Fundamental Concepts of Statistics

Exercise session 3

1. Find expressions for the approximate mean and variance of \( Y = g(X) \) with \( g(x) = \log x \)

2. If \( X \) is uniformly distributed on \([10, 20]\) find the approximate and exact mean and variance of \( 1/X \) and compare them.

3. Find the approximate mean and variance of \( Y = \sqrt{X} \) when \( X \) is a random variable following a Poisson distribution with mean \( \lambda \).

4. If \( X \) is distributed as \( N(75, 100) \), find \( P(X < 60) \) and \( P(70 < X < 100) \).

5. If \( X \) is distributed as \( N(\mu, \sigma^2) \), find \( b \) such that \( P(-b < \frac{X-\mu}{\sigma} < b) = 0.90 \).

6. If \( X \) is distributed as \( N(\mu, \sigma^2) \) so that \( P(X < 89) = 0.90 \) and \( P(X < 94) = 0.95 \), find \( \mu \) and \( \sigma^2 \).

7. If \( X \) is distributed as \( N(5, 10) \), find \( P(0.04 < (X - 5)^2 < 38.4) \).

8. If \( X \) is distributed as \( N(1, 4) \), find \( P(1 < X^2 < 9) \).

9. If \( X \) is distributed as \( N(\mu, \sigma^2) \), show that \( E[|X - \mu|] = \sigma \sqrt{2/\pi} \).