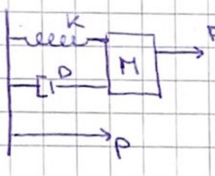
EXERCISES PROCESS AUTOMATION

Equation

① Mass-spring damper ① Linear dinamic syst. $Ma + Dv + Kp = F$



$a = \dot{v}$ $\quad M\dot{v} + Dv + Kp = F$

$v = \dot{p}$

$x_1 = p \quad x_2 = v;$

$u = F \quad y = p$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{K}{M}x_1 - \frac{D}{M}x_2 + \frac{1}{M}u \\ y = x_1 \end{cases}$$

② Compute matrix A of din. syst.

$$A = \begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \end{array}\begin{bmatrix} 0 & 1 \\ -\frac{K}{M} & -\frac{D}{M} \end{bmatrix}$$

③ Discuss stability for $M>0, K\geq 0, D\geq 0$

$$\boxed{\varphi(s) \triangleq \det(sI - A)} = \det\begin{bmatrix} s & -1 \\ \frac{K}{M} & s+\frac{D}{M} \end{bmatrix} = s^2 + \frac{D}{M}s + \frac{K}{M}$$

$\varphi(s) = 0; \quad s^2 + \frac{D}{M}s + \frac{K}{M} = 0; \quad Ms^2 + Ds + K = 0$

$$s = \frac{-D \pm \sqrt{D^2 - 4\cdot M\cdot K}}{2M}$$

→ $K=0; D=0; S=0$ not asymptotically stable

→ $K=0 \quad D>0 \quad \begin{cases} s_1 = 0 \\ s_2 = -\frac{D}{M} \end{cases}$ " " "

→ $K>0; D=0 \quad s = I_j\sqrt{\frac{K}{M}}$

not asymptotically stable

→ $D^2 - 4MK < 0 \Rightarrow \boxed{D < 2\sqrt{KM}}$

→ $K>0 \quad M>0 \Rightarrow D < 2\sqrt{KM}$ Asymptotically stable

$$s = \frac{-D \pm j\sqrt{4KM - D^2}}{2M} = -\frac{D}{2M} \pm j\frac{\sqrt{4KM - D^2}}{2M}$$

$M>0$
$K>0$
$D>0$

→ $K>0, M>0 \quad D = 2\sqrt{KM}$

$s = -\frac{D}{2M}$ Asypt. stable $\quad s = \frac{-D \pm \sqrt{D^2 - 4KM}}{2M}$ Asymp. stabile

④ $M>0, K>0, D>0$, complete equilibrium state corresponding to a constant input force

$u = \bar{u}$ constant $\forall t$

$$\begin{cases} \bar{x}_2 = 0 \\ -\frac{K}{M}\bar{x}_1 - \frac{D}{M}\bar{x}_2 + \frac{1}{M}\bar{u} = 0 \end{cases} \quad \begin{cases} \bar{x}_2 = 0 \\ -\frac{K}{\cancel{M}}\bar{x}_1 - \cancel{\frac{D}{M}\bar{x}_2} + \frac{1}{\cancel{M}}\bar{u} = 0 \end{cases} \quad \begin{cases} \bar{x}_2 = 0 \\ \bar{x}_1 = \frac{\bar{u}}{K} \end{cases}$$

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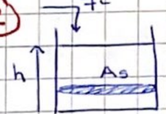
② ① Linear dinamic system (input $u = q_i$, output $y = h$)



$u = q_i \quad x = h$

$y = h$

$\frac{d}{dt}(A_s h) = q_i$, A_s costant → $\dot{h} = \frac{q_i}{A_s}$

$$\begin{cases} \dot{x} = \frac{1}{A_s}u \\ y = x \end{cases}$$

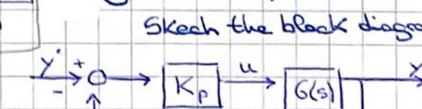
② Compute the transfer function $\quad \dot{x}(t) \rightarrow \underbrace{s\,x(s)}_{x(t)}$

$G(s)$

$$\begin{cases} s\,x(s) = \frac{1}{A_s}\cdot u(s) \\ y(s) = x(s) \end{cases} \rightarrow Y(s) = \frac{1}{A_s\cdot s}U(s)$$

③ Control system with proportional controller

Skech the block diagram feedback



④ Expression of transfer function of closed loop system.

