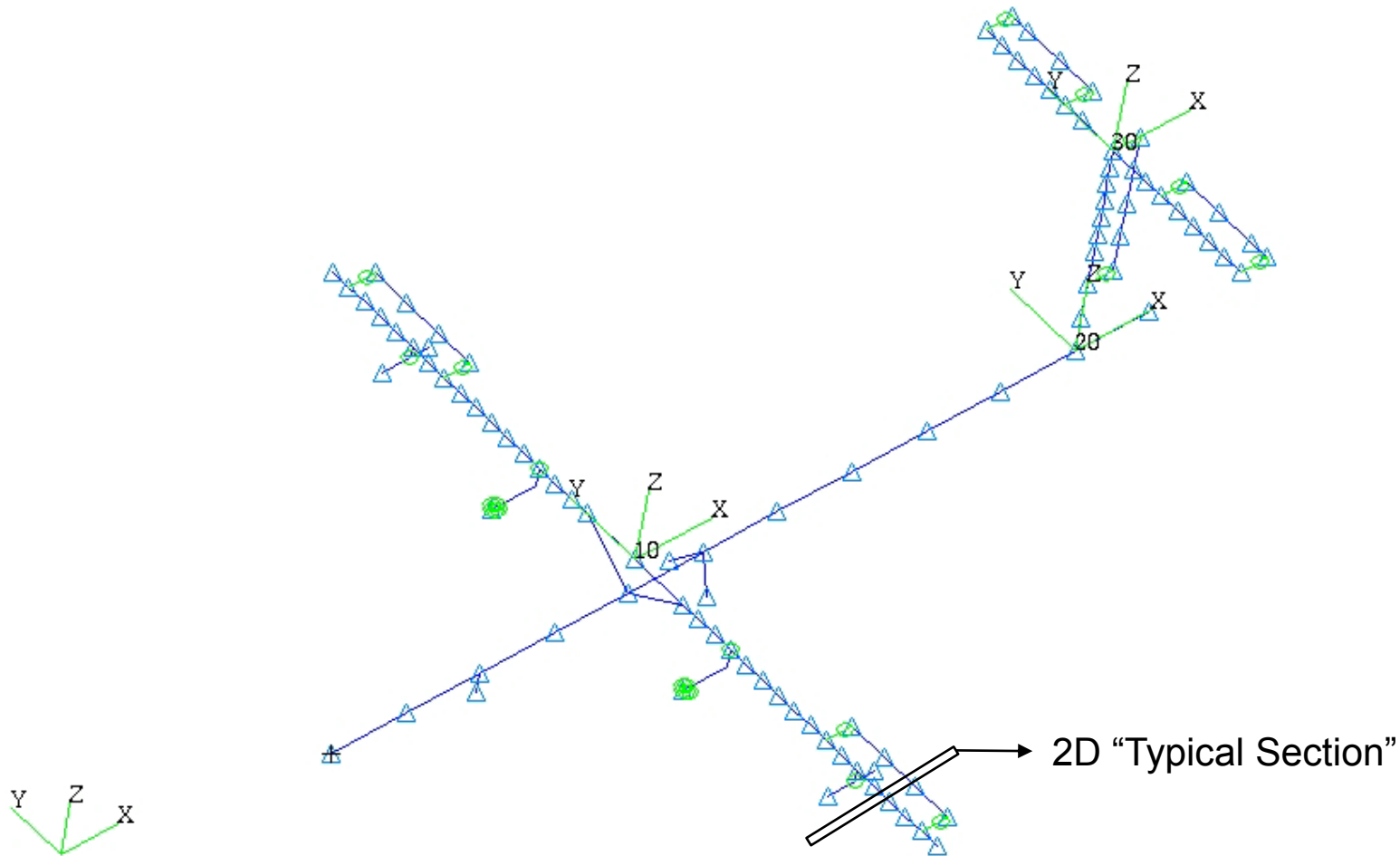
☐ Aeromodel →



Video

NON-DAMPED OSCILLATORY MOTION
of
NORMAL MODES
due to
COUPLING (AEROELASTIC INSTABILITY)

Generic Transport Aircraft (**GTA**) FE Model →



“ It has been said that the development of unsteady airload theory for oscillating wings did more to promote misunderstanding of the flutter phenomenon than any other factor”

(Bisplinghoff, “Principles of Aeroelasticity”)

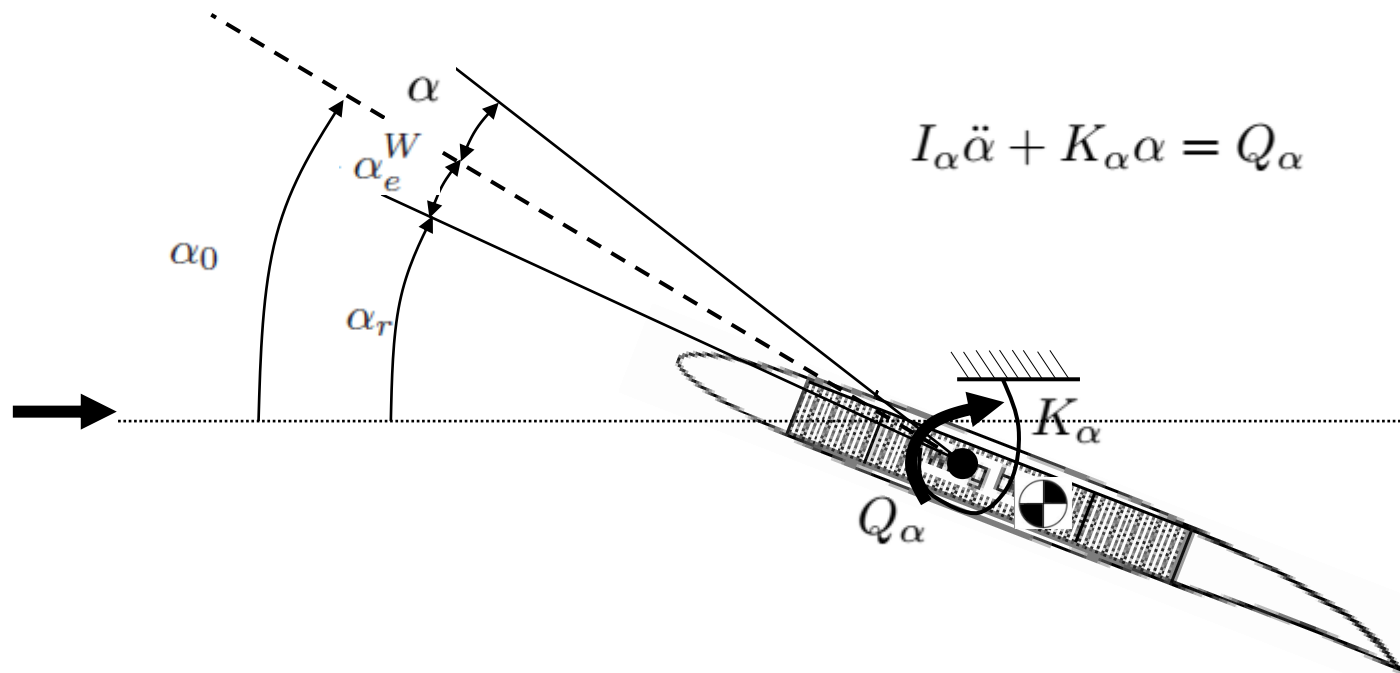


Then, let's start today with a *physical explanation* of the flutter with focus on:

- ✓ UNSTEADY AERODYNAMICS
- ✓ COALESCENCE OF MODES
- ✓ CONTROL SURFACE FLUTTER

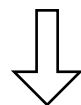
□ Let's start with ...

- ✓ 1DOF: Pure rotation around the elastic axis
- ✓ Let's assume we know the expression of the aerodynamic moment (next slide)
- ✓ Let's play with the equations !



$$I_{\alpha} \ddot{\alpha} + K_{\alpha} \alpha = Q_{\alpha}$$

$$\left[I_{\alpha} - \frac{\partial Q_{\alpha}}{\partial \ddot{\alpha}} \right] \ddot{\alpha} - \frac{\partial Q_{\alpha}}{\partial \dot{\alpha}} \dot{\alpha} + \left[K_{\alpha} - \frac{\partial Q_{\alpha}}{\partial \alpha} \right] \alpha = 0$$



$$a_0 p^2 + a_1 p + a_2 = 0$$

$$a_2 \leq 0 \left(K_{\alpha} \leq \frac{\partial Q_{\alpha}}{\partial \alpha} \right) \Rightarrow \text{Torsional divergence}$$

$$a_1 \leq 0 \left(\frac{\partial Q_{\alpha}}{\partial \dot{\alpha}} \geq 0 \right) \Rightarrow \text{Dynamic instability of } \textit{Flutter}$$



Theodore Theodorsen, the NACA 's Norwegian import, complained that many *LMAL engineers were weak in mathematics.*



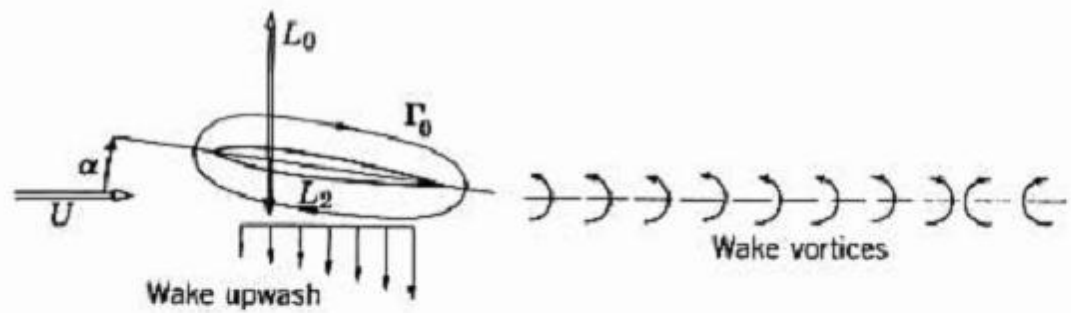
Notes: photo from <http://history.nasa.gov/SP-4305/ch4.htm>

PHYSICAL EXPLANATION OF FLUTTER

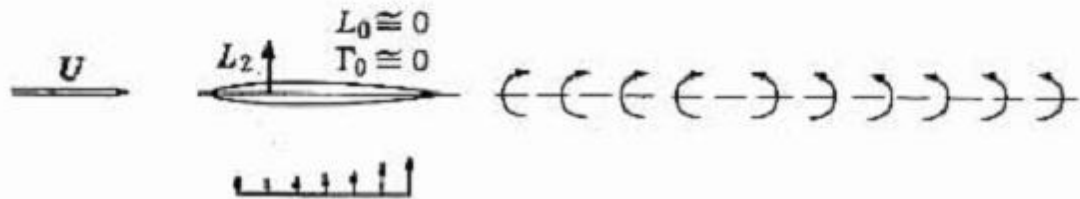
Unsteady
Aerodynamics – The
influence of the wake

□ For this example ...

