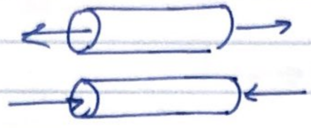
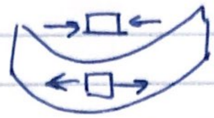
Chapter 9 – TRANSFORMATION OF STRESSES/PRINCIPAL STRESSES

normal stresses



Chp 4.

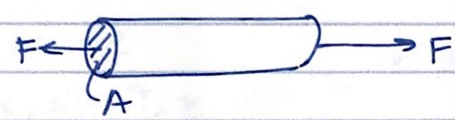


Chp 6.

shear stresses

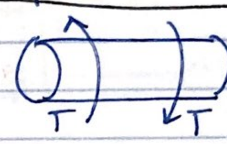
Torsion

Simple Tension



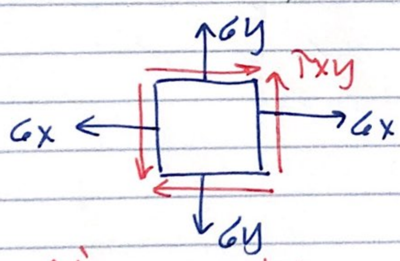
Failure criteria $\sigma_x = \frac{F}{A} = \frac{\sigma_{yield}}{FS}$

Simple Torsion



$\tau = \frac{Tr}{J} = \tau_{allow}$

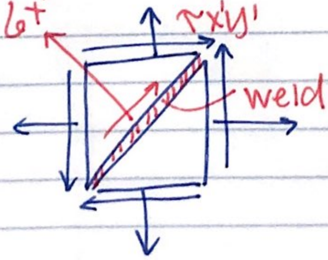
Bilateral Tension

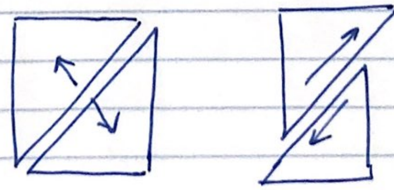


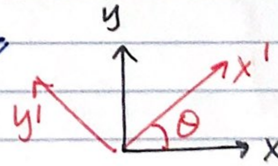
$\sigma_x = 100$

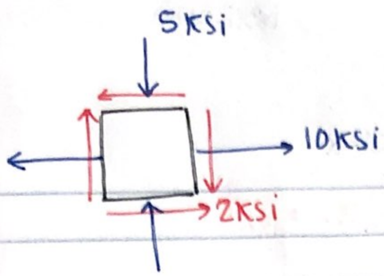
$\sigma_y = 90$

$\frac{\sigma_{yield}}{FOS} = 105$

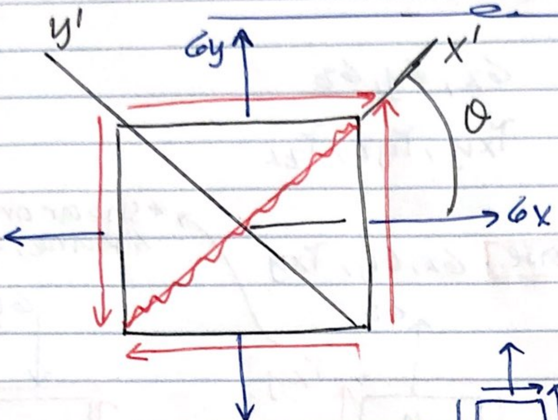








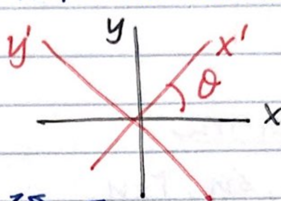
$$\begin{cases} \sigma_x = 10 \text{ ksi} \\ \sigma_y = -5 \text{ ksi} \\ \tau_{xy} = -2 \text{ ksi} \end{cases}$$

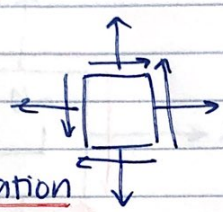


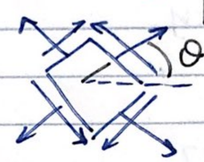
given: σ_x, σ_y, τ_{xy}, θ

Find: $\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$

transformation of stresses





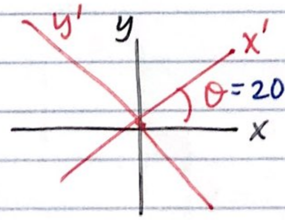


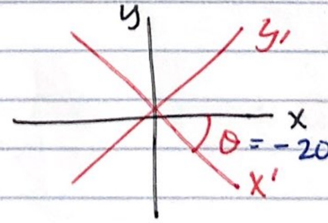
• Transformation Equation

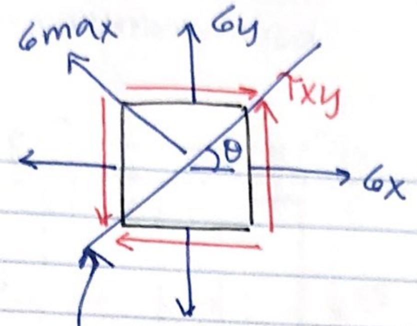
$$\boxed{\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\theta + \tau_{xy} \cdot \sin 2\theta}$$

$$\boxed{\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\theta - \tau_{xy} \cdot \sin 2\theta}$$