$Y(s) = G(s)\cdot u = G(s)\cdot K_p(y^\circ(s) - y(s))$

$$\left(1 + \frac{K_p}{sA_s}\right)Y(s) = \frac{K_p}{A_s s}y^\circ(s) \rightarrow Y(s) = \frac{K_p/sA_s}{1 + \frac{K_p}{sA_s}}y^\circ = \underbrace{\frac{K_p}{sA_s + K_p}}_{F(s)}y^\circ(s)$$

$A_s > 0 \quad K_p > 0$

closed loop syst.

asympt. stable

③ Consider transfer function of 1^{st} order system $G(s) = \frac{Y(s)}{U(s)} = \frac{\mu}{1+sT}$

$s \longleftrightarrow \frac{d}{dt}$

$(1+sT)Y(s) = \mu\, U(s)$

$Y(s) + T\, s\, Y(s) = \mu\, U(s)$

$T \cdot \dot{y}(t) + y(t) = \mu\, u(t)$

$u(t) - \text{step}(t) = 1, \ \forall t \geq 0 \ , \ y(t)\,?$

$T\dot{y}(t) + y(t) = \mu \quad t \geq 0$

① Compute analytical express. of the output y when the input u is subjected to a step variation

$$\dot{y}(t) = \frac{\mu}{T} e^{-\frac{t}{T}} \xleftarrow{y(0)=0} y(t) = \mu\left(1 - e^{-\frac{t}{T}}\right) \quad t \geq 0$$

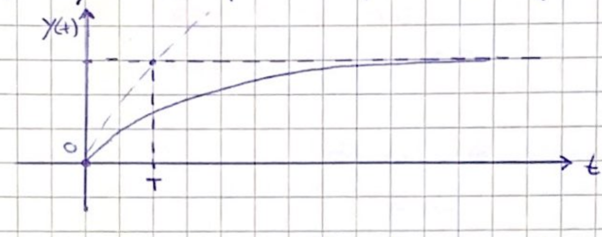
② Sketch the plot of the output y as computed as previus item (assume μ an T ⊕)

$y(0) = 0$

$t \to \infty \to y(t) = \mu$

$t \to \infty \to \dot{y}(t) \to 0$

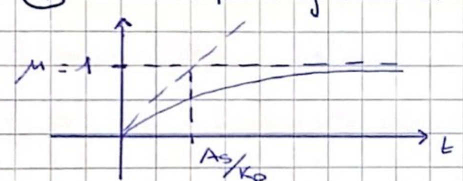
$\dot{y}(0) = \frac{\mu}{T}$



③ Making reference to transfer function on Ex.2 recognize in the form of this exercise

$G(s) = \frac{\mu}{1+sT} \qquad F(s) = \frac{K_P}{A_s\, s + K_P} = \frac{1}{s\frac{A_s}{K_P} + 1} \qquad \mu = 1 \quad T = \frac{A_s}{K_P}$

④ Sketch plot of level h when reference h° is a step



④ Consider the control law of a PID controller $u(t) = \underbrace{K_P\, e(t)}_{1} + \underbrace{K_I \int_0^t e(\tau)\, d\tau}_{2} + \underbrace{K_D \frac{de(t)}{dt}}_{3}$

Expression of transfer function from the error e to the control variable u.

