Problem 8.27 Discuss the convexity of the following functions:

(i) $f(x) = (x-2)x^{2/3}$; (ii) $f(x) = |x|e^{|x|}$; (iii) $f(x) = \log(x^2 - 6x + 8)$.

Problem 8.28

(i) Sketch the graph of the function $f(x) = x + \log|x^2 - 1|$.

(ii) Based on the previous graph, plot function $g(x) = |x| + \log|x^2 - 1|$ and $h(x) = \big|x + \log|x^2 - 1|\big|$.

Problem 8.29 Sketch a plot of the following functions:

(i) $f(x) = e^x \sin x$;

(ii) $f(x) = \sqrt{x^2 - 1} - 1$;

(iii) $f(x) = xe^{1/x}$;

(iv) $f(x) = x^2 e^x$;

(v) $f(x) = (x-2)x^{2/3}$;

(vi) $f(x) = (x^2 - 1)\log\left(\dfrac{1+x}{1-x}\right)$;

(vii) $f(x) = \dfrac{x}{\log x}$;

(viii) $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$;

(ix) $f(x) = \dfrac{e^{1/x}}{1-x}$;

(x) $f(x) = \log[(x-1)(x-2)]$;

(xi) $f(x) = \dfrac{e^x}{x(x-1)}$;

(xii) $f(x) = 2\sin x + \cos 2x$;

(xiii) $f(x) = \dfrac{x-2}{\sqrt{4x^2+1}}$;

(xiv) $f(x) = \sqrt{|x-4|}$;

(xv) $f(x) = \dfrac{1}{1+e^x}$;

(xvi) $f(x) = \dfrac{e^{2x}}{e^x - 1}$;

(xvii) $f(x) = e^{-x}\sin x$;

(xviii) $f(x) = x^2 \sin\dfrac{1}{x}$.

Problem 8.30 Draw the graph of the following functions:

(i) $f(x) = \min\{\log|x^3 - 3|, \log|x+3|\}$;

(ii) $f(x) = \dfrac{1}{|x|-1} - \dfrac{1}{|x-1|}$;

(iii) $f(x) = \dfrac{1}{1+|x|} - \dfrac{1}{1+|x-a|}$, $(a > 0)$;

(iv) $f(x) = x\sqrt{x^2 - 1}$;

(v) $f(x) = \arctan\log|x^2 - 1|$;

(vi) $f(x) = 2\arctan x + \arcsin\left(\dfrac{2x}{1+x^2}\right)$.

Problem 8.31 Plot the function

$$f(x) = \begin{cases} \dfrac{e^{1/x}}{1+x}, & x \neq 0, \\ 0 & x = 0, \end{cases}$$

and discuss how many real solutions has the equation $\dfrac{e^{1/x}}{1+x} = x^3$.

Problem 8.32 Given the function $f(x) = \dfrac{1+x}{3+x^2}$ plot the functions $g(x) = \sup\limits_{y>x} f(y)$ and $h(x) = \inf\limits_{y>x} f(y)$.

Problem 8.33 Determine the equations of the tangents to $f(x) = \log(1+x^2)$ at its inflection points and plot them along with the graph of $f(x)$.