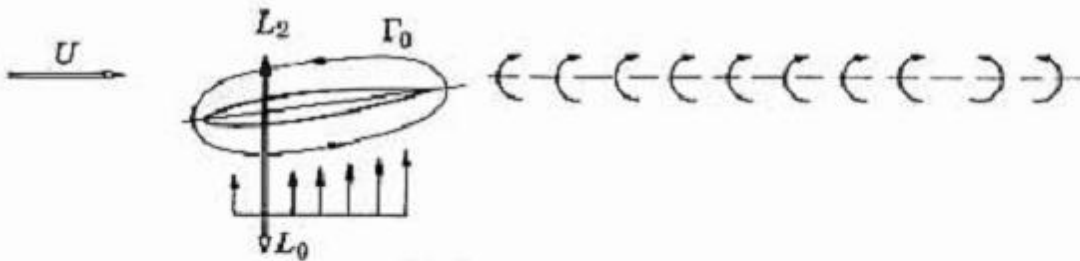
- ✓ Rotational axis ahead of $\frac{1}{4}$ -chord line
- ✓ Reduced frequency k less than 0.0435



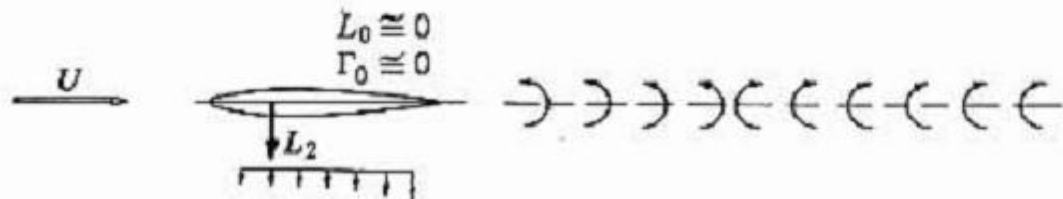
(a) Maximum positive α



(b) $\alpha = 0$, $\dot{\alpha} < 0$



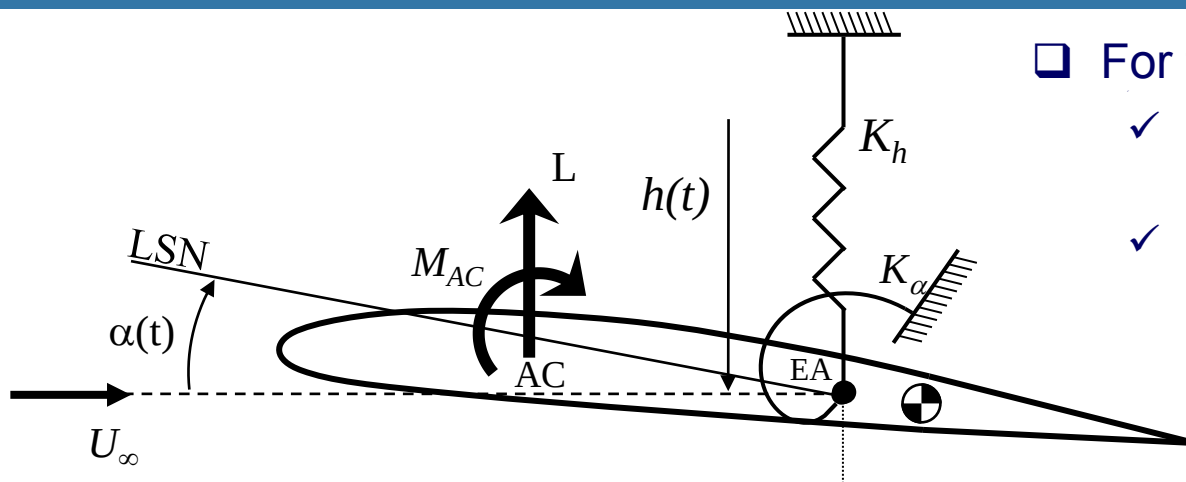
(c) Maximum negative α



(d) $\alpha = 0$, $\dot{\alpha} > 0$

PHYSICAL EXPLANATION OF FLUTTER – COALESCENCE OF MODES

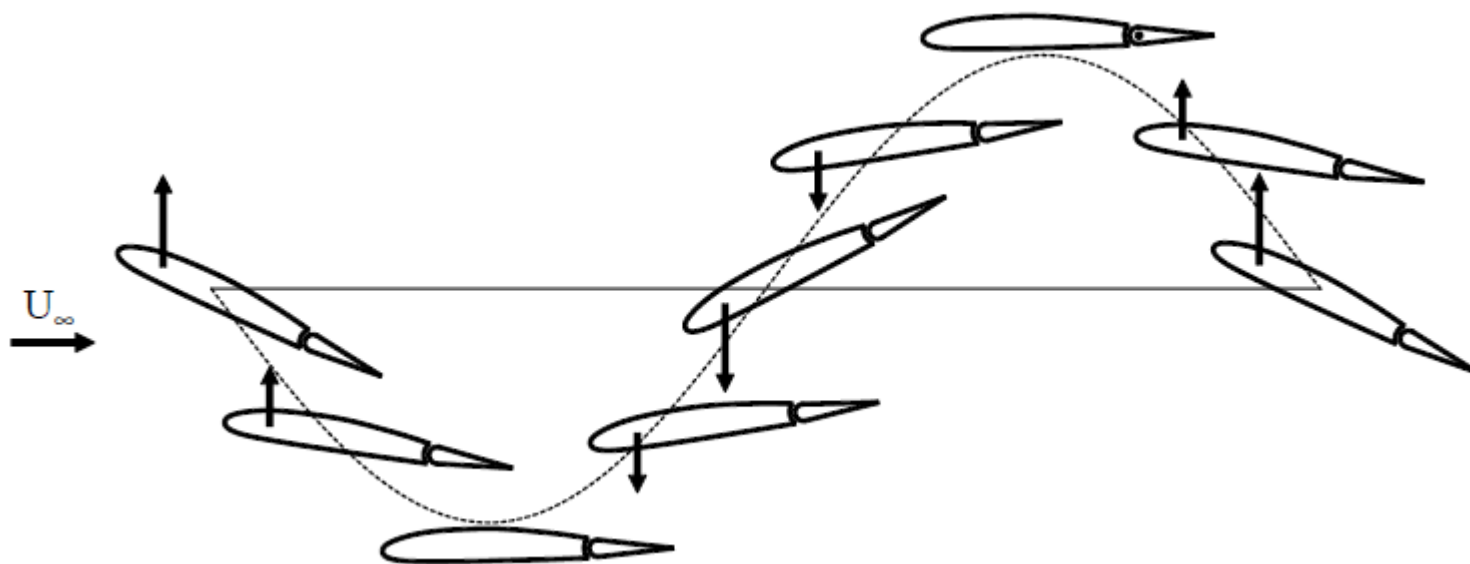
“Typical Section” with heaving (bending) + pitching (torsion)



□ For this example ...

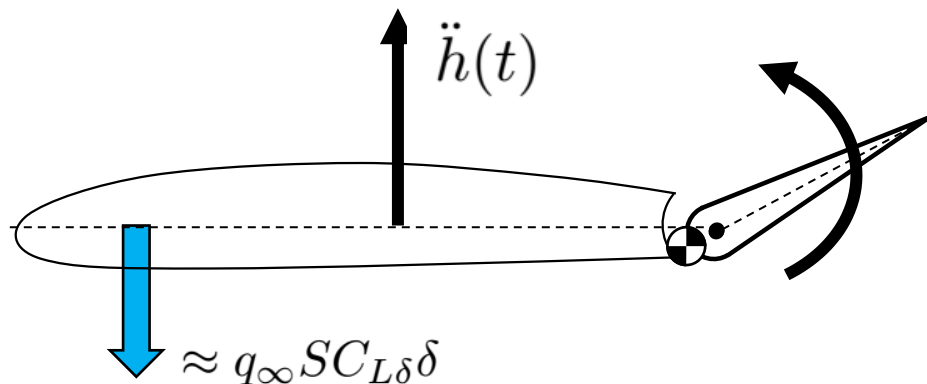
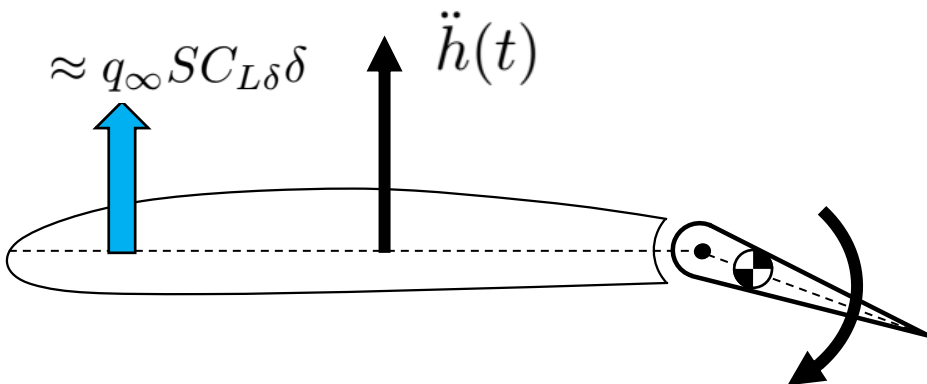
- ✓ Lift is determined by the instantaneous angle of attack
- ✓ Lift acts at the aerodynamic center

