Fundamental Concepts of Statistics

Exercise session 6

1. Two random samples of size n are taken from two populations, and two proportions p_1 and p_2 are estimated. It is expected that both population proportions are close to 0.5. What should the sample size n be so that the standard deviation of the difference $\hat{p}_1 - \hat{p}_2$ will be less than 0.02?

2. The value of a population mean increases linearly through time:

$$\mu(t) = \alpha + \beta t,$$

while the variance remains constant. Independent samples of size n are taken at times $t = 1, 2, 3$.

a) Find the conditions on w_1, w_2 and w_3 such that

$$\hat{\beta} = w_1 \bar{X}_1 + w_2 \bar{X}_2 + w_3 \bar{X}_3$$

(with $\bar{X}_i$ denoting the sample average at time i, $i = 1, 2, 3$) is an unbiased estimator of β.

b) What values of w_1, w_2 and w_3 minimize the variance of $\widehat{\beta}$ subject to the constraint that the estimator is unbiased?

3. Is $\overline{X}^2$ an unbiased estimator of μ_X^2? If not, what is the bias?

4. The typical estimator for a population proportion p is the sample proportion $\widehat{p}_1 = \frac{X}{n}$ where X is the number of successes in a random sample of size n. However, in the case when the population proportion p is small and the sample size n is small, one might easily get zero successes and an estimate 0 for the proportion p. To remedy this, the Wilson estimator is proposed as $\widehat{p}_2 = \frac{X+2}{n+4}$. Find the bias and mean squared error of both estimators, and show whether $\widehat{p}_2$ is consistent.