PID $\longrightarrow$ ① PROPORTIONAL ② INTEGRAL ③ DERIVATE ACTION

$s \longleftrightarrow \frac{d}{dt}$

$\frac{1}{s} \longrightarrow \int_0^t$

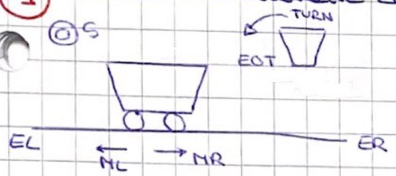
$U(s) = K_P\, e(s) + \frac{K_I}{s} e(s) + K_D\, s\, e(s)$

$U(s) = \left(K_P + \frac{K_I}{s} + K_D s\right) e(s)$

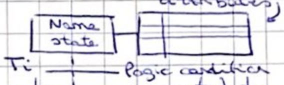
$$U(s) = \frac{K_D s^2 + K_P s + K_I}{s}\, e(s)$$

EXERCISES — WRAP-UP EXERCISES

(1) Consider the automatic cart



SUM-UP SFC:



→ STEPS or STATES with some actions associated to.

→ TRANSITIONS with some logic condition

→ DIRECT LINKS BETWEEN STATE AND TRANSITIONS

RULES:

→ A STEP OR STATE CAN BE ACTIVE OR INACTIVE

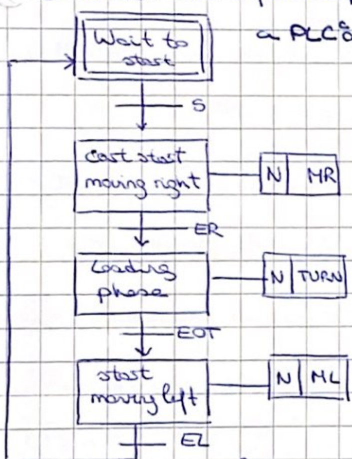
→ ACTIONS ARE ONLY EXECUTED FOR ACTIVE STATES

→ STEPS CAN BE ACTIVATED WHEN ALL THE ABOVE STEPS ARE ACTIVE AND THE CONNECTING TRANSITION IS SUPERABLE

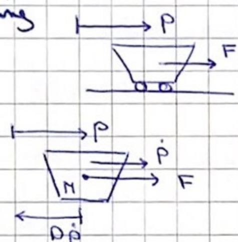
→ WHEN A TRANSITION IS PASSED, ALL TH ABOVE STATES ARE DEACTIVATED AND THE ONES BELOW ARE ACTIVATED AT ONCE

→ IF MORE TRANSITIONS ARE SUPERABLE THEN THEY ARE ALL PASSED SIMULTANEOUSLY.

① Sketch SFC represent program of a PLC controlling



② Consider the motion: write the transfer function from a force applied



$$M\ddot{p} = F - D\dot{p}$$

$$M\ddot{p} + D\dot{p} = F \rightarrow (Ms^2+Ds)\,p(s) = F(s)$$

$$\frac{d}{dt} \longleftrightarrow s \qquad G(s) = \frac{P(s)}{F(s)} =$$

$$= \frac{1}{Ms^2+Ds} = \frac{1}{s(Ms+D)}$$

Transf. function

③ Sketch block diagram where cart controlled by proport. controller.

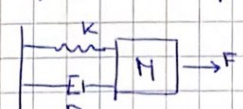
$$p(s) = G(s)\cdot K\cdot(p^\circ(s) - p(s))$$

$$(1+G(s)\cdot K)\,p(s) = G(s)K\,p^\circ(s)$$

$$\frac{p(s)}{p^\circ(s)} = \frac{KG(s)}{1+KG(s)} = \frac{\frac{K}{Ms^2+Ds}}{1+\frac{K}{Ms^2+Ds}} = \frac{\frac{K}{\cancel{Ms^2+Ds}}}{\frac{Ms^2+Ds+K}{\cancel{Ms^2+Ds}}} = \frac{K}{Ms^2+Ds+K}$$