$$\alpha_{qs} \approx \alpha + \frac{\dot{h}}{U_\infty}$$

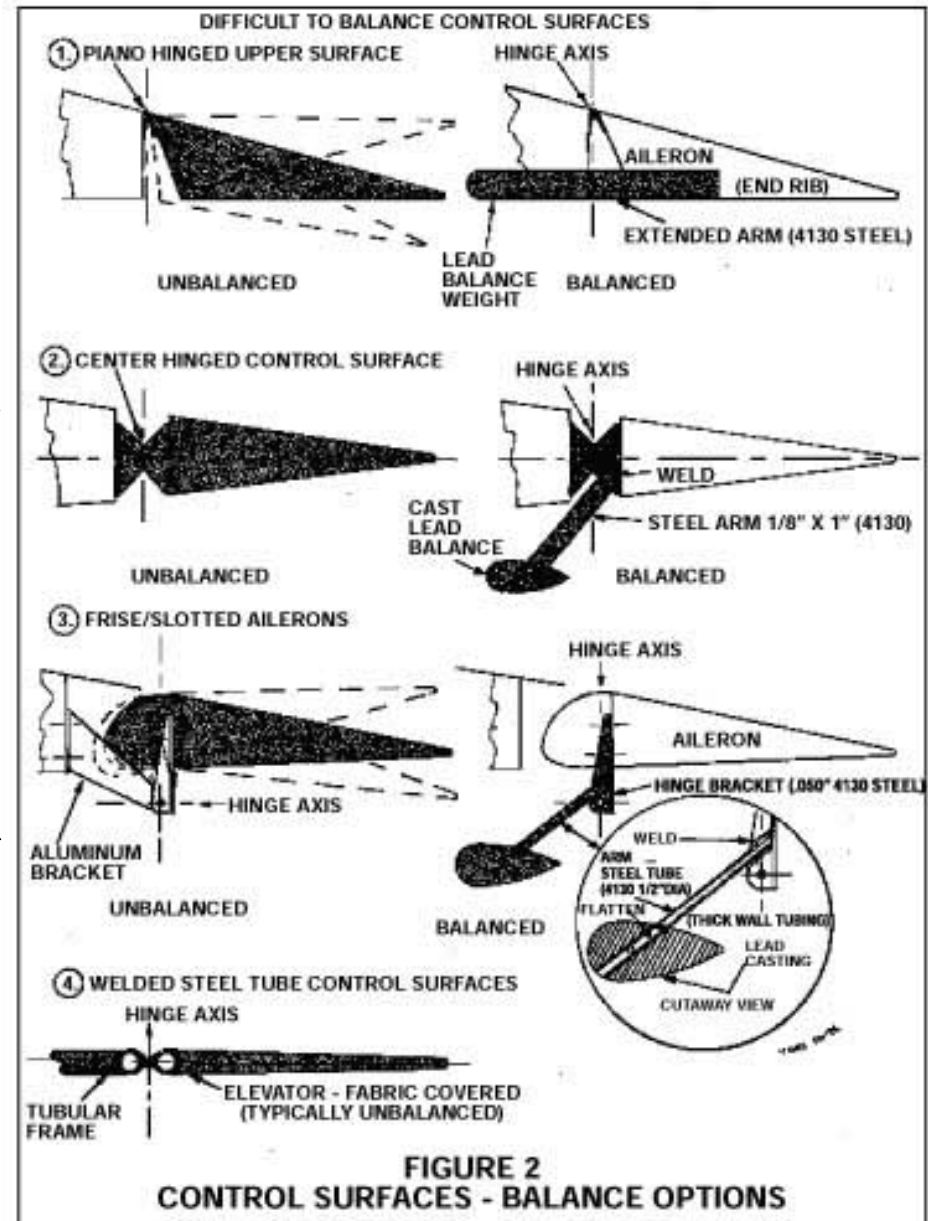


PHYSICAL EXPLANATION OF FLUTTER – CONTROL SURFACE

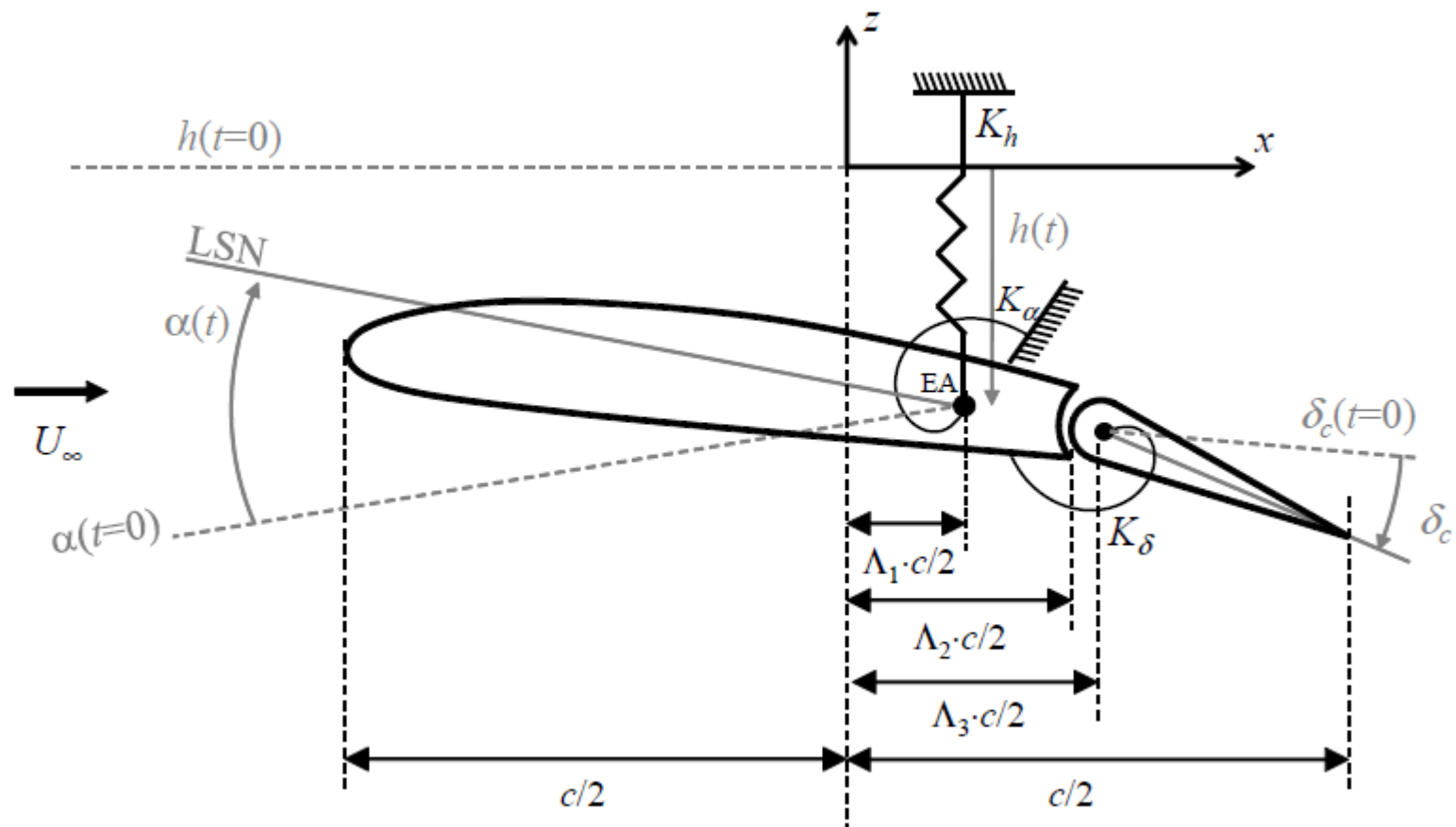
Control Surface deflection coupled with bending / torsion



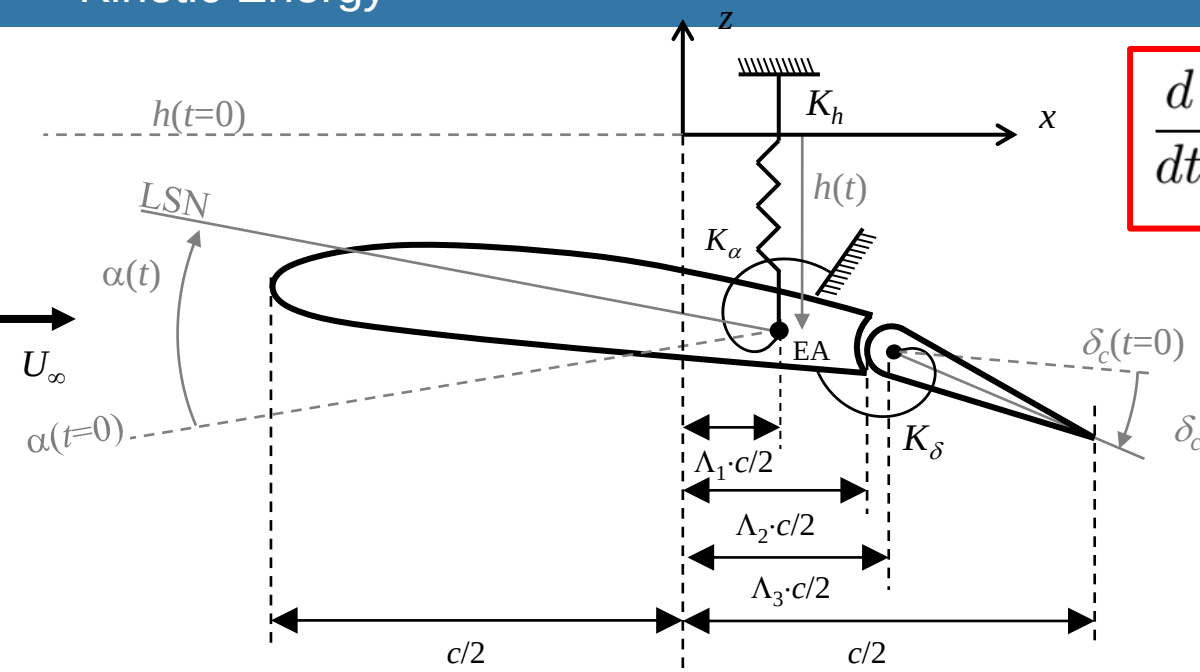
Mass balance $\rightarrow$ move forward the CS CoG



$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) + \frac{\partial U}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = Q_i^A \quad Q_i = \sum_{k=1}^N \frac{\partial \vec{r}_k}{\partial q_i} \cdot \vec{F}_k$$



Kinetic Energy



$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_j} \right) + \frac{\partial U}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} = Q_j$$

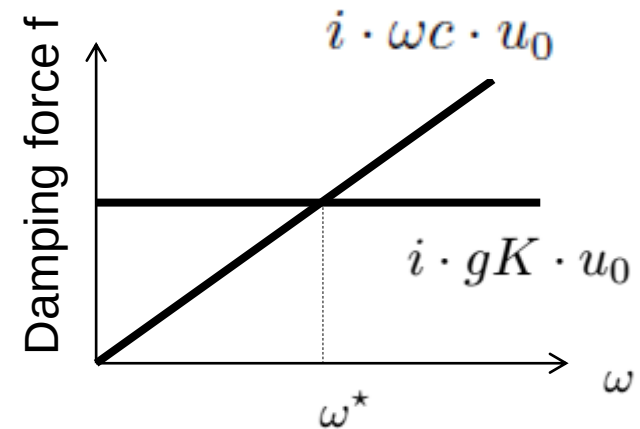
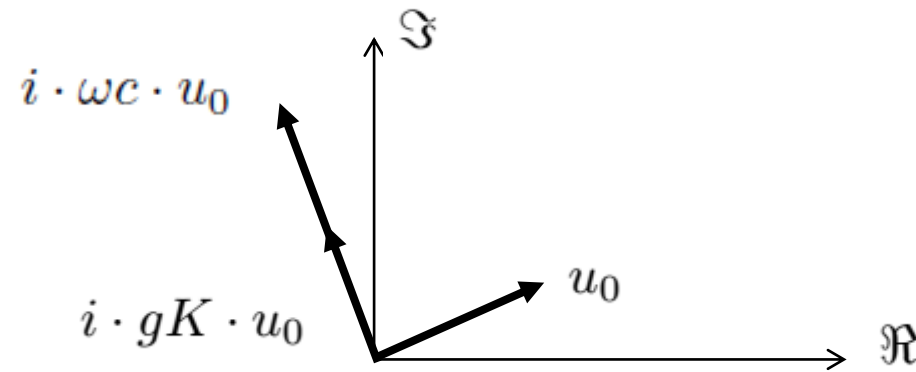
$$T = \left[\frac{1}{2} M \dot{h}^2 + \frac{1}{2} I_\alpha \dot{\alpha}^2 + \frac{1}{2} I_\delta \dot{\delta}_c^2 + S_\alpha \dot{h} \dot{\alpha} + S_\delta \dot{h} \dot{\delta}_c + \left(I_\delta + \Lambda_{31} \frac{c}{2} S_\delta \right) \dot{\alpha} \dot{\delta}_c \right]$$

$$I_\delta = \int_{-c/2}^{+c/2} \left(x - \Lambda_3 \frac{c}{2}\right)^2 H\left(x - \Lambda_2 \frac{c}{2}\right) dm \quad S_\delta = \int_{-c/2}^{+c/2} (x - \Lambda_3 c) H\left(x - \Lambda_2 \frac{c}{2}\right) dm$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_j} \right) + \frac{\partial U}{\partial q_j} + \boxed{\frac{\partial D}{\partial \dot{q}_j}} = Q_j$$

❑ **Viscous** damping $\rightarrow$ proportional to the speed $f_v = b\dot{u}$
(Fluid viscosity, dampers,...)

