

Ejercicios (Series numéricas)

8.1. Escribiéndolas como series telescópicas, estudiar las siguientes series:

a) $\sum_{n=1}^{\infty} \frac{1}{2^n} \cdot \frac{n+2}{n(n+1)}$. (Descomponer $\frac{n+2}{n(n+1)}$ en fracciones simples.)

b) $\sum_{n=1}^{\infty} 3^n \operatorname{sen}^3 \frac{a}{3^n}$. (Obsérvese que $\operatorname{sen} x = 3 \operatorname{sen} \frac{x}{3} - 4 \operatorname{sen}^3 \frac{x}{3}$.)

c) $\sum_{n=1}^{\infty} 2^{n-1} \tan^2 \frac{a}{2^n} \tan \frac{a}{2^{n-1}}$ (Utilizar que $\tan x = \frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}}$.)

d) $\sum_{n=1}^{\infty} \operatorname{sen} \frac{1}{2^n} \cos \frac{3}{2^n}$. (Tener en cuenta que $\cos x \operatorname{sen} y = \frac{1}{2}(\operatorname{sen}(x+y) - \operatorname{sen}(x-y))$.)

e) $\sum_{n=1}^{\infty} \frac{1}{4^n \cos^2(x/2^n)}$, $0 < x < \pi/2$. (Usar que $\frac{1}{4 \cos^2 a} = \frac{1}{\operatorname{sen}^2 2a} - \frac{1}{4 \operatorname{sen}^2 a}$.)

8.2. Estudiar el carácter de las siguientes series:

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|---|---|--|
| 1) $\sum_{n=1}^{\infty} \frac{\operatorname{sen}^4 n}{n^2}$, | 8) $\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^{-n^3}$, | 15) $\sum_{n=1}^{\infty} \frac{1}{n} \operatorname{sen} \frac{1}{n}$, |
| 2) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} - 2/3}$, | 9) $\sum_{n=1}^{\infty} \frac{1}{\log n}$, | 16) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n}$, |
| 3) $\sum_{n=1}^{\infty} \frac{1+n^2}{n!}$, | 10) $\sum_{n=1}^{\infty} \frac{1}{na+b}$,
($a, b \neq (0, 0)$), | 17) $\sum_{n=1}^{\infty} \frac{n(n+1)}{n^2+2n}$, |
| 4) $\sum_{n=1}^{\infty} \cos^n \left(a + \frac{b}{n}\right)$,
$0 < a < \pi/2$, | 11) $\sum_{n=1}^{\infty} \frac{\operatorname{sen} nx}{n^2}$, | 18) $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^{n+1/n}$, |
| 5) $\sum_{n=1}^{\infty} \frac{n^2+1}{na^n}$,
$a \neq 0$, | 12) $\sum_{n=1}^{\infty} \frac{1}{n-3/2}$, | 19) $\sum_{n=1}^{\infty} \frac{\log(n+1) - 1}{(1+n)^2}$, |
| 6) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$, | 13) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$, | 20) $\sum_{n=1}^{\infty} \frac{1}{3 - \cos(1/n)}$, |
| 7) $\sum_{n=1}^{\infty} \frac{3^n}{n^2+1}$, | 14) $\sum_{n=1}^{\infty} \frac{1 + \operatorname{sen}^2 nx}{n^2}$, | 21) $\sum_{n=1}^{\infty} \left(\frac{x}{n}\right)^n n!$, |

