Compound Interest Law

The final capital obtained at the end of every period can be determined by means of the capital available at the beginning of the period

$$\begin{cases} C_1 = C_0 + C_0 i = C_0 (1+i) \\ C_2 = C_1 + C_1 i = C_1 (1+i) = C_0 (1+i)^2 \\ C_3 = C_2 + C_2 i = C_2 (1+i) \ = C_0 (1+i)^3 \\ C_4 = C_3 + C_3 i = C_3 (1+i) \ = C_0 (1+i)^4 \\ C_n = C_0 (1+i)^n \end{cases}$$

The Final Capital is calculated using the **Compound Interest Law**

$$C_n = C_0 (1+i)^n$$

Effective Interest Rate

$$\begin{cases} C_1 = C_0(1+i) \Rightarrow i = \dfrac{C_1 - C_0}{C_0} \\ C_2 = C_1(1+i) \Rightarrow i = \dfrac{C_2 - C_1}{C_1} \\ \cdots \\ C_n = C_{n-1}(1+i) \Rightarrow i = \dfrac{C_n - C_{n-1}}{C_{n-1}} \end{cases}$$

The EAIR reflects the total Interest produced at the end of every period by every monetary unit available at the beginning of the period

" i **" is called Effective Interest Rate of the financial operation**

The Effective Interest rate is a measurement of :

Profiability: if my financial operation is an investment

Cost: if it is about asking for financing

Total Interest in Compound

$$\left.\begin{aligned} C_{n+1} &= C_n(1+i) \\ C_n &= C_{n-1}(1+i) \end{aligned}\right\} subtract:$$

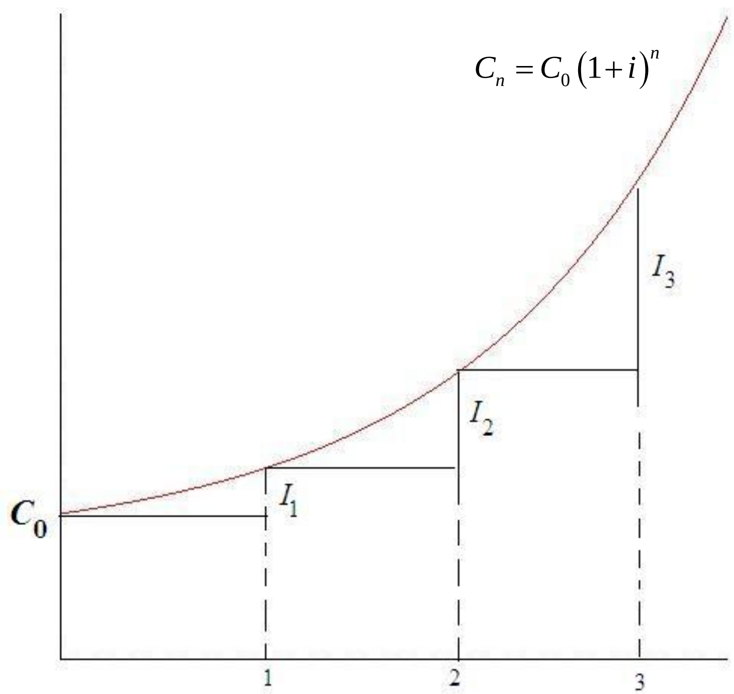
$$C_{n+1} - C_n = C_n(1+i) - C_{n-1}(1+i)$$

$$\underbrace{C_{n+1} - C_n}_{I_{n+1}} = \underbrace{(C_n - C_{n-1})}_{I_n}(1+i) \Rightarrow \boxed{I_{n+1} = I_n(1+i)}$$

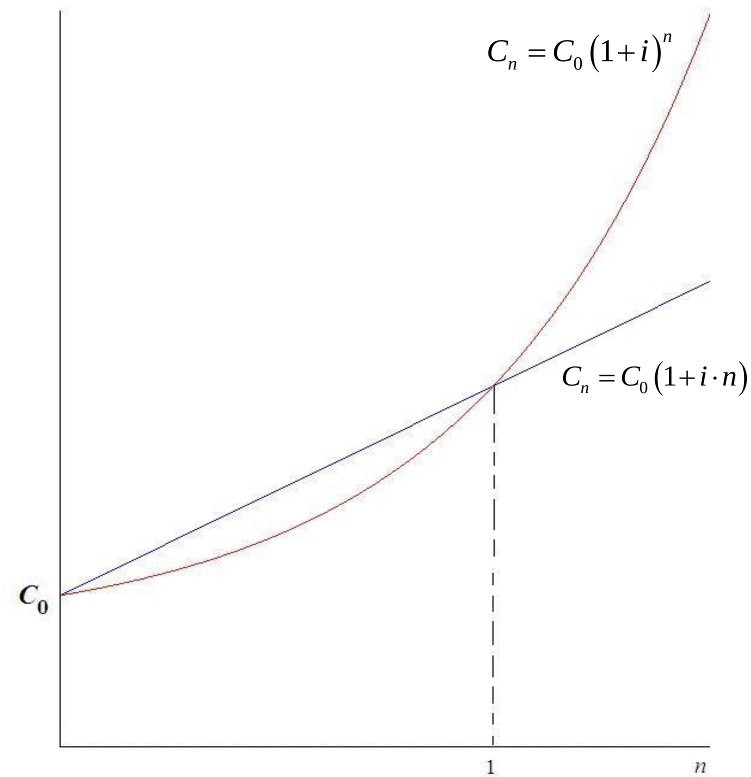
The Interests from each period grow geometrically and the ratio is