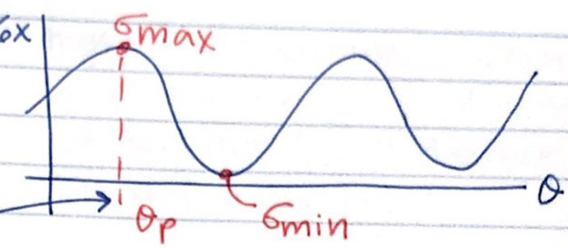
$$\boxed{\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \cdot \sin 2\theta + \tau_{xy} \cdot \cos 2\theta}$$







$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\theta + \tau_{xy} \cdot \sin 2\theta$$



$$\frac{d\sigma_{x'}}{d\theta} = 0 \Rightarrow \theta_p \text{ direction of principal plane.}$$

$$\frac{d\sigma_{x'}}{d\theta} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) 2\sin 2\theta + 2\tau_{xy} \cdot \cos 2\theta = 0$$

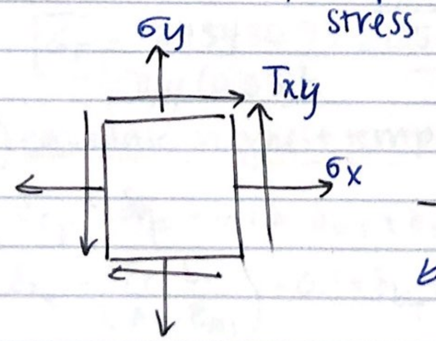
$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}$$

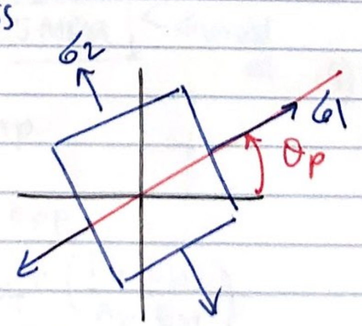
$$\theta_p = \frac{1}{2} \tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

$$\boxed{\sigma_{x'}\Big|_{\theta=\theta_p} = \sigma_{max} = \sigma_1}$$

$$\boxed{\begin{matrix}\sigma_1 \\ \sigma_2\end{matrix} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$\sigma_2 = \sigma_{min}$ principal stress



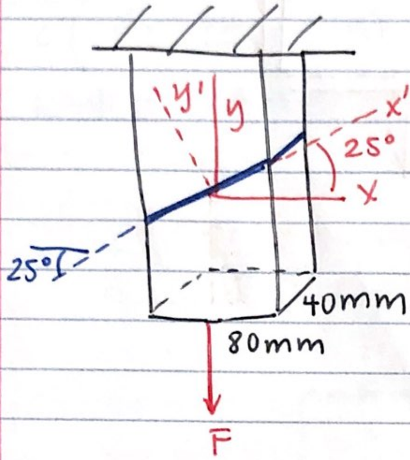


* no shear stress on max/min princi planes

11/25 - ME 132

- failure

normal strength > shear strength



- glue: normal strength: 800 KPa
  shear strength: 600 KPa

$\sigma_x = 0 \quad \sigma_y = \frac{F}{A} \quad \tau_{xy} = 0$

$A = (0.04)(0.08)\,m^2$

$$\sigma_{y'} = \frac{\cancel{\sigma_x}^{0} + \sigma_y}{2} - \frac{\cancel{\sigma_x}^{0} - \sigma_y}{2}\cos 2\theta - \cancel{\tau_{xy}}^{0} \cdot \sin 2\theta$$

$$\sigma_{y'} = \frac{F}{2A} + \frac{F}{2A}\cos 2\theta = \frac{F}{2A}(1 + \cos 50)$$

$$\sigma_{y'_{max}} = 800\,KPa = \frac{F_{max}}{2(0.04)(0.08)}(1 + \cos 50)$$

$$\tau_{xy'} = -\frac{\cancel{\sigma_x}^{0} - \sigma_y}{2}\sin 2\theta + \cancel{\tau_{xy}}^{0}\cos 2\theta$$