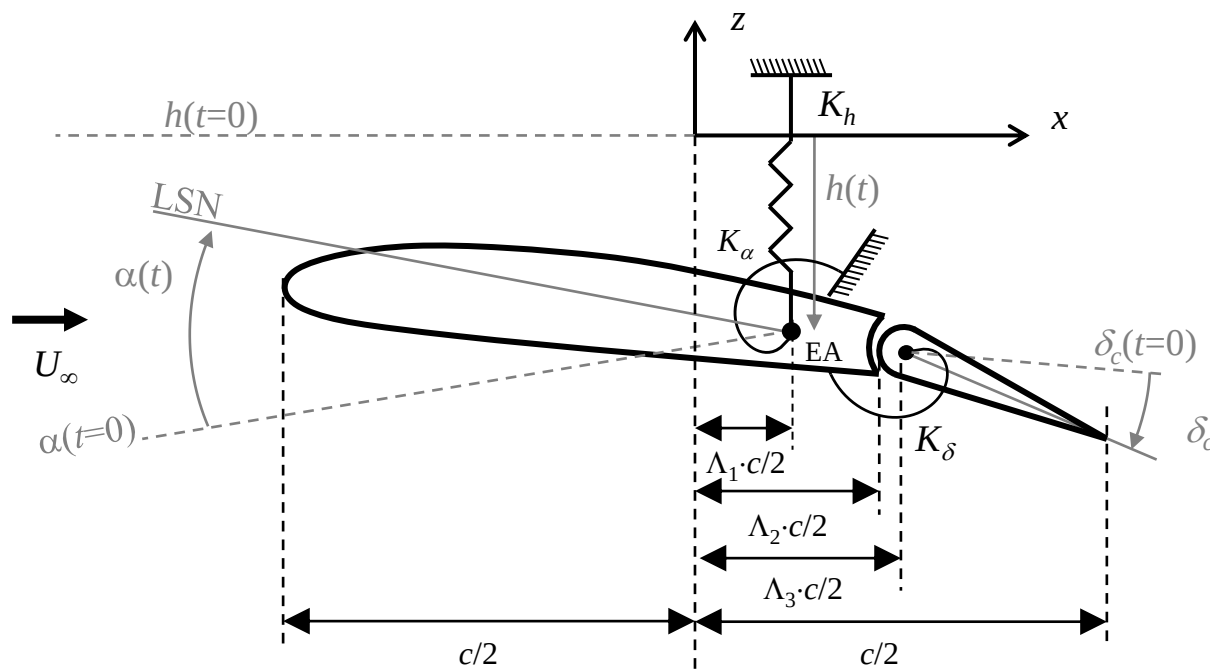
❑ **Structural** damping $\rightarrow$ proportional to the displacement $f_s = i \cdot g \cdot K \cdot u$
(Mechanical friction)



$$D_s = \frac{1}{2} \frac{gK}{\omega} \dot{u}^2 \Rightarrow \frac{\partial D_s}{\partial \dot{u}} = \frac{gK}{\omega} \dot{u} \Rightarrow f_s = i \cdot gK \cdot u_0$$

$$D = \frac{1}{2} \frac{g_h}{\omega} k_h \dot{h}^2 + \frac{1}{2} \frac{g_\alpha}{\omega} k_\alpha \dot{\alpha}^2 + \frac{1}{2} \frac{g_\delta}{\omega} k_\delta \dot{\delta}^2 = \frac{1}{2} \frac{g_h}{\omega} M \omega_h^2 \dot{h}^2 + \frac{1}{2} \frac{g_\alpha}{\omega} I_\alpha \omega_\alpha^2 \dot{\alpha}^2 + \frac{1}{2} \frac{g_\delta}{\omega} I_\delta \omega_\delta^2 \dot{\delta}^2$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_j} \right) + \boxed{\frac{\partial U}{\partial q_j}} + \frac{\partial D}{\partial \dot{q}_j} = Q_j$$



$$U = \frac{1}{2} k_h h^2 + \frac{1}{2} k_\alpha \alpha^2 + \frac{1}{2} k_\delta \delta^2 = \frac{1}{2} M \omega_h^2 h^2 + \frac{1}{2} I_\alpha \omega_\alpha^2 \alpha^2 + \frac{1}{2} I_\delta \omega_\delta^2 \delta^2$$

FORMULATION OF LAGRANGE EQUATIONS

Non-dimensional formulation

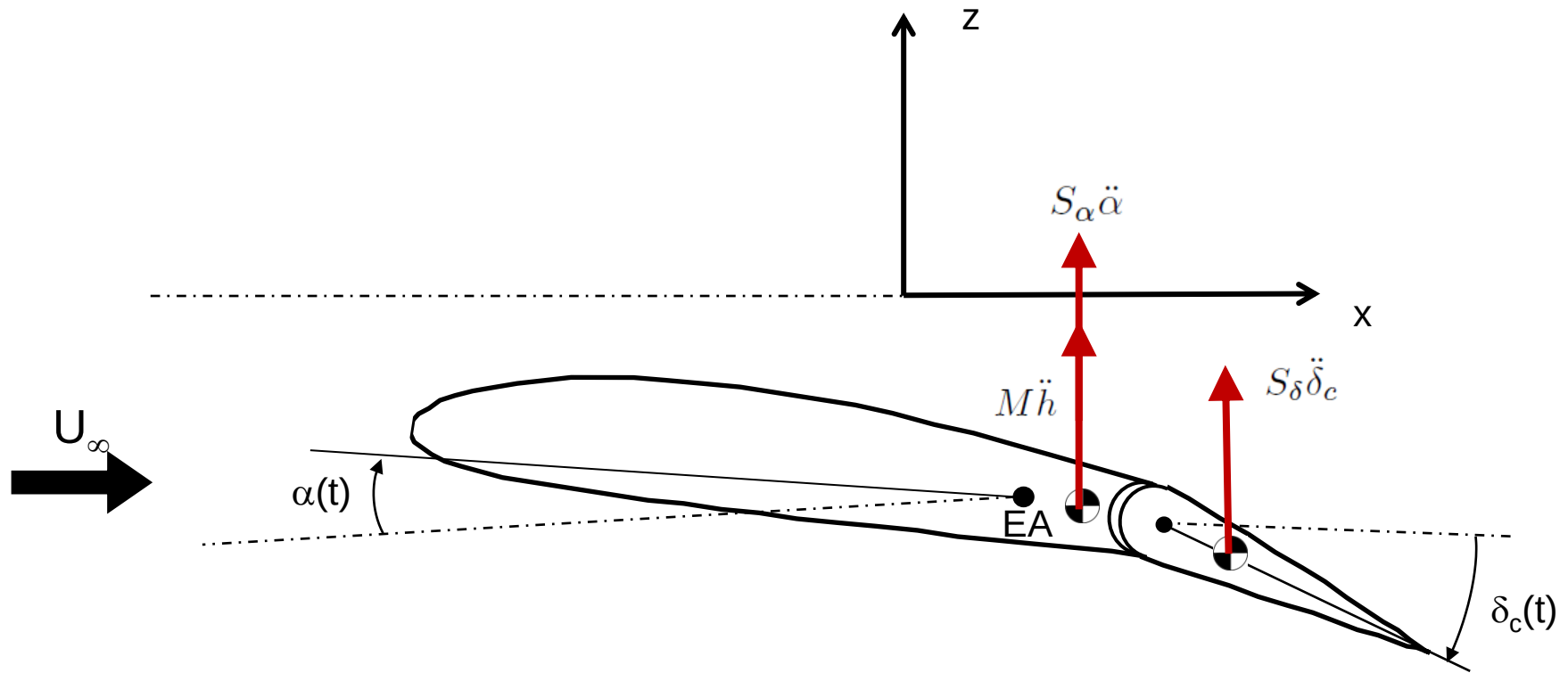


$$\begin{bmatrix} M & S_\alpha & S_\delta \\ S_\alpha & I_\alpha & I_\delta + \Lambda_{31} \frac{c}{2} S_\delta \\ S_\delta & I_\delta + \Lambda_{31} \frac{c}{2} S_\delta & I_\delta \end{bmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \\ \ddot{\delta}_c \end{Bmatrix} + \begin{bmatrix} M\omega_h^2 & 0 & 0 \\ 0 & I_\alpha\omega_\alpha^2 & 0 \\ 0 & 0 & I_\delta\omega_\delta^2 \end{bmatrix} \begin{Bmatrix} h \\ \alpha \\ \delta_c \end{Bmatrix} + \frac{1}{\omega} \begin{bmatrix} g_h M\omega_h^2 & 0 & 0 \\ 0 & g_\alpha I_\alpha\omega_\alpha^2 & 0 \\ 0 & 0 & g_\delta I_\delta\omega_\delta^2 \end{bmatrix} \begin{Bmatrix} \dot{h} \\ \dot{\alpha} \\ \dot{\delta}_c \end{Bmatrix} = \begin{Bmatrix} Q_h \\ Q_\alpha \\ Q_\delta \end{Bmatrix}, \quad (3.15)$$

$$\begin{bmatrix} 1 & x_\alpha & x_\delta \\ x_\alpha & r_\alpha^2 & r_\delta^2 + \Lambda_{31}x_\delta \\ x_\delta & r_\delta^2 + \Lambda_{31}x_\delta & r_\delta^2 \end{bmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \\ \ddot{\delta}_c \end{Bmatrix} + \omega_\alpha^2 \begin{bmatrix} \left(\frac{\omega_h}{\omega_\alpha}\right)^2 & 0 & 0 \\ 0 & r_\alpha^2 & 0 \\ 0 & 0 & r_\delta^2 \left(\frac{\omega_\delta}{\omega_\alpha}\right)^2 \end{bmatrix} \begin{Bmatrix} h \\ \alpha \\ \delta_c \end{Bmatrix} + \frac{\omega_\alpha^2}{\omega} \begin{bmatrix} g_h \left(\frac{\omega_h}{\omega_\alpha}\right)^2 & 0 & 0 \\ 0 & g_\alpha r_\alpha^2 & 0 \\ 0 & 0 & g_\delta r_\delta^2 \left(\frac{\omega_\delta}{\omega_\alpha}\right)^2 \end{bmatrix} \begin{Bmatrix} \dot{h} \\ \dot{\alpha} \\ \dot{\delta}_c \end{Bmatrix} = \begin{Bmatrix} \frac{Q_h}{M \left(\frac{c}{2}\right)} \\ \frac{Q_\alpha}{M \left(\frac{c}{2}\right)^2} \\ \frac{Q_\delta}{M \left(\frac{c}{2}\right)^2} \end{Bmatrix}, \quad (3.17)$$