④ Discuss stability closed-loop system

Like before



$$G(s) = \frac{1}{Ms^2+Ds+K}$$

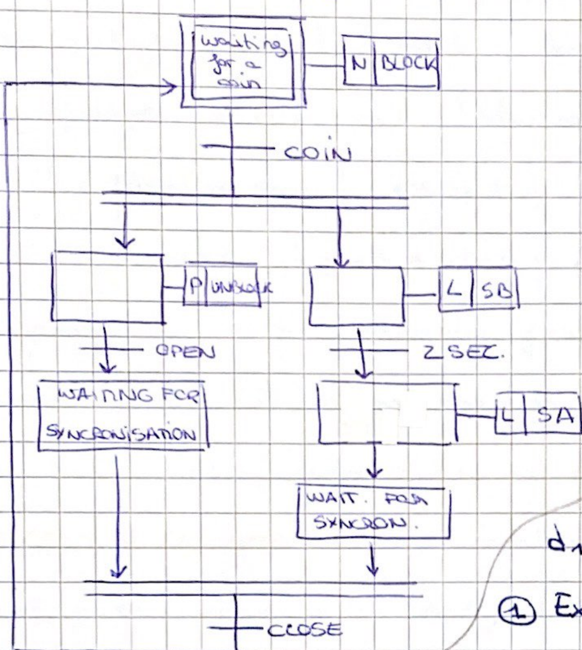
Asymptotic stability $\forall K > 0$

$M > 0 \quad D > 0$

$K < 0$ $\quad S_{1,2} = \frac{-D \pm \sqrt{D^2-4MK}}{2M}$ $\quad$ System is unstable

$K = 0$ $\quad S_{1,2} = \frac{-D \pm D}{2M} <$ $-D/M$, 0 system not asympto. stable

(2) Consider simple vending machine. Sketch SFC might represent program of a PLC



(3) Consider 2 degrees of freedom planar robot despicted in the picture.
1^{st} prismatic joint that allows allows for translational motion for first on 2^{nd} revolut.
d_1, θ_2 joint variables a_2 length second link. P position end-effector

(1) Explain what are dir. and inv Kinematics

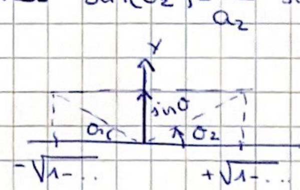
- DIRECT KINEMATICS finds the positiong and orientation of the end-effector given the joint variables. $P = \begin{bmatrix} P_x \\ P_y \end{bmatrix} = f(d_1, \theta_2)$
- INVERSE KINEATICS finds the joint variable given the position and orientation of the end-effector $\begin{bmatrix} d_1 \\ \theta_2 \end{bmatrix} = g(P_x, P_y)$

(2) Expression direct Kinematics

$$\begin{cases} P_x = d_1 + a_2 \cos(\theta_2) \\ P_y = a_2 \cdot \sin(\theta_2) \end{cases}$$

(3) Inverse Kinematics $\sin(\theta_2) = \frac{P_y}{a_2}$ $\sin(\theta_2) = \sin(\pi - \theta_2)$

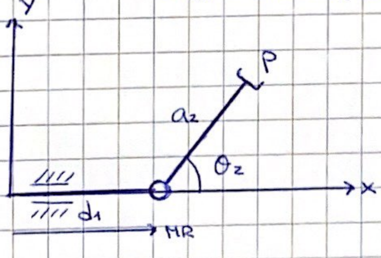
$\cos(\theta_2) = \pm\sqrt{1 - \sin^2(\theta_2}$



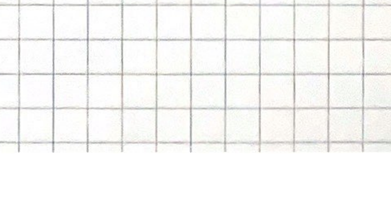
$\theta_{2,1} = \text{atan2}(\sin(\theta_2), \cos(\theta_2))$