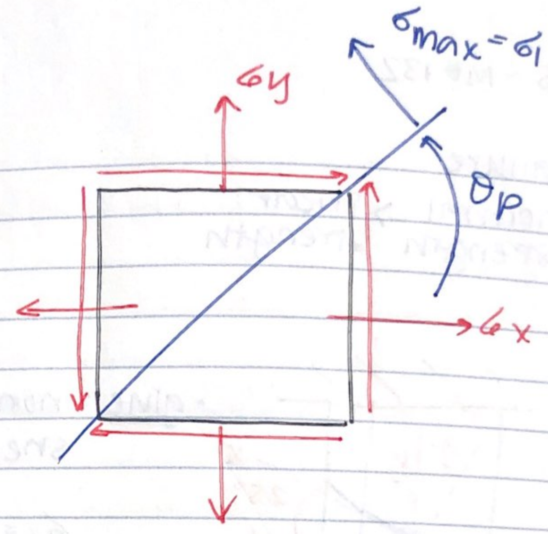
$$\tau_{xy'} = \frac{F'_{max}}{2(0.04)(0.08)} \cdot \sin(50) = 600\,KPa$$

choose smallest F_{max}

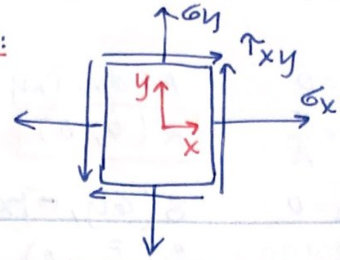
Principal Stresses

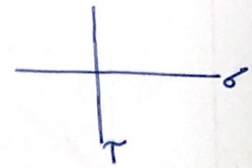
- Principal stresses
Principal planes

given: $\sigma_x, \sigma_y, \tau_{xy}$
Find: σ_1, σ_2



• given:



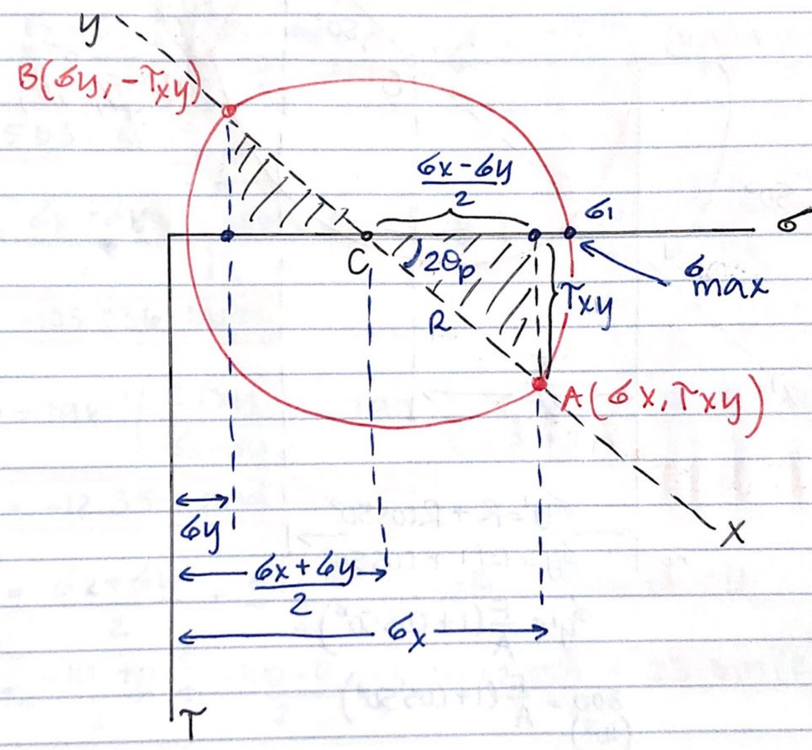


MOHR CIRCLE

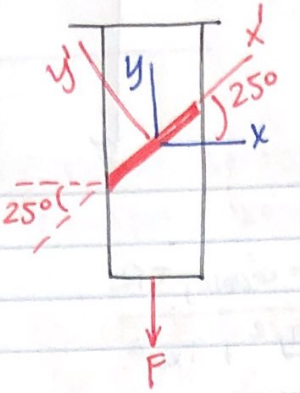
1. on a σ-τ axis,
2. locate $A(\sigma_x, \tau_{xy})$
3. locate $B(\sigma_y, -\tau_{xy})$
4. connect A & B
5. $C\left(\frac{\sigma_x+\sigma_y}{2}, 0\right)$ center
6. radius = CA = CB
7. $\sigma_1 = C + R = \sigma_{avg} + R$

$$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$2\theta_p = \tan^{-1}\left(\frac{\tau_{xy}}{\frac{\sigma_x-\sigma_y}{2}}\right)$$



