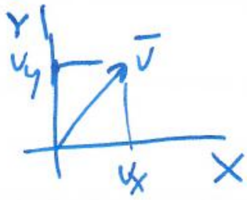
VECTORS

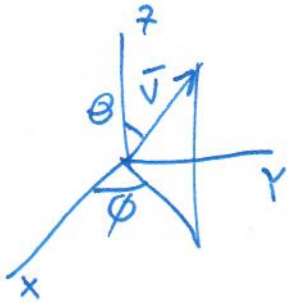
Basics

$$\vec{v} = \bar{v}_x + \bar{v}_y = v_x \vec{i} + v_y \vec{j}$$

2D $\begin{cases} v_x = v\cos\theta \\ v_y = v\sin\theta \end{cases}$

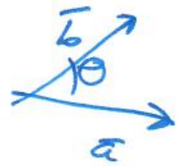


3D $\begin{cases} v_x = v\sin\theta\cos\phi \\ v_y = v\sin\theta\sin\phi \\ v_z = v\cos\theta \end{cases}$



Dot product

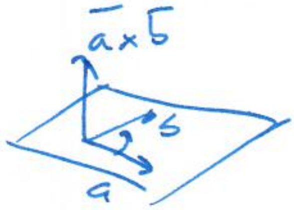
$$\bar{a}\cdot\bar{b} = a_x b_x + a_y b_y + a_z b_z$$
$$= |a|\,|b|\cos\theta$$



Meaning
↓
projection

Cross product

$\bar{a}\times\bar{b}$

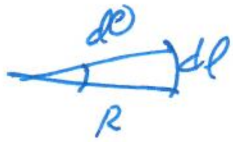


$$\bar{a}\times\bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$|\bar{a}\times\bar{b}| = |a|\,|b|\sin\theta$$

Angle

$$\theta = \frac{l}{R}$$



$$d\theta = \frac{dl}{R}$$

Solid Angle

$$\Omega = \frac{S}{R^2} \qquad d\Omega = \frac{dS}{R^2}$$

Surfaces as vectors

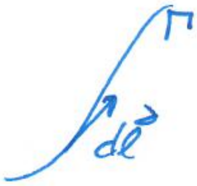
$\vec{S}$

$\vec{S} = |S| \vec{u}_s$

$$\vec{S} = \iint d\vec{S} = \iint dS\, \vec{u}_s$$

Line Integral

$$I = \int_\Gamma \vec{v} \cdot d\vec{l}$$



$\Gamma \equiv$ path

Example : work

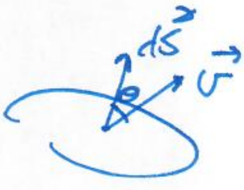
Circulation

Γ is a closed path

$$C = \oint \vec{v} \cdot d\vec{l}$$

Flux

$$\Phi = \iint \vec{v} \cdot d\vec{S}$$



$$\begin{cases} \Phi \text{ Max} : \theta = 0 \\ \Phi \text{ Min} : \theta = \frac{\pi}{2} \end{cases}$$

Nabla operator

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

Divergence

$$\vec{\nabla}\cdot\vec{v} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)(v_x, v_y, v_z)$$

$$= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Gradient

$$\vec{\nabla}\cdot\bar{\Phi} = \left(\frac{\partial \Phi}{\partial x}, \frac{\partial \Phi}{\partial y}, \frac{\partial \Phi}{\partial z}\right)$$

Curl

$$\bar{\nabla}\times\bar{v} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$

Divergence theorem / Gauss theorem

$$\oiint \vec{v}\cdot d\vec{S} = \iiint_V (\bar{\nabla}\cdot\bar{v})\, dV$$

Stokes theorem

$$\oint \vec{v}\cdot \vec{dl} = \iint_S (\bar{\nabla}\times\bar{v})\, d\vec{S}$$

Conservative field $\Rightarrow (\bar{\nabla}\times\bar{v}) = 0$