Amortization Systems (types of loans)

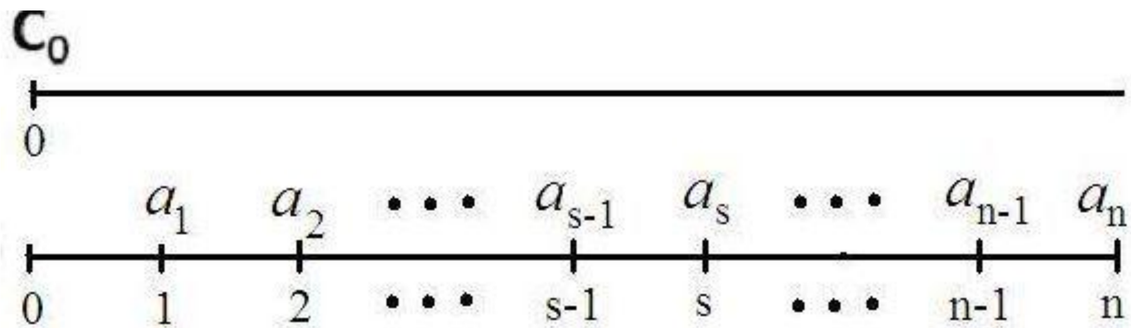
The most common methods to pay off the money lent by a financial Institution are the following:

1. French Loan
2. American Loan
3. Italian Loan
4. Geometric Loan

French Loan

The French Loan is characterized by :

- The interest rate is constant.
- The payments are periodic and constant.



At time point 0 the financial value of the amount of money lent by the creditor coincides exactly with the value of all the payments that the debtor should make. So,the financial equivalence at time point 0 (t=0) is:

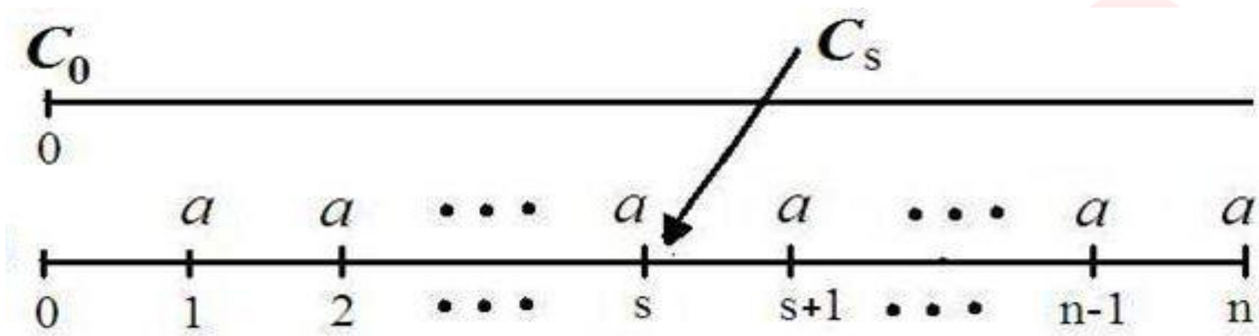
$$C_0 = a \cdot a_{\overline{n|}i} \Rightarrow$$

Payments :

$$a = \frac{C_0}{a_{\overline{n|}i}}$$

French Loan

The outstanding capital or mathematical reserve to the right can be determined by any of these three methods: retrospective, prospective or recurrent .