$\theta_{2,2} = \text{atan2}(\sin(\theta_2, \cos(\theta_2))$

$d_1 = P_x - a_2 \cdot \cos(\theta_2)$ ⟨ $d_{1,1} = P_x - a_2 \cos(\theta_{2,1})$; $d_{1,2} = P_x - a_2 \cos(\theta_{2,2})$

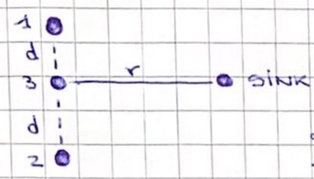


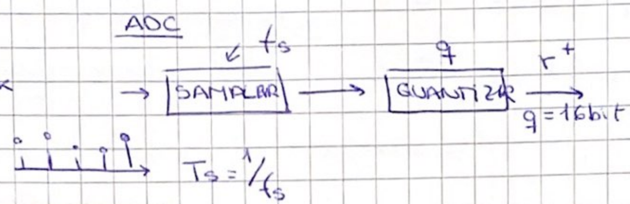
(4) Two solutions



EXERCISES   COMMUNICATION

A wireless sensor network is composed of three nodes and a sink. The three nodes temperature sensors with sampling frecuency $f_s = 1$ [KHz] and quantization level $q = 16$ bit. What is the raw data rate generated by each node?





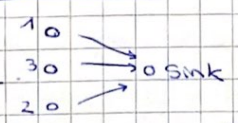
$N = 1000$; $\boxed{r = N \times q} = 1000 \cdot 16 \text{ bit} = 16 \text{ Kbit/s}$

The three nodes send remotely all the collected samples with a single packet transmission every second (the packet contains all the samples collected in the last second). Assume that the energy for acquiring one temperature sample is $E_s = 1$ [uJ/sample], the energy required to operate the TX/RX circuitry is $E_c = 50$ [nJ/bit], the energy required to support sufficient transmission output power $E_{tx}(d) = Kd^2$ [nJ/bit] being $K = 1$ [nJ/bit/m²], the energy for processing one temperature sample $E_p = 0'5$ [uJ/sample], $d = 5$ [m] and $r = 10$m
Find out the energy consumed by each one of the three sensors in one second in two cases: 1. The sensors send the temperature samples directly to the sink.
2. Sensors 1 and 2 send their packets to sensor 3; sensor 3 performs the average of all the temperature samples within the packets (from 1, 2 and its own) and sends a single packet to the sink (packet size $L = 4$ byte) Find out the network lifetime in the two previous cases, if all the nodes have an initial energy budget $E_0 = 1$ J

| Sample 1 | Sample 2 | ... | Sample N |

$\ell = N \times 16$ $\ell = 16$ Kbits

CASE 1

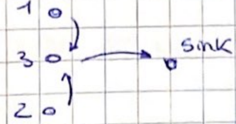


$E_1 = \underbrace{E_2}_{1} \cdot N + \ell \cdot E_c + \ell \cdot E_{Tx}\left(\sqrt{d^2 + r^2}\right)$

$E_2 = E_1 = 3'8$ mJ $E_3 = E_s \times N + \ell \times E_c + \ell \times E_{Tx}(r) = 3'4$ mJ

CASE 2

$E_1 = E_2 = E_s \cdot N + \ell E_c + \ell E_{Tx}$



$E_3 = \underbrace{E_s \cdot N}_{\text{sampling}} + \underbrace{\ell \cdot E_c + \ell \cdot E_c}_{\text{from 1 and 2}} + \underbrace{3 \cdot N \cdot E_p}_{\text{processing}} + \underbrace{L\, E_c + L\, E_{Tx}(r)}_{\text{transmitting}} \simeq 4'1$

CASE 1 : LIFETIME $= \dfrac{E_b}{E_1} = \dfrac{1000}{3'8} = 263$ s

CASE 2 LIFETIME $= \dfrac{E_b}{E_3} = \dfrac{1000}{4'1} = 243$ s