Ex/



$\sigma_x = 0$

$\sigma_y = \dfrac{F}{A}$

$\tau_{xy} = 0$

$A(\sigma_x, \tau_{xy})$

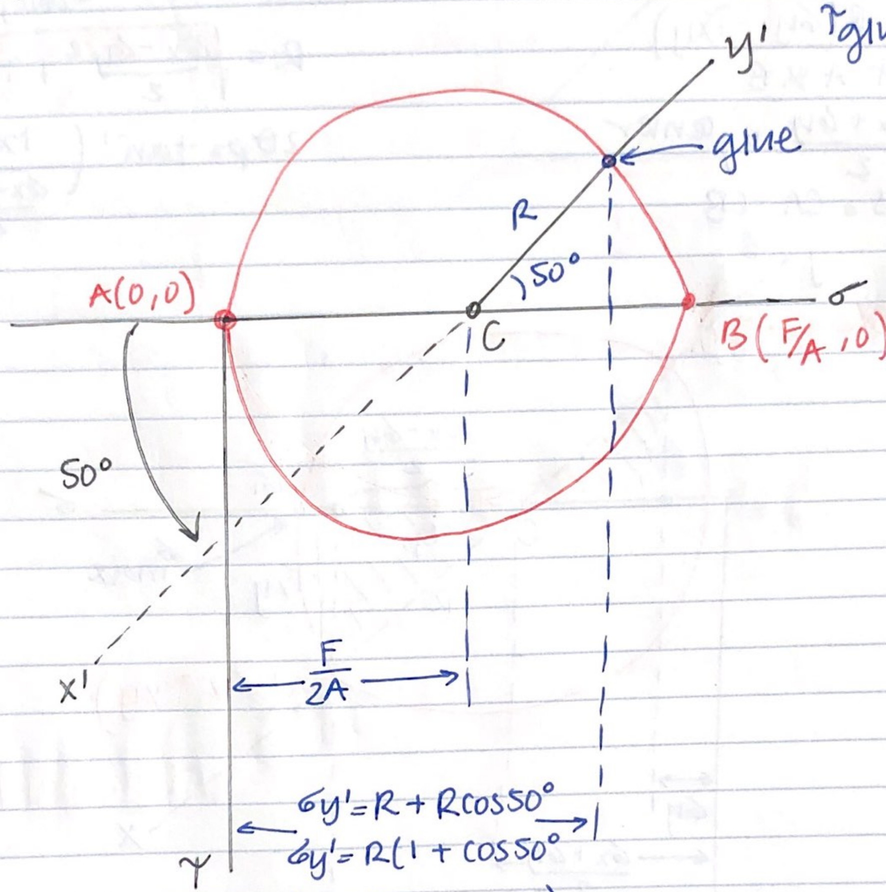
$\boxed{A(0,0)}$

$B(\sigma_y, -\tau_{xy})$

$B\left(\dfrac{F}{A}, 0\right)$

$600 = \dfrac{F}{2} \sin 50$

$\tau_{glue} = \dfrac{F}{2A} \sin 50$



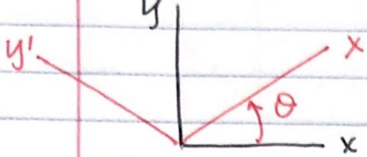
$\sigma_{y'} = R + R\cos 50°$

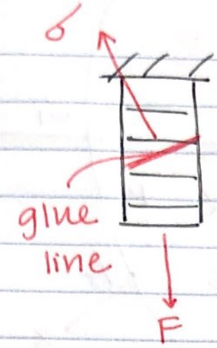
$\sigma_{y'} = R(1 + \cos 50°)$

$\sigma_{y'} = \dfrac{F}{A}(1 + \cos 50°)$

$\underset{(10^3)}{800} = \dfrac{F}{A}(1 + \cos 50°)$

ME 146 - 10/26





$$\begin{cases} \sigma_y = \frac{F}{A} \\ \sigma_x = 0 \\ \tau_{xy} = 0 \end{cases}$$

- Mohr Circle of Stresses

given $\sigma_x, \sigma_y, \tau_{xy}$

$A(\sigma_x, \tau_{xy})$

$B(\sigma_y, -\tau_{xy})$

$C(\sigma_{avg}, 0)$

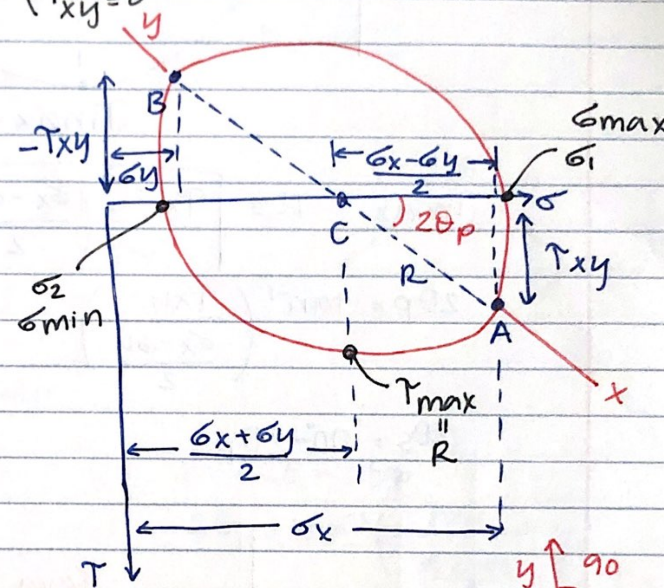
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

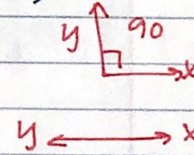
$$R = \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$$

