Fundamental Concepts of Statistics

Exercise session 7

1. Let $X_1, ..., X_n$ be a random sample from a Poisson distribution with mean λ and $T = \sum_{i=1}^{n} X_i$. Show that the distribution of $X_1, \ldots, X_n$ given T is independent of λ so that T is sufficient for λ.

2. Let $X_1, ..., X_n$ be a random sample from the distribution with density

$$f(x;\theta) = \frac{\theta}{(1+x)^{1+\theta}},\ x > -1 \text{ and } \theta > 0.$$

Find a sufficient statistic for θ.

3. Suppose that X is binomially (n, p) distributed.

a) Show that the MLE of p is $\hat{p} = X/n$.

b) Show that this MLE attains the Cramr-Rao lower bound.

4. Suppose that $X_1, ..., X_n$ is a random sample from geometric distribution Geo(p) with

$$P(X = x) = p(1-p)^{x-1},\ x = 1, 2, \ldots$$

Expected value of geometric distribution is $1/p$.

a) Find the MLE of p.

b) Find the asymptotic variance of the MLE.

5. The Pareto distribution is defined through

$$f(x;\theta) = \theta x_0^{\theta} x^{-\theta-1},\ x > x_0.$$

Assume that x_0 is given. Consider $X_1, X_2, \ldots, X_n$ iid sample from this distribution.

a) Find the MLE of θ

b) Find the asymptotic variance of the MLE.

6. Let $X_1, ..., X_n$ be a random sample from the Rayleigh distribution with parameter $\theta > 0$:

$$f(x;\theta) = \frac{x}{\theta^2} e^{-x^2/(2\theta^2)},\ x > 0.$$

Find the MLE of θ and the asymptotic variance of the MLE given that $E(X_i^2) = 2\theta^2$.

7. Let $X_1, ..., X_n$ be a random sample from uniform distribution $U[0, \theta]$. Find the MLE of θ.