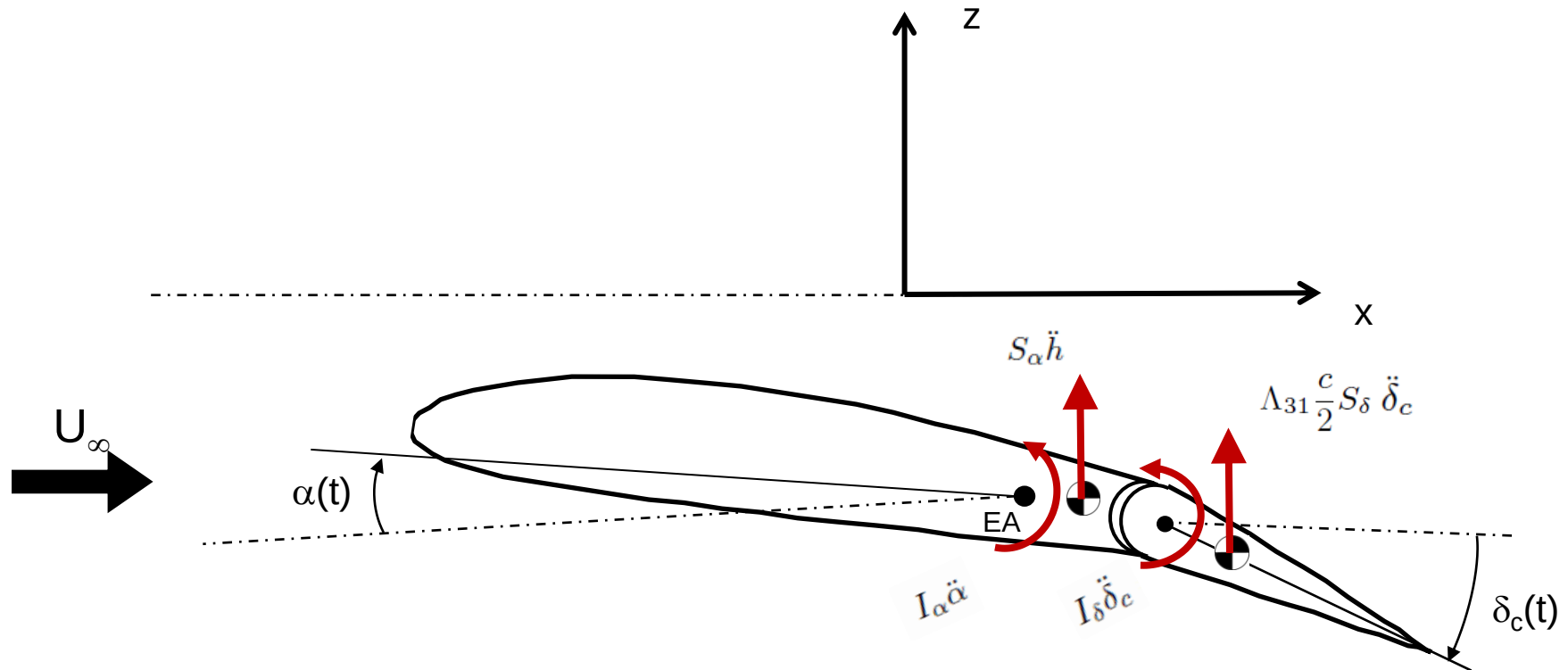
FORMULATION OF LAGRANGE EQUATIONS

h-DOF INERTIA FORCES



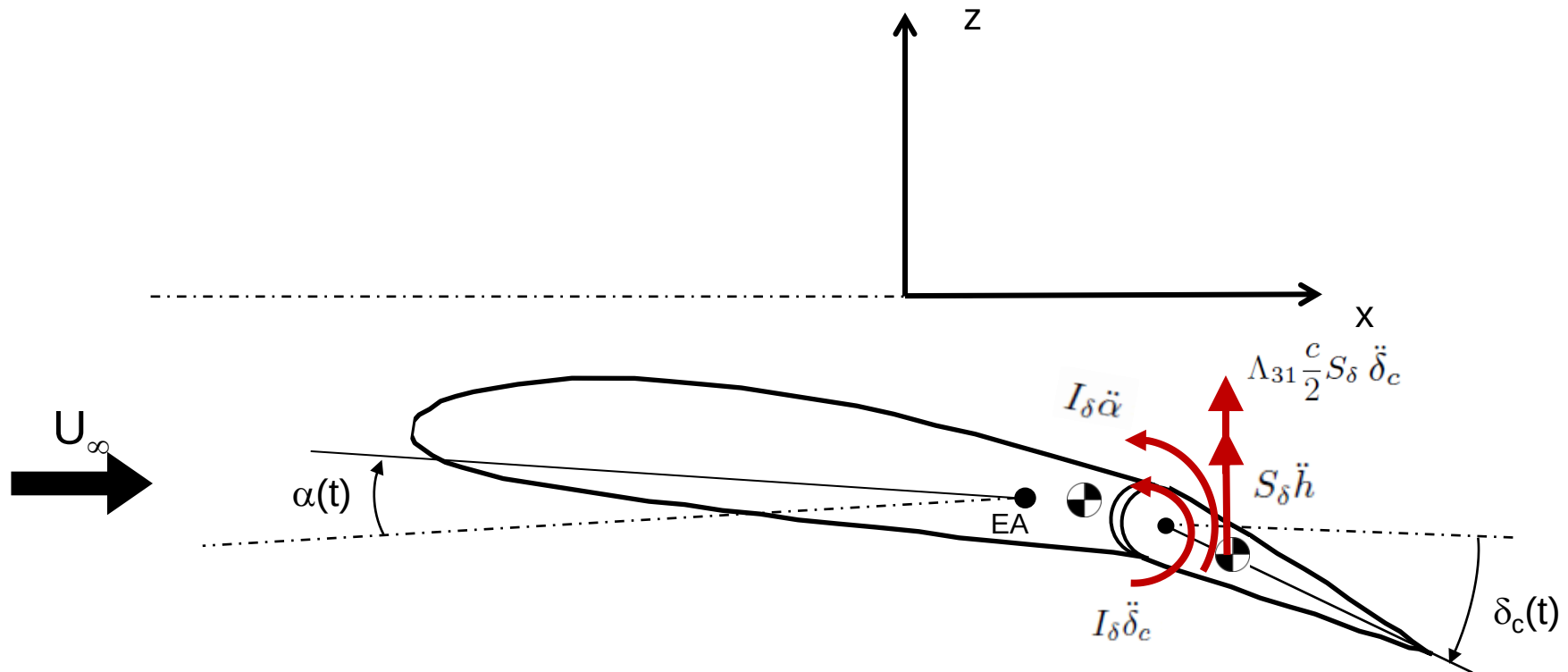
FORMULATION OF LAGRANGE EQUATIONS

α -DOF INERTIA FORCES



FORMULATION OF LAGRANGE EQUATIONS

δ_c -DOF INERTIA FORCES



AEROELASTIC EQUATION IN FREQUENCY-DOMAIN

Harmonic response is assumed



$$\left(-\omega^2 [M_{ij}] + i\omega [F_{ij}] + [K_{ij}] - \frac{\frac{1}{2}\rho_\infty U_\infty^2}{M} [Q_{ij}] \right) \begin{Bmatrix} \frac{h_0}{b} \\ \alpha_0 \\ \delta_0 \end{Bmatrix} = 0$$

$$\left[-[M_{ij}] - \frac{1}{2\pi\mu k^2} [Q_{ij}] + \left(\frac{\omega_\alpha}{\omega} \right)^2 (1 + ig_\alpha) [\tilde{K}] \right] \begin{Bmatrix} \frac{h_0}{b} \\ \alpha_0 \\ \delta_0 \end{Bmatrix} = 0$$

$$\left| [\tilde{K}]^{-1} \left([M_{ij}] + \frac{1}{2\pi\mu k^2} [Q_{ij}] \right) - \left(\frac{\omega_\alpha}{\omega} \right)^2 (1 + ig_\alpha) [I] \right| = 0$$

$$k = \frac{\omega b}{U_\infty}$$

$$\mu = \frac{M}{\pi\rho_\infty b^2}$$

Solution Method #01: Set equal to zero both real and imaginary parts to obtain “k” and “ω”



What happens if rigid body modes are involved in the motion ?



Alternative method: invert the mass matrix

Solution Method #02: Vg-METHOD

$$\left| [\tilde{K}]^{-1} \left([M_{ij}] + \frac{1}{2\pi\mu k^2} [Q_{ij}] \right) - \left(\frac{\omega_\alpha}{\omega} \right)^2 (1 + ig) [I] \right| = 0$$

NOTE: The structural damping is substituted by a generic “g” and the problem is converted into an EIGENVALUE PROBLEM with iteration in “k” till “g” matches the structural damping “g_α”

$$\rho_\infty = cte \Rightarrow \mu = cte$$

k

$$[A] = [\tilde{K}]^{-1} \left([M_{ij}] + \frac{1}{2\pi\mu k^2} [Q_{ij}] \right)$$

$$|[A] - \lambda [I]| = 0$$

$$\omega_m = \frac{\omega_\alpha}{\sqrt{\Re(\lambda_m)}}$$

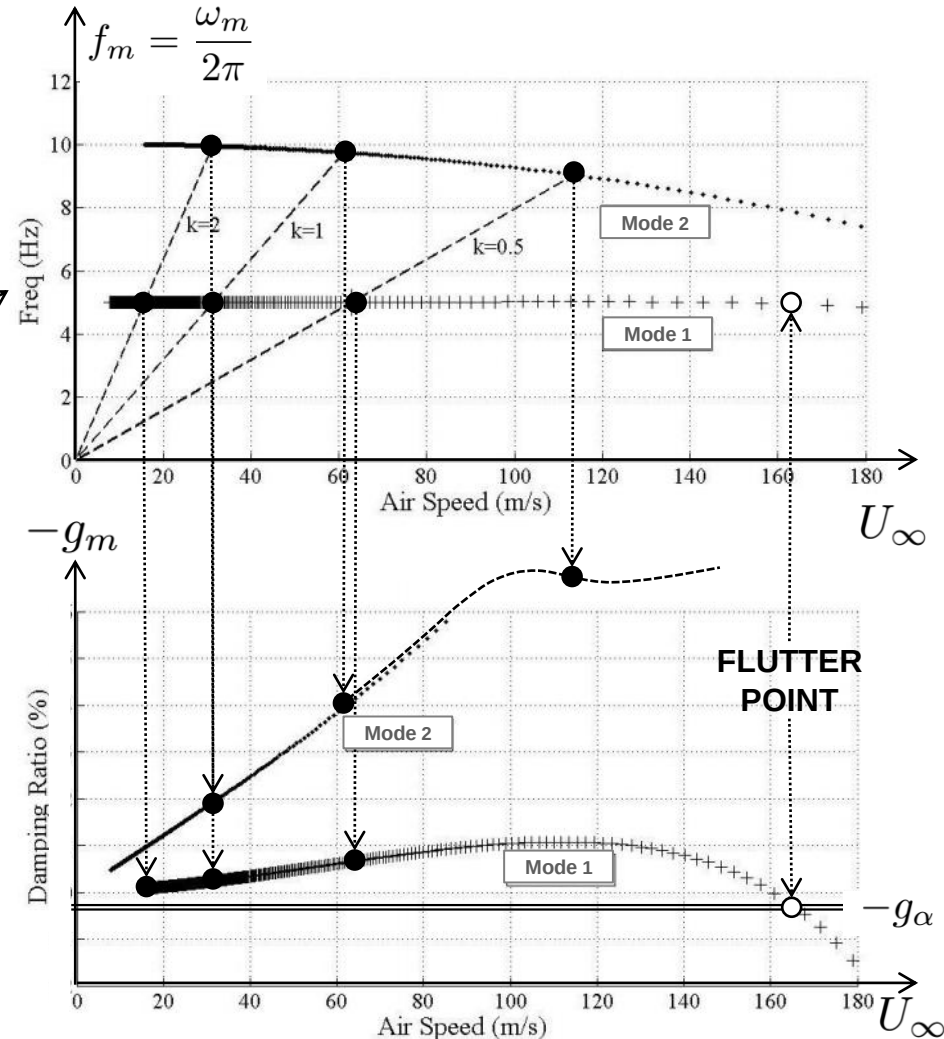
$$U_{\infty(m)} = \frac{\omega_m b}{k}$$