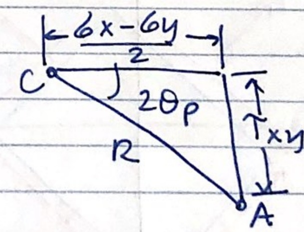
$\sigma_1 = C + R \qquad \sigma_2 = C - R$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$$

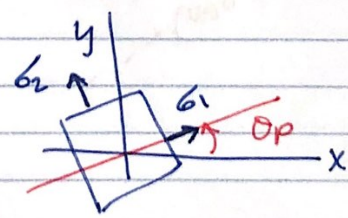






$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}$$

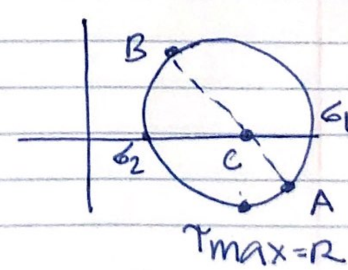
$$\theta_p = \frac{1}{2} \tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$



*principal planes

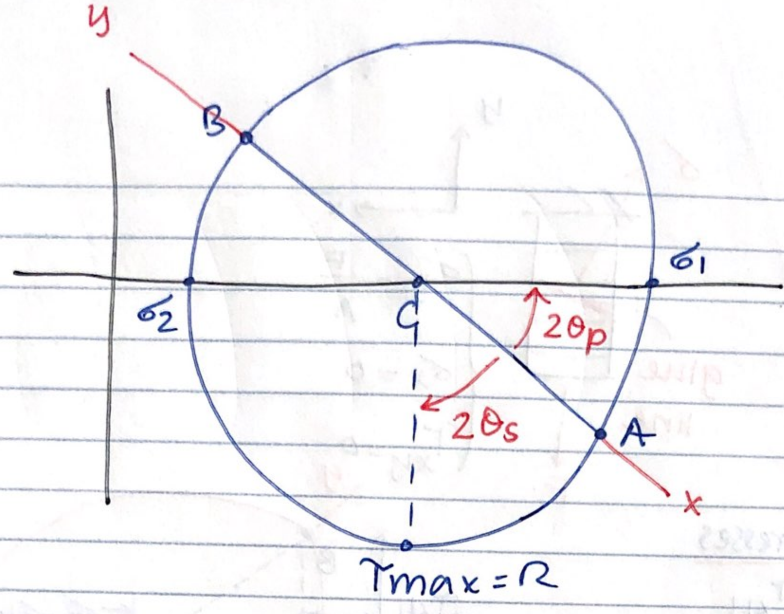
NO SHEAR

because on σ axis



Max shear plane

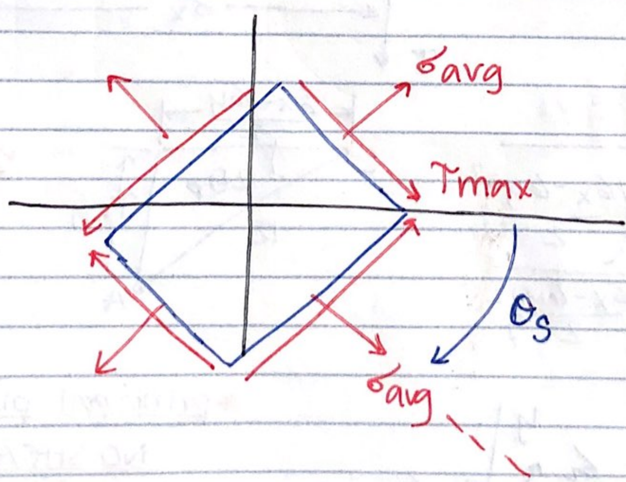
$\tau_{max} = R = \tau_{xy}^2 + ($



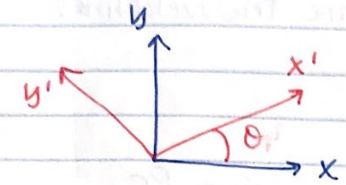
$$\tau_{max} = R = \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$$

$$2\theta_p = \tan^{-1}\left(\frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}\right)$$

$$2\theta_s = 90^\circ - 2\theta_p$$



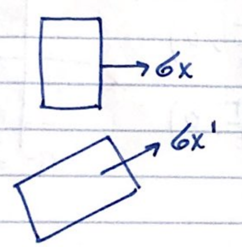
* Ex/ given $\sigma_x, \sigma_y, \tau_{xy}$
Find $\sigma_{x'}, \sigma_{y'}, \tau_{x'y'}$