$$\begin{array}{lll}
22) \sum_{n=1}^{\infty} \frac{1}{n(1 + \frac{1}{2} + \dots + \frac{1}{n})}, & 27) \sum_{n=1}^{\infty} \frac{1}{(\log n)^p}, & 32) \sum_{n=1}^{\infty} \frac{(-1)^n (n+1)}{n!}, \\
23) \sum_{n=1}^{\infty} \frac{1 + \frac{1}{2} + \dots + \frac{1}{n}}{n^3 \log n}, & 28) \sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}, & 33) \sum_{n=1}^{\infty} \frac{(n^2+1)x^n}{(n+1)!}, \\
24) \sum_{n=1}^{\infty} \frac{1}{(\log n)^{2n}}, & 29) \sum_{n=1}^{\infty} \frac{\log n}{n^p}, & 34) \sum_{n=1}^{\infty} e^{1/n^2} - e^{1/(n^2+1)}, \\
25) \sum_{n=1}^{\infty} \log \frac{n+1}{n}, & 30) \sum_{n=1}^{\infty} \log(1 + \frac{x}{n}), & 35) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}, \\
26) \sum_{n=1}^{\infty} e^{-\sqrt{n^2+1}}, & 31) \sum_{n=1}^{\infty} \frac{(-1)^n}{1 + \frac{1}{2} + \dots + \frac{1}{n}}, & 36) \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^{2n}.
\end{array}$$

8.3. Hallar la suma, si converge, de las siguientes series:

$$\begin{array}{ll}
a) \sum_{n=2}^{\infty} \frac{4n-1}{(n+2)(n-1)^2}, & m) \sum_{n=3}^{\infty} \frac{3n^2+8n+6}{(n+2)!}, \\
b) \sum_{n=1}^{\infty} \frac{1}{n(n+1)}, & n) \sum_{n=1}^{\infty} \frac{n-1}{n!(n+2)}, \\
c) \sum_{n=2}^{\infty} \frac{2n+3}{n(n-1)(n+2)}, & ñ) \sum_{n=1}^{\infty} \frac{n^3-1}{n!}, \\
d) \sum_{n=2}^{\infty} \frac{1}{n^2-1}, & o) \sum_{n=1}^{\infty} \frac{n^2+1}{(n+1)!}, \\
e) \sum_{n=1}^{\infty} \frac{1}{4n^2+16n+7}, & p) \sum_{n=2}^{\infty} \frac{n^2+5n+7}{(n+2)!}, \\
f) \sum_{n=2}^{\infty} \frac{1}{(n+1)^2-4}, & q) \sum_{n=1}^{\infty} (-1)^n \frac{2n+1}{n(n+1)}, \\
g) \sum_{n=1}^{\infty} \frac{3n^2+7n+6}{n(n+1)(n+2)(n+3)}, & r) \sum_{n=1}^{\infty} \frac{n(n+1)}{2^n}, \\
h) \sum_{n=1}^{\infty} \frac{1}{(n-1+\sqrt{3})(n-2+\sqrt{3})(n+\sqrt{3})}, & s) \sum_{n=1}^{\infty} \frac{n^2}{3^n}, \\
i) \sum_{n=1}^{\infty} \frac{n^2+3n+1}{n^2(n+1)^2}, & t) \sum_{n=1}^{\infty} (n+1)x^n, \\
j) \sum_{n=1}^{\infty} \frac{n^2(n+1)^2}{n!}, & u) \sum_{n=1}^{\infty} (-1)^n \frac{n^2-n}{3^n}, \\
k) \sum_{n=1}^{\infty} \frac{3^n(n-3)}{n!}, & v) \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}+(n+1)\sqrt{n}}, \\
l) \sum_{n=1}^{\infty} \frac{n^3-n+1}{n!3^n}, & w) \sum_{n=1}^{\infty} \left(\log\left(\frac{n}{n+1}\right)^n - \frac{1}{2n} + 1 \right).
\end{array}$$

8.4. Hallar la suma de las siguientes series:

a) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots,$

b) $\frac{1}{4} - \frac{1}{3} + \frac{1}{8} - \frac{1}{9} + \frac{1}{12} - \frac{1}{15} + \frac{1}{16} - \frac{1}{21} + \frac{1}{20} - \dots,$

c) $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} - \frac{1}{2} + \frac{1}{9} + \frac{1}{11} + \frac{1}{13} + \frac{1}{15} - \frac{1}{4} + \dots,$

d) $1 + \frac{1}{3} - \frac{1}{2} - \frac{1}{4} - \frac{1}{6} + \frac{1}{5} + \frac{1}{7} - \frac{1}{8} - \frac{1}{10} - \frac{1}{12} + \dots,$

e) $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2} + \dots$