$$g_m = \frac{\Im(\lambda_m)}{\Re(\lambda_m)}$$

$g_m < g_\alpha \rightarrow$ NO FLUTTER

$g_m = g_\alpha \rightarrow$ FLUTTER

$g_m > g_\alpha \rightarrow$ DIVERGENT

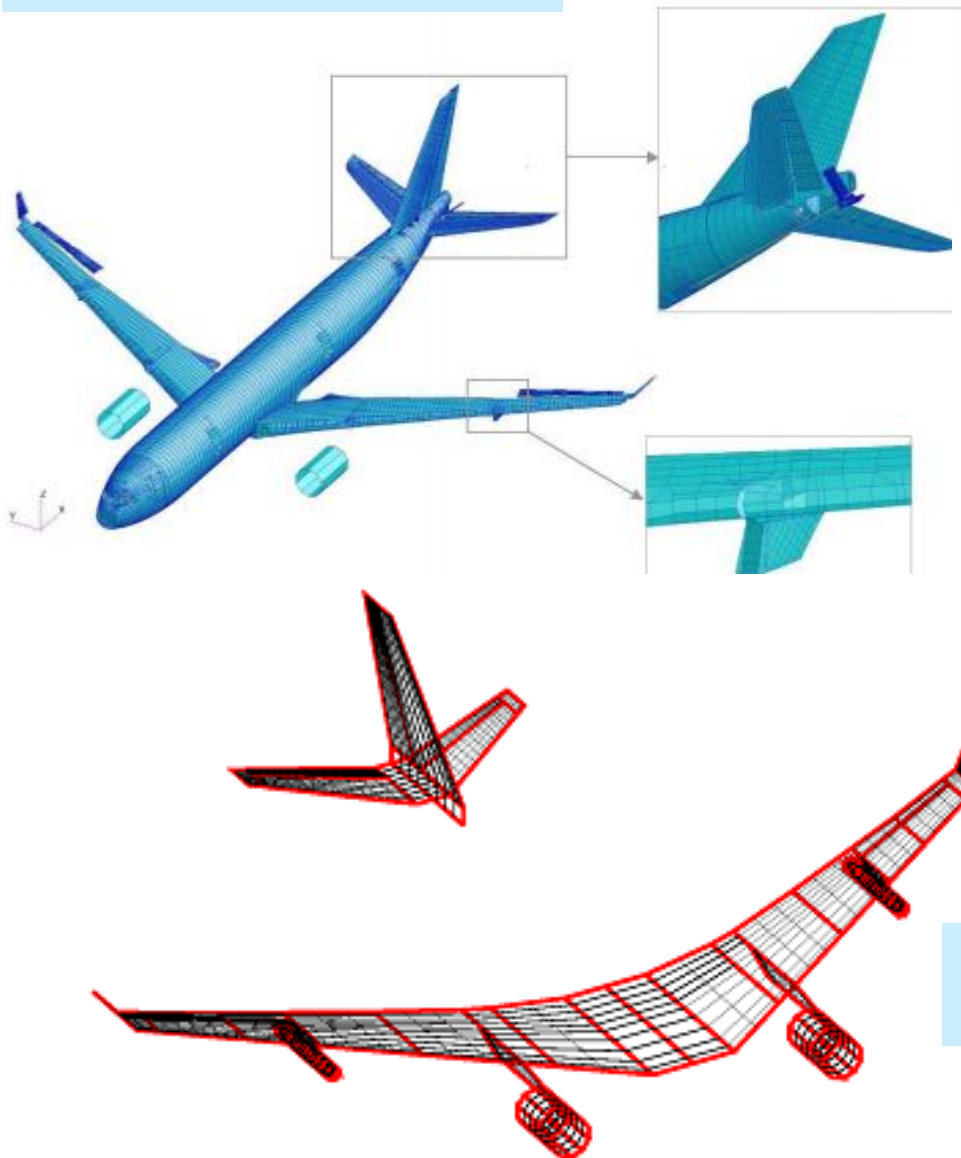


... AND THE REAL SITUATION : AIRCRAFT DESIGN

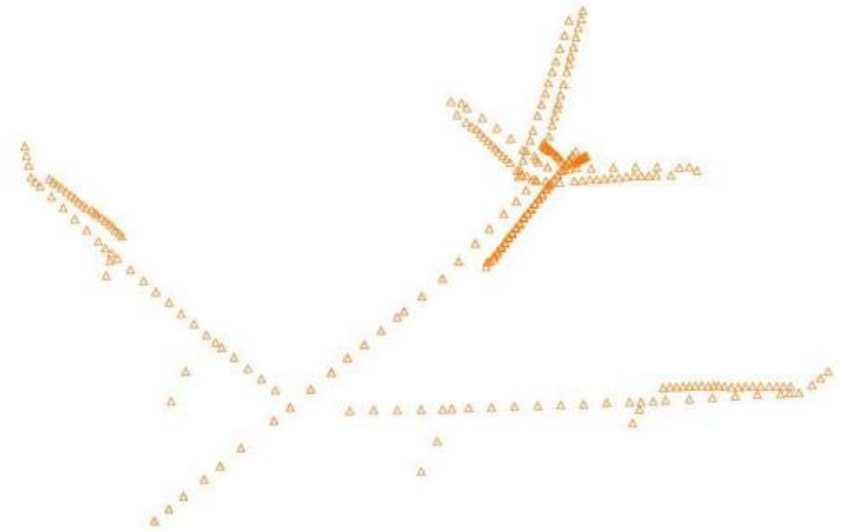
AEROELASTIC MODEL OF THE A330-MRTT TANKER AIRCRAFT



FINITE ELEMENT MODEL



LUMPED MASS MODEL

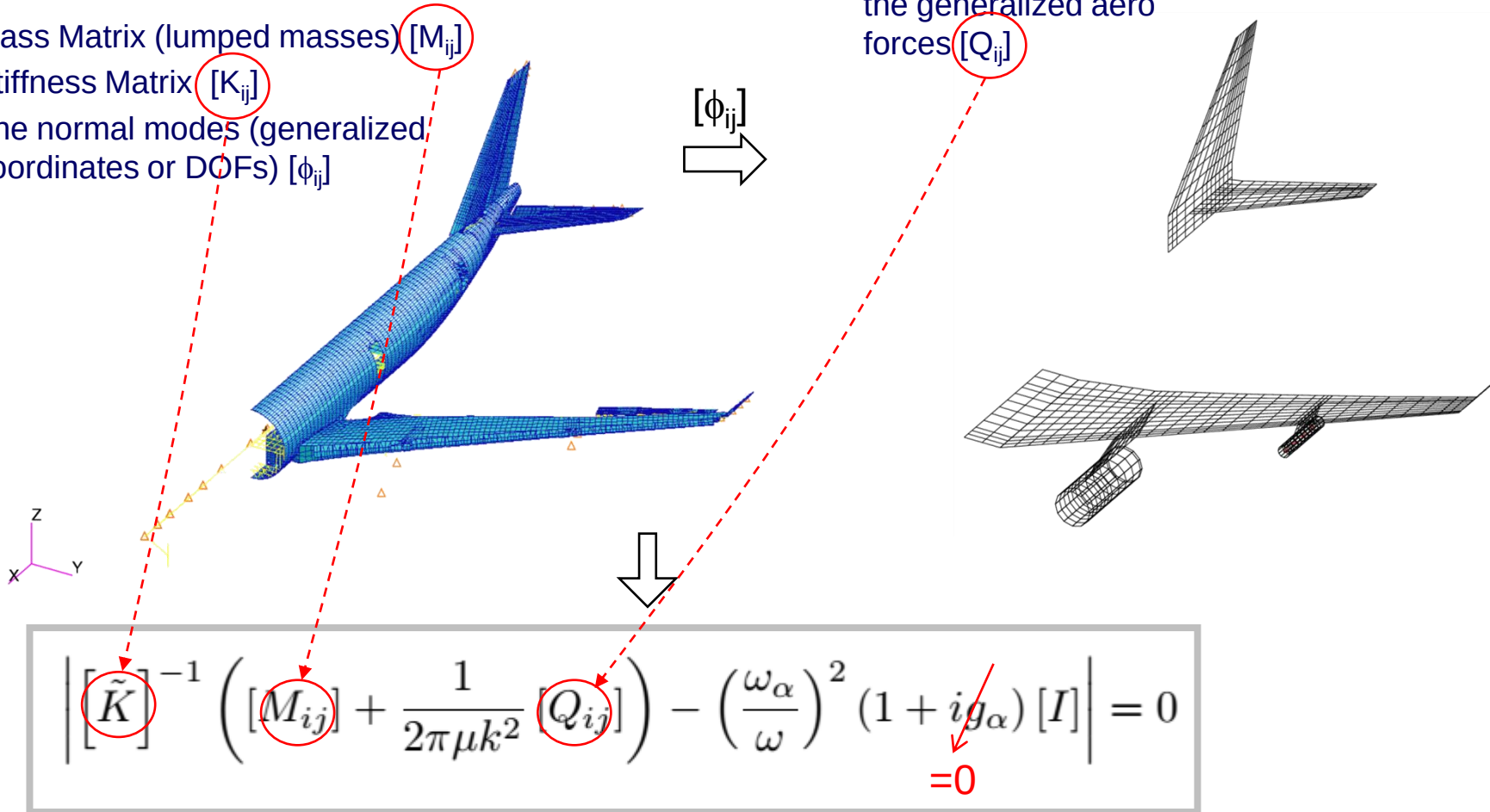


UNSTEADY SUBSONIC AERODYNAMICS (DOUBLET-LATTICE METHOD)

Finite Element Model to obtain

1. Mass Matrix (lumped masses) $[M_{ij}]$
2. Stiffness Matrix $[K_{ij}]$
3. The normal modes (generalized coordinates or DOFs) $[\phi_{ij}]$

Aerodynamic Doublet-Lattice Model to obtain the generalized aero forces $[Q_{ij}]$



Notes: see videos of flutter calculations

... AND THE REAL SITUATION : AIRCRAFT DESIGN

FLUTTER EQUATION AND VG METHOD



$$[M_{aa}] \{\ddot{u}_a\} + [B_{aa}] \{\dot{u}_a\} + [K_{aa}] \{u_a\} = \{Q_a^A(u_a, \dot{u}_a, \ddot{u}_a)\} + \{Q_a^E(t)\}$$

$$\downarrow$$

$$\{Q_a^A(u_a, \dot{u}_a, \ddot{u}_a)\} = [G_{ka}]^T [Q_{kk}] [G_{ka}] [\phi_a] [u_h]$$

$$[\phi_a]^T [M_{aa}] [\phi_a] \{\ddot{u}_h\} + [\phi_a]^T [B_{aa}] [\phi_a] \{\dot{u}_h\} + [\phi_a]^T [K_{aa}] [\phi_a] \{u_h\} =$$

$$= [\phi_a]^T [G_{ka}]^T [Q_{kk}] [G_{ka}] [\phi_a] [u_h] + [\phi_a]^T \{Q_a^E(t)\}$$

$$[M_{hh}] \{\ddot{u}_h\} + [B_{hh}] \{\dot{u}_h\} + [K_{hh}] \{u_h\} = \frac{1}{2} \rho_\infty U_\infty^2 [Q_{hh}(k, M_\infty)] [u_h] + \{Q_h^E(t)\}$$

$$-\omega^2 [M_{hh}] \{\tilde{u}_h\} + i\omega [B_{hh}] \{\tilde{u}_h\} + [K_{hh}] \{\tilde{u}_h\} = \frac{1}{2} \rho_\infty U_\infty^2 [Q_{hh}(k, M_\infty)] [\tilde{u}_h]$$

$$\left(-\omega^2 [M_{hh}] + i\omega [B_{hh}] + [K_{hh}] - \frac{1}{2} \rho_\infty U_\infty^2 [Q_{hh}(k, M_\infty)] \right) \{\tilde{u}_h\} = 0$$

$$\left(-\omega^2 [M_{hh}] + i\omega [B_{hh}] + (1 + ig) [K_{hh}] - \frac{1}{2} \rho_\infty U_\infty^2 [Q_{hh}(k, M_\infty)] \right) \{\tilde{u}_h\} = 0$$

$$\left(-\omega^2 [M_{hh}] + (1 + ig) [K_{hh}] - \frac{1}{2} \rho_\infty U_\infty^2 [Q_{hh}(k, M_\infty)] \right) \{\tilde{u}_h\} = 0$$