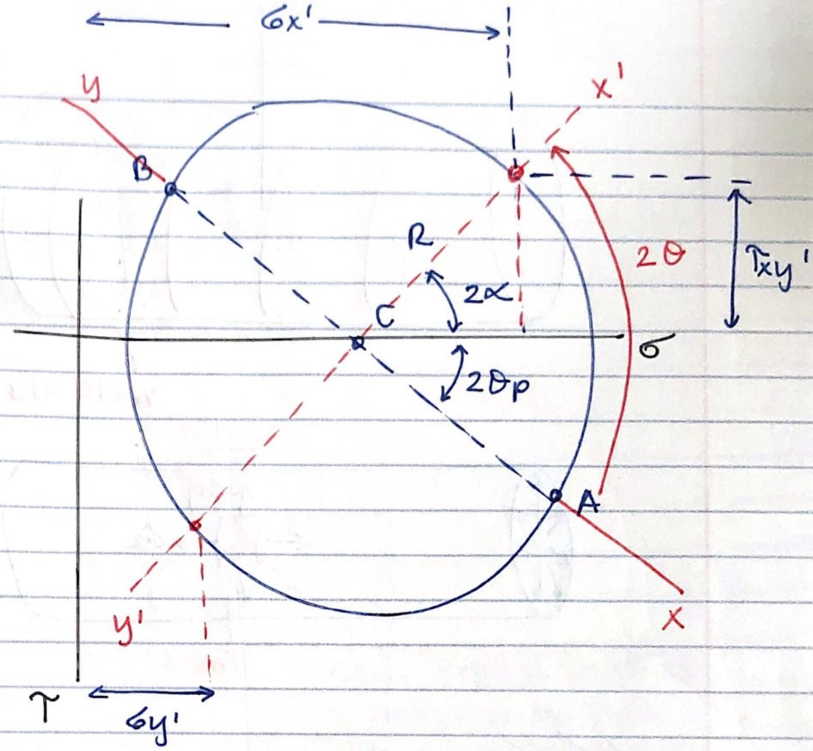


$A(\sigma_x, \tau_{xy})$

$B(\sigma_y, -\tau_{xy})$

$C(\sigma_{avg}, 0)$





$$2\alpha = 2\theta - 2\theta_p$$

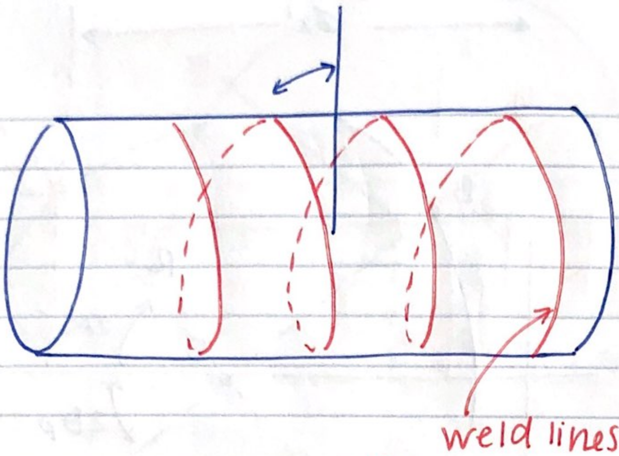
$$2\theta_p = \tan^{-1}\left(\frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}\right)$$

$$\sigma_{x'} = C + R\cos 2\alpha$$

$$\sigma_{y'} = C - R\cos 2\alpha$$

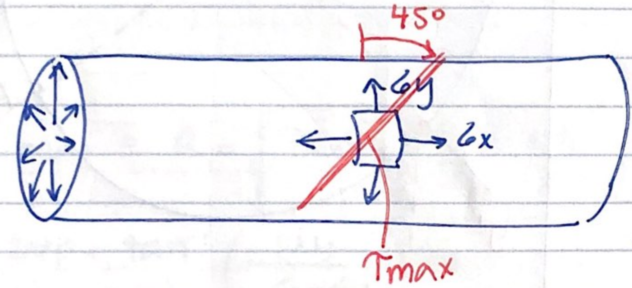
$$\tau_{x'y'} = R \cdot \sin 2\alpha$$

How much stress are the weld lines



$\sigma x = \frac{pr}{2t}$

$\sigma y = \frac{pr}{t}$

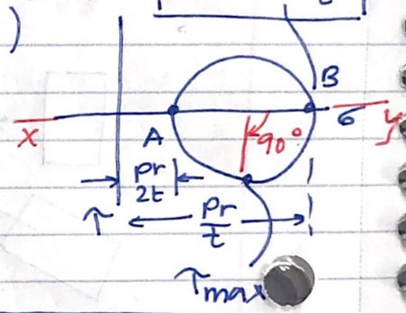


$A(\sigma x, \tau xy)$

$A\left(\frac{pr}{2t}, 0\right)$

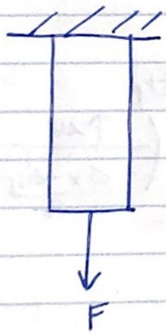
$B\left(\frac{pr}{t}, 0\right)$

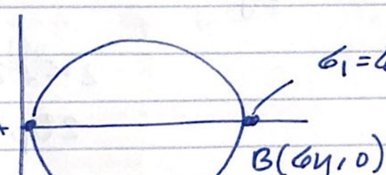
$\boxed{\sigma_1 = \sigma y = \frac{pr}{t}}$