$$(1+i)$$

Compound Interest Graph



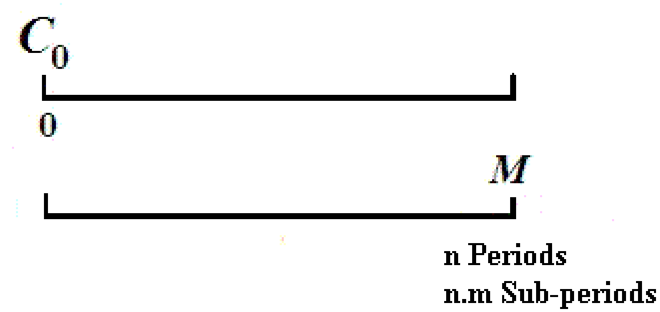
Final value
(Compound Interest)



Comparison of both laws: simple and compound interest law

<u>Equivalent Interest Rates</u>

Two types of interest rates are said to be equivalent or indifferent using whichever chosen. They will produce the same final value, investing the same amount of money for the same period of time



$$i \Rightarrow M = C_0(1+i)^n \qquad i_m \Rightarrow M = C_0(1+i_m)^{m \cdot n}$$

$$i \approx i_m \Rightarrow \cancel{C_0}(1+i)^{\cancel{n}} = \cancel{C_0}(1+i_m)^{m \cdot \cancel{n}} \quad \Rightarrow 1+i = (1+i_m)^m$$

EQUIVALENT INTEREST RATES IN COMPOUND CAPITALIZATION

Equivalent Interest Rates

Two types of interest rates are said to be equivalent or indifferent using whichever chosen. They will produce the same final value, investing the same amount of money for the same period of time

EQUIVALENT INTEREST RATES IN COMPOUNDING

In compound the equivalent interest rates are not related proportionally . The expression which connects both is:

$$(1+i) = (1+i_m)^m \Rightarrow \begin{cases} i = (1+i_m)^m - 1 \\ i_m = (1+i)^{\frac{1}{m}} - 1 \end{cases}$$

ANNUAL PERCENTAGE RATES (APRs) and EFFECTIVE INTEREST RATES (EAIRs)

- Nominal Annual Interest Rate j(m): Indicates that the interest is added to the capital to produce more interest

- m = indicates the number of times that the interest is added to the capital to produce more interest along the year (m ≤ 1).
- For instance m = 2 means that the interest is added to the capital twice a year, that is, at the end of every semester.
-

$$j(m) = m \cdot i_m \qquad i_m = \frac{j(m)}{m}$$

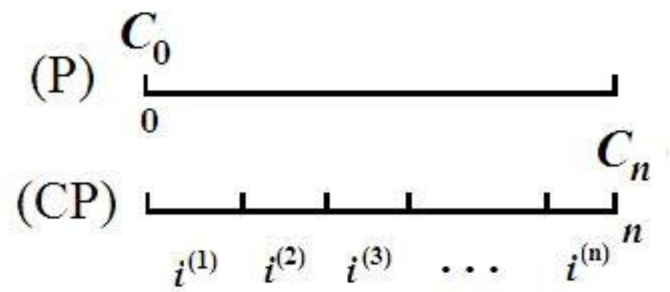
- Example :

A nominal interest rate of $j(2)=0.1$ means that the interest is added to the capital at the end of every semester. For this reason, it is called nominal annual interest rate, compounding six monthly .

EAIR will be calculated this way: $i_2 = \frac{j(2)}{2} = \frac{0,1}{2} = 0,05 = 5\%$

$$i = (1+i_2)^2 - 1 = (1+0,05)^2 - 1 = 0,1025 = 10,25\%$$

VARIBLE INTEREST RATE WITH COMPOUNDING



$i^{(1)}$) Effective Interest rate for the first period

$i^{(2)}$ Effective Interest rate for the second period

i^3 Effective Interest rate for the third period

i^n Effective Interest rate for the n period

$$C_n = C_0\left(1+i^{(1)}\right)\left(1+i^{(2)}\right)\left(1+i^{(3)}\right)\cdots\left(1+i^{(n)}\right)$$