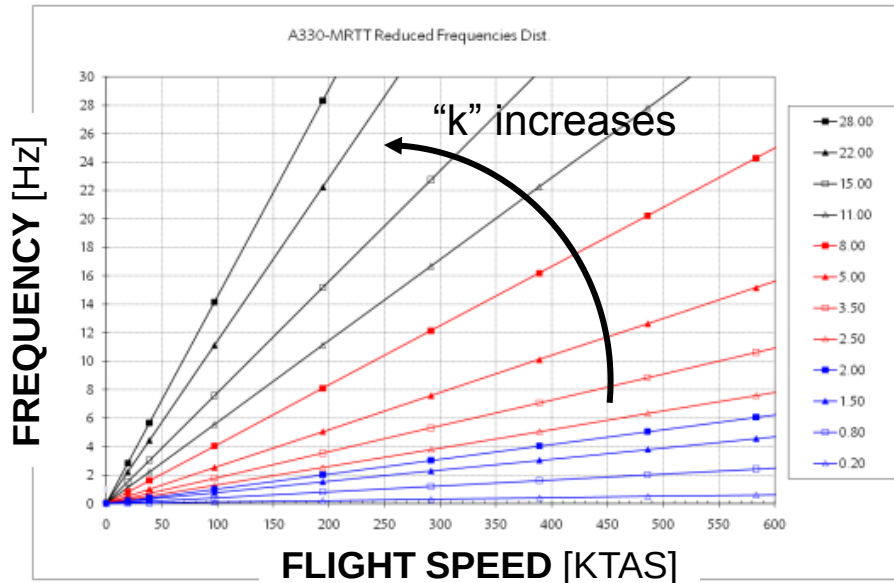


Contrary to the “typical section”, the aerodynamic forces depend on Mach number and the Vg method will require iterations to match density (flight altitude), Mach number, and Flight Speed.

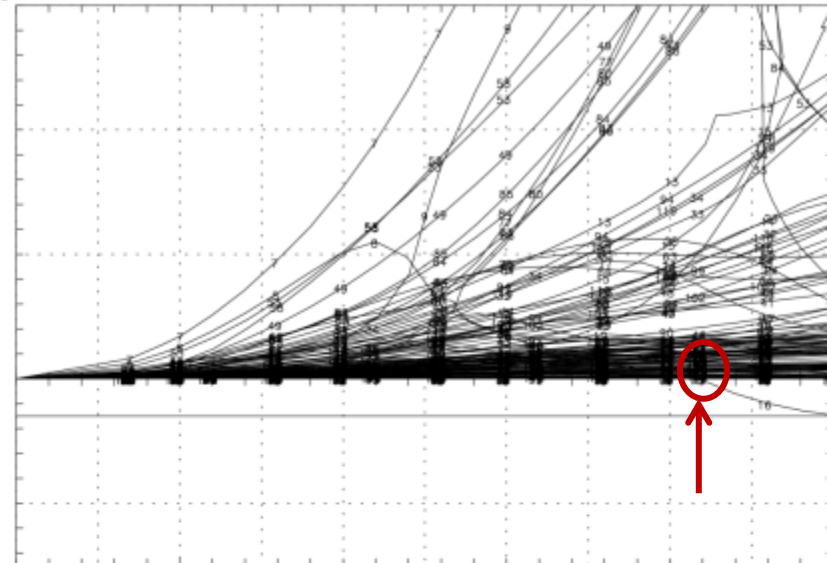
... AND THE REAL SITUATION: FLUTTER EQUATION AND VG METHOD

$$\left| [M_{hh}] + \frac{1}{2} \frac{\rho_{\infty} U^2}{\omega^2} [Q_{hh}(k, M_{\infty})] - \frac{(1 + ig)}{\omega^2} [K_{hh}] \right| = 0$$

$$\left| [M_{hh}] + \frac{\rho_{\infty} b^2}{2k^2} [Q_{hh}(k, M_{\infty})] - \frac{(1 + ig)}{\omega^2} [K_{hh}] \right| = 0$$

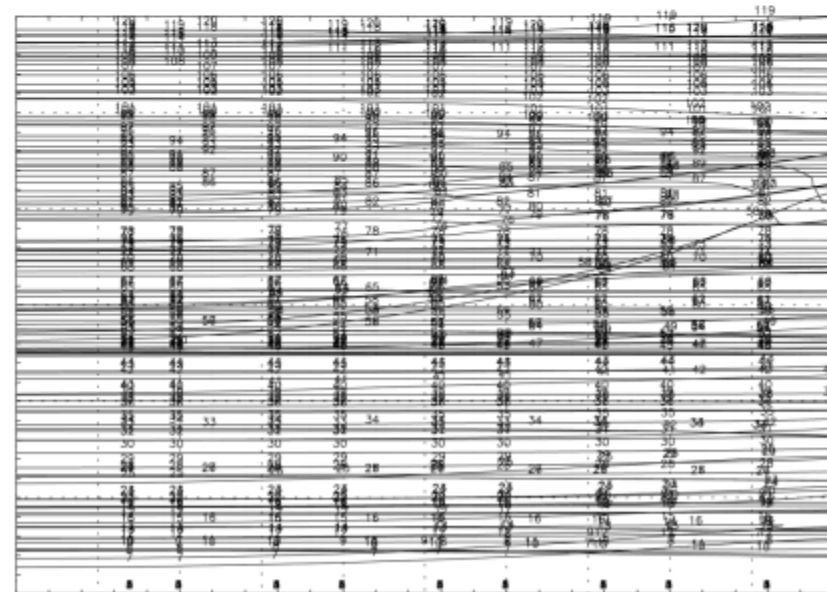


Damping (g)



Speed (Knots EAS)

Frequency (Hz)

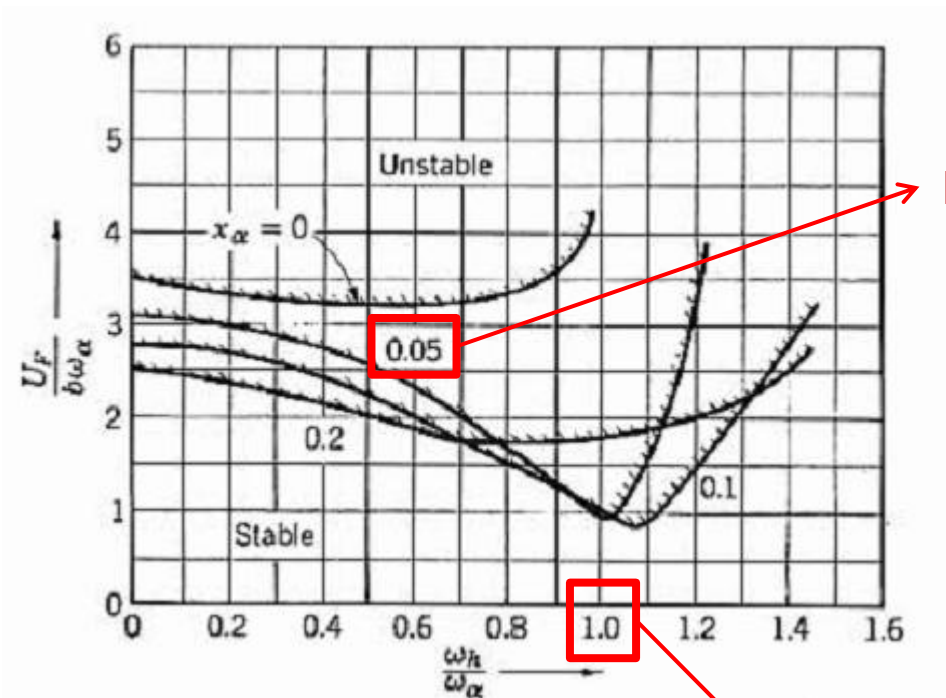


Speed (Knots EAS)