$\boxed{\tau max = R = \frac{pr}{4t} \text{ in plane}}$

$\boxed{\sigma avg = \frac{pr}{2t} + \frac{pr}{4t} = \frac{3pr}{4t}}$





$\sigma_1 = \sigma y = \frac{F}{A}$

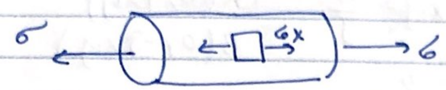
$B(\sigma y, \cancel{-\tau xy}\ 0)$

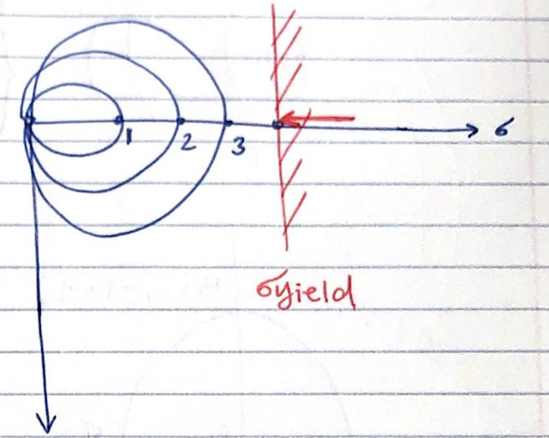
$A(\cancel{\sigma x}\ 0, \cancel{\tau xy}\ 0)$

