Fundamental Concepts of Statistics

Exercise session 5

1. Consider the maximum $U_{(n)}$ of n simulated uniform (0,1) i.i.d. random variables $U_1, \dots, U_n$.

a) Show that $n(1-U_{(n)})$ converges in distribution to a standard exponential distribution with distribution function $F(y) = 1 - e^{-y}$ $(y > 0)$ as $n \to \infty$. [To this end compute $P(n(1-U_{(n)}) > y)$ and take the limit for $n \to \infty$.]

b) How to adapt this result if the random variables are uniformly distributed on $(0, a)$ for some $a > 0$. [Hint: how to transform the given case to the uniform (0,1) case.]

2. A straight rod is formed by connecting three sections A,B and C, each of which is manufactured on a different machine. The length of section A, in inches, has the normal distribution with mean 20 and variance 0.04. The length of section B, in inches, has the normal distribution with mean 14 and variance 0.01. The length of section C, in inches, has the normal distribution with mean 26 and variance 0.04. The three sections are joined so that there is an overlap of 2 inches at each connection. What is the probability that the rod will be between 55.7 and 56.3 inches long?

3. Define for $n \in \mathbb{N}$ random variable X_n for which

$$P\left(X_n = -\frac{1}{n}\right) = P\left(X_n = \frac{1}{n}\right) = \frac{1}{2}$$

Does the sequence of random variables converge in distribution? If so find the limiting distribution and show convergence.

4. Prove that

$$X_n \xrightarrow{P} \delta_C \Longleftrightarrow X_n \xrightarrow{d} \delta_C$$

where the distribution δ_C is characterized by $P(\delta_C = C) = 1$ for some constant $C \in \mathbb{R}$.

Hint: $|a - b| > d \Longleftrightarrow a - b > d$ or $b - a > d$.