CRS/MOD KEAS HZ (FT) MODAL COMPONENTS IN FLUTTER POINTS CRS: G=0.03 G=0.03

INFLUENCE OF PARAMETERS ON FLUTTER SPEED

RATIO ω_h/ω_α



Moderate aft CG location

Ratio bending/torsion around 1.0

 Assuming incompressible flow

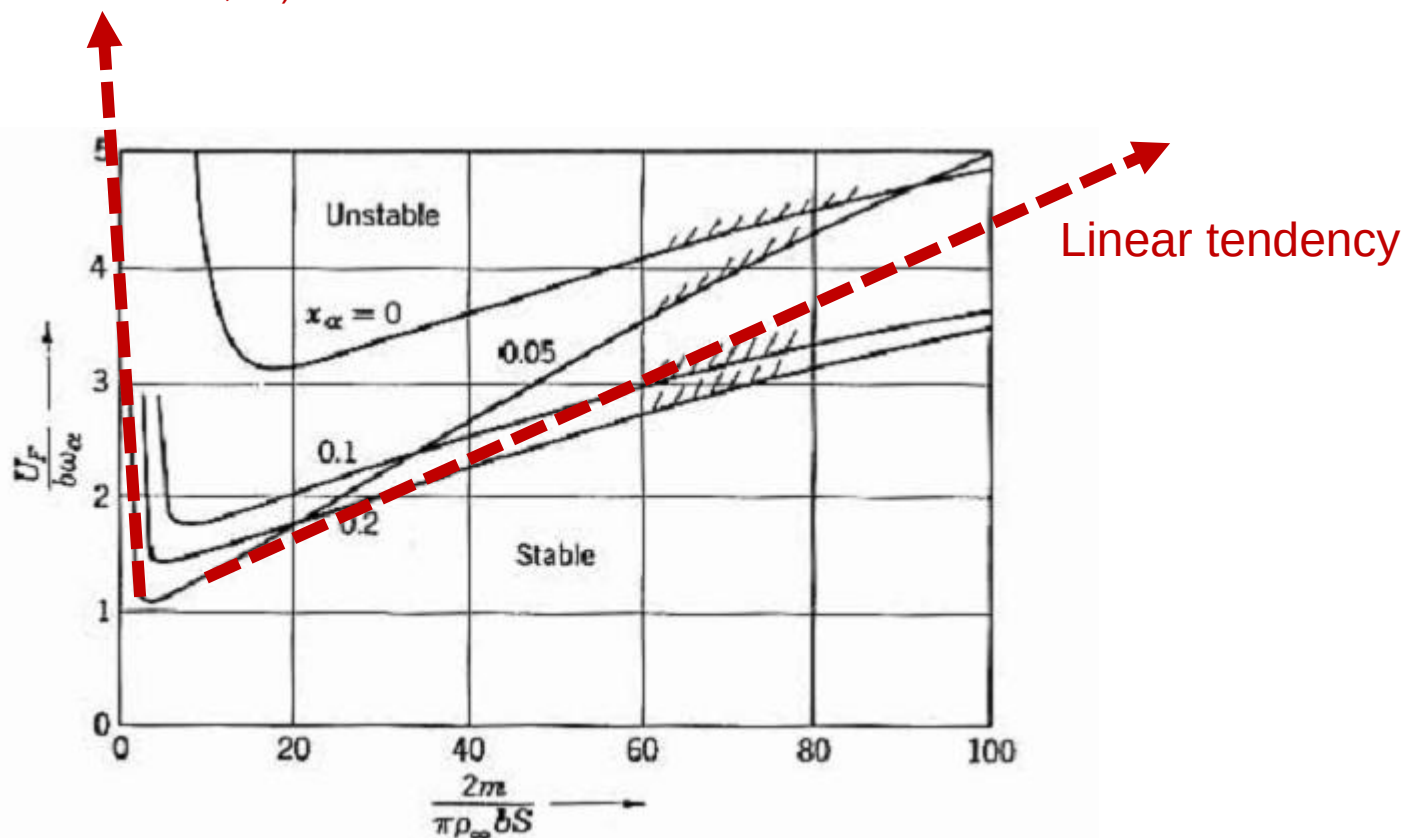
INFLUENCE OF PARAMETERS ON FLUTTER SPEED

Mass parameter μ



Dense medium

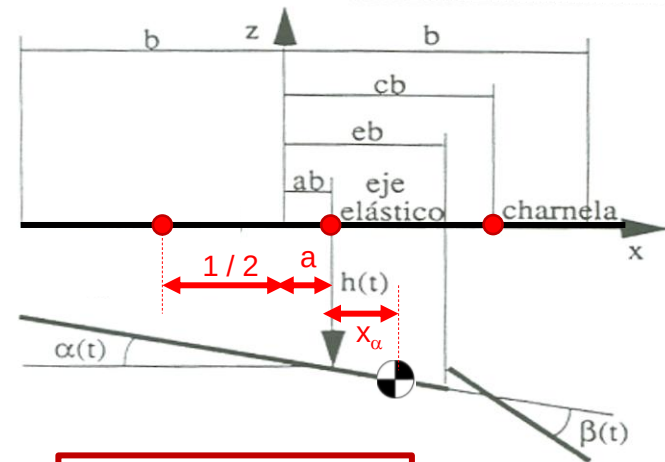
(Light personal airplanes at low altitude, submerged lifting surfaces on high-speed ships and submarines, ...)



Assuming incompressible flow

INFLUENCE OF PARAMETERS ON FLUTTER SPEED

Distance CoG to AC : $\frac{1}{2} + a + x_\alpha$



$$\omega_h/\omega_\alpha = 0$$

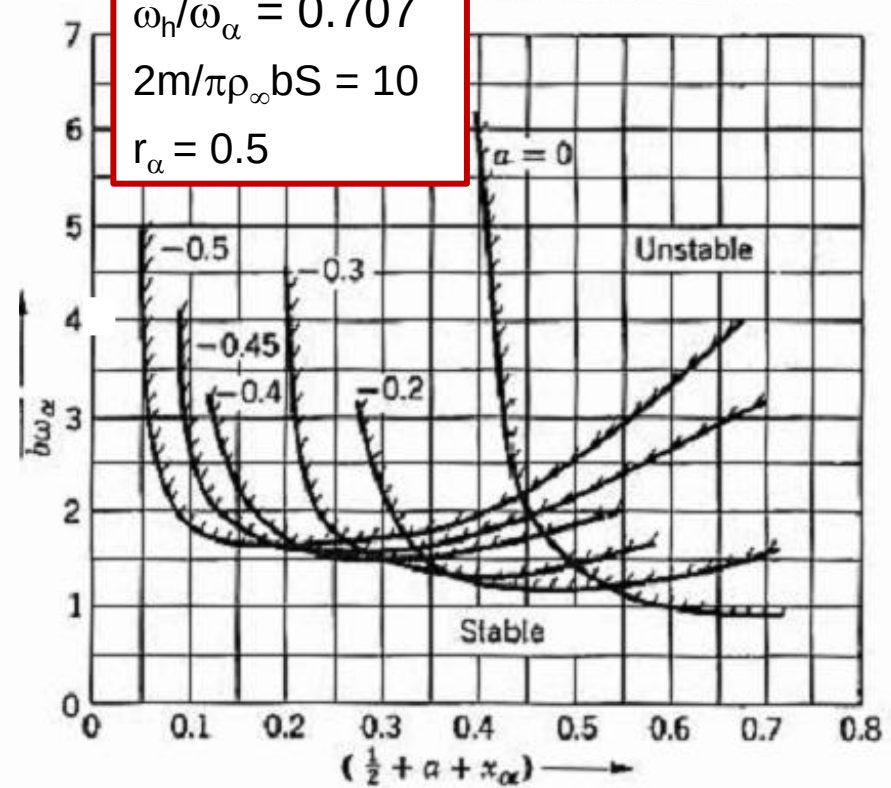
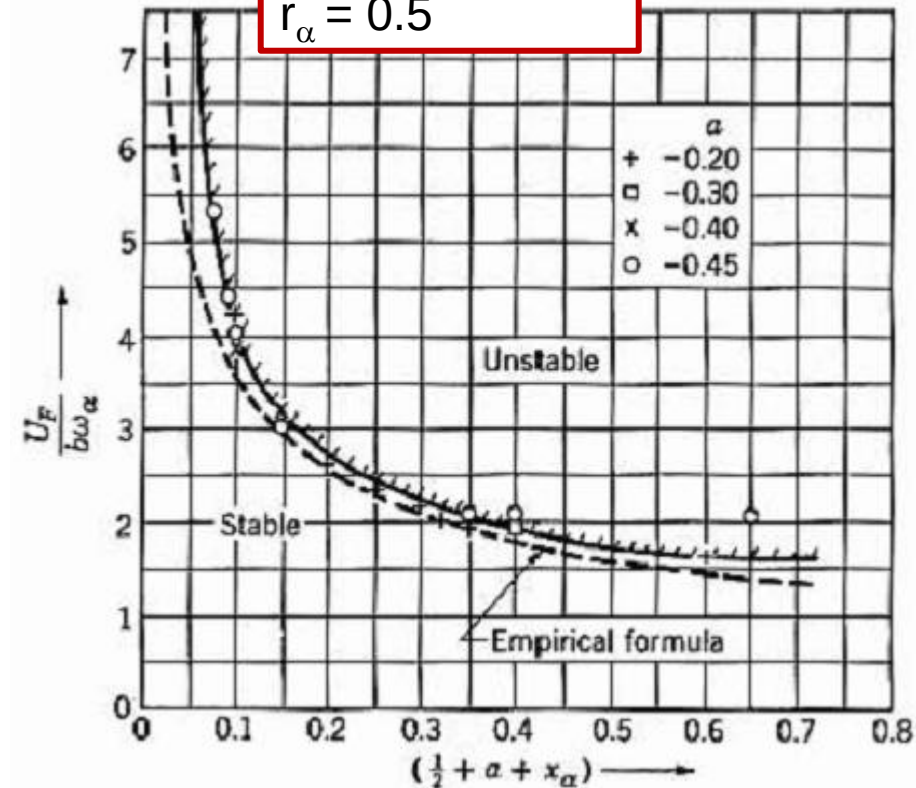
$$2m/\pi\rho_\infty bS = 10$$

$$r_\alpha = 0.5$$

$$\omega_h/\omega_\alpha = 0.707$$

$$2m/\pi\rho_\infty bS = 10$$

$$r_\alpha = 0.5$$



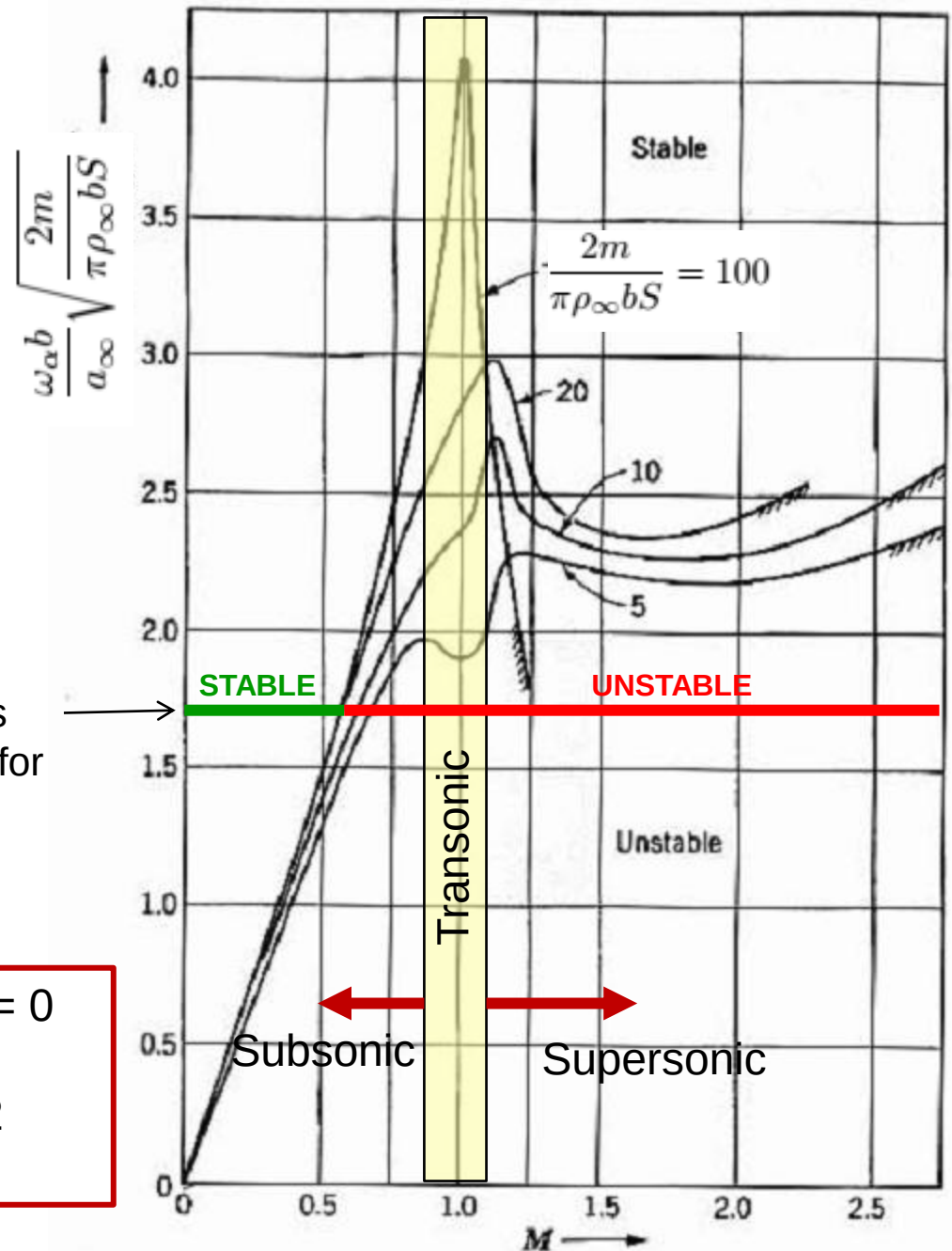
Principle of mass balancing: a CoG ahead of the Elastic Axis (EA) suppresses flutter completely, at least for the forward EA positions ($a < 0$)

INFLUENCE OF PARAMETERS ON FLUTTER SPEED

Effect of Mach number

Flight at a fixed ambient state corresponds to a certain horizontal straight line; no flutter is expected if this line falls entirely within the stable zone for all Mach numbers of interest

$$\begin{aligned} \omega_h/\omega_\alpha &= 0 \\ a &= 0 \\ x_\alpha &= 0.2 \\ r_\alpha &= 0.5 \end{aligned}$$



Next weeks... the wild side of the Aeroelasticity:

UNSTEADY AERODYNAMICS

“Escuela Técnica Superior de Ingeniería Aeronáutica y del Espacio”

UNIVERSIDAD POLITÉCNICA DE MADRID