$A(0,0)$

10/26 – ME 132

• Pure Tension

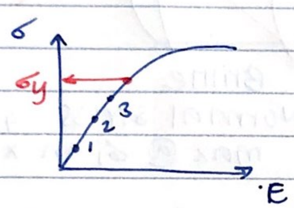


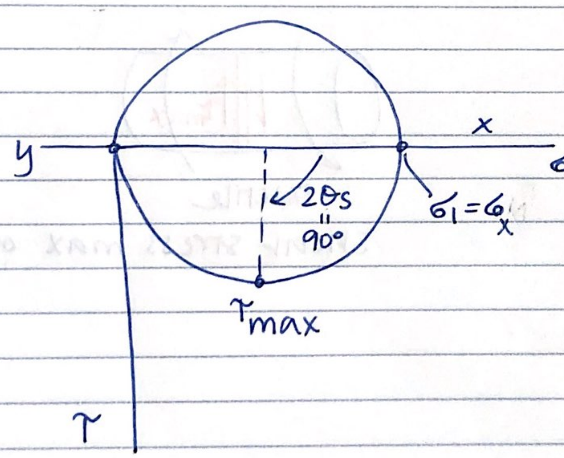


$\sigma_x = \sigma$ $\quad$ $A(\sigma, 0)$

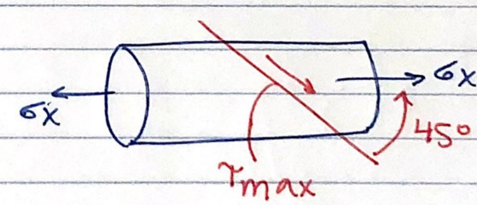
$\sigma_y = 0$ $\quad$ $\boxed{B(0,0)}$

$\tau_{xy} = 0$





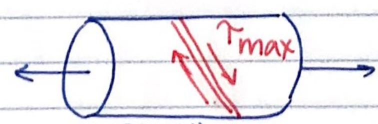
* ANY STRESS THAT IS APPLIED IN THE ABSENCE OF SHEAR IS PRINCIPAL STRESS



• Ductile materials

Low shear strength

High normal strength



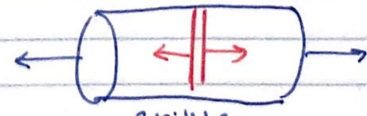
Ductile

Will break @ shear max = 45°

• Brittle materials

Low Normal strength

High shear strength



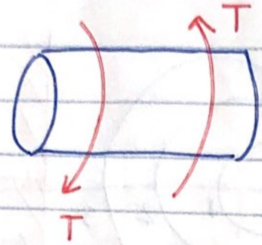
Brittle

Will break @ max normal strength

p. 458, 459

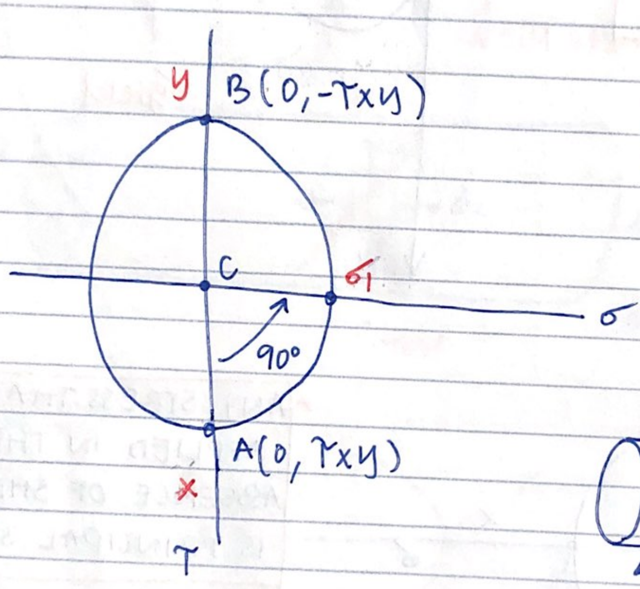
Example Problems 9.5, 9.6

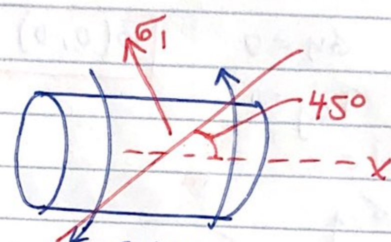
- **Pure Torsion**



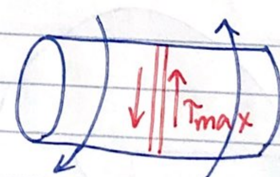
$$\begin{cases} \tau_{xy} = \frac{Tr}{J} \\ \sigma_x = 0 \\ \sigma_y = 0 \end{cases}$$

$A(0, \tau_{xy})$
$B(0, -\tau_{xy})$



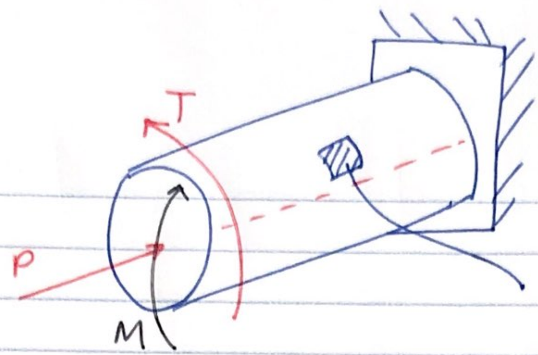


Brittle
Normal stress max @ σ_1 45° in x-axis

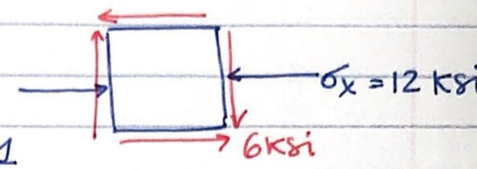


Ductile
Shear stress max on x-axis

* EX/



$\sigma_x = -12$ ksi
$\sigma_y = 0$
$\tau_{xy} = -6$ ksi



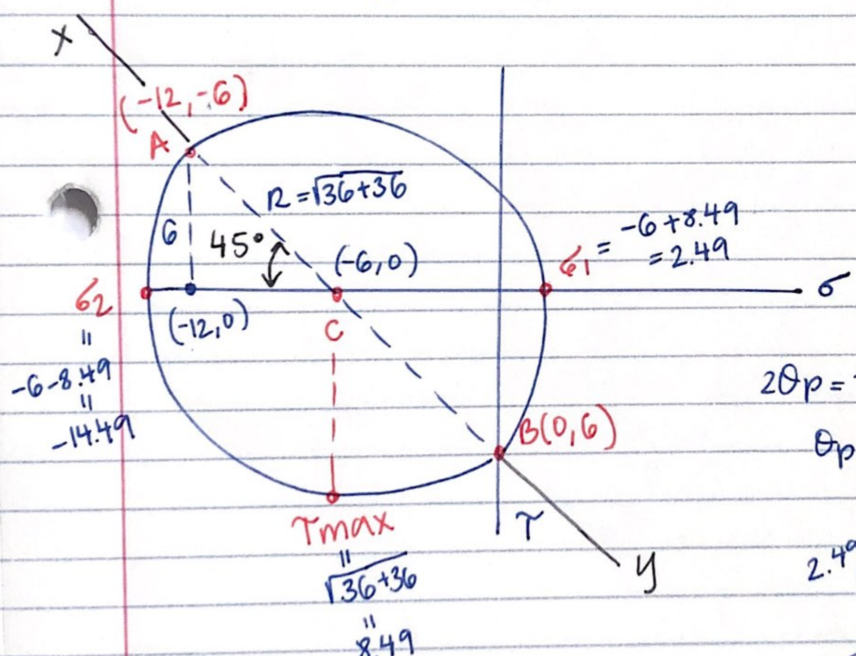
$\boxed{\tau_{xy} = \frac{Tr}{J} = -6 \text{ ksi}}$

$\sigma_x = \frac{-P}{A} - \frac{My}{I}$

$\boxed{\sigma_x = -12 \text{ ksi}}$

$\boxed{\sigma_y = 0}$

$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-12+0}{2} = -6$ ksi



$2\theta_p = \tan^{-1}\left(\frac{6}{6}\right) = 45°$

$\theta_p = 22.5°$