Retrospective M. → $$C_s = C_0(1+i)^s - a \cdot S_{\overline{s|}i}$$

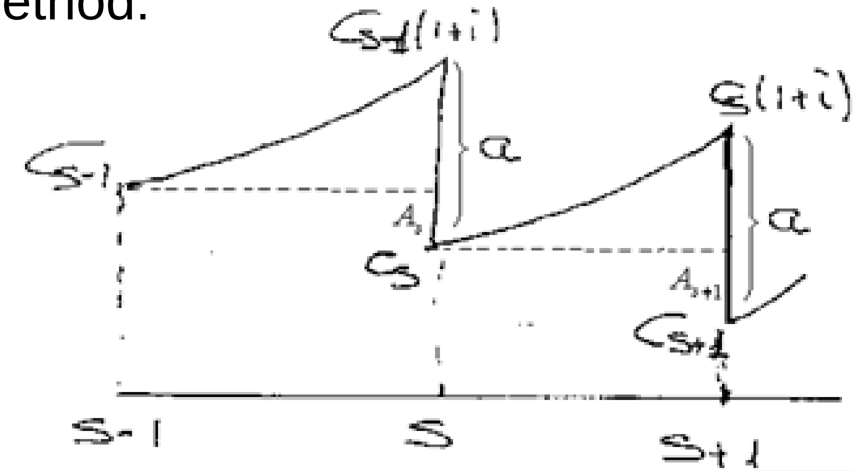
Prospective M. → $$C_s = a \cdot a_{\overline{n-s|}i}$$

Recurrent M. → $$C_s = C_{s-1}(1+i) - a$$

French loan

PRINCIPAL REPAYMENT

Let's calculate the outstanding capital at the end of periods s and s+1, using the recurrent method:



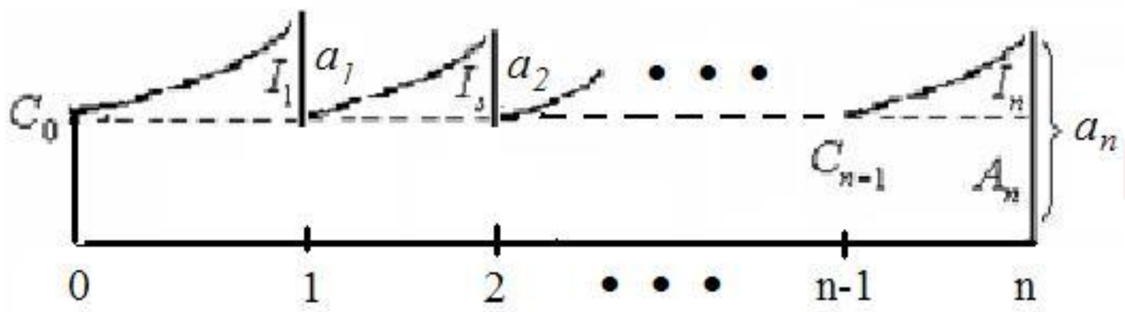
$$\left.\begin{aligned} C_s &= C_{s-1}(1+i) - a \\ C_{s+1} &= C_s(1+i) - a \end{aligned}\right\} \Longrightarrow C_s - C_{s+1} = (C_{s-1} - C_s)(1+i)$$

$$A_{s+1} = A_s(1+i) \Longrightarrow A_s = A_1(1+i)^{s-1}$$

The principal repayments vary geometrically and the ratio is (1+*i*)

American Loan

The debtor will only return the interest of every period at the end of the period. Except in the last period where not only does the debtor pay the interest but also the loan principal too.



The outstanding capital at the end of every period will coincide with the loan principal. And the principal paid on the loan at the end of every period will be 0. This will be from the first period to the penultimate but not in the last . At the end of the last period the borrower will repay the interest together wit the loan principal.

OUTSTANDING CAPITAL

$$C_s = C_0 \quad s = 1, \ldots, n-1$$

$$C_n = 0$$

PRINCIPAL PAID ON THE LOAN

$$M_s = 0 \quad s = 1, \ldots, n-1$$

$$M_n = C_0$$

American Loan

All the Payments made at the end of the periods will coincide with the interest of every period except the last one.

The last payment can be broken into two parts:

1. The interest of this last period
2. The loan principal

The principal repayments will be 0 in every period except in the last one . In the last period the principal repayment will be equal to loan principal.

Payments
$a_s = I_s = C_0 \cdot i_s \quad (\mathrm{s} = 1, \ldots, \mathrm{n}-1)$
$a_n = I_n + C_0 = C_0 \cdot i_n + C_0$

Principal Repayments
$A_s = 0 \quad (\mathrm{s} = 1, \ldots, \mathrm{n} - 1)$
$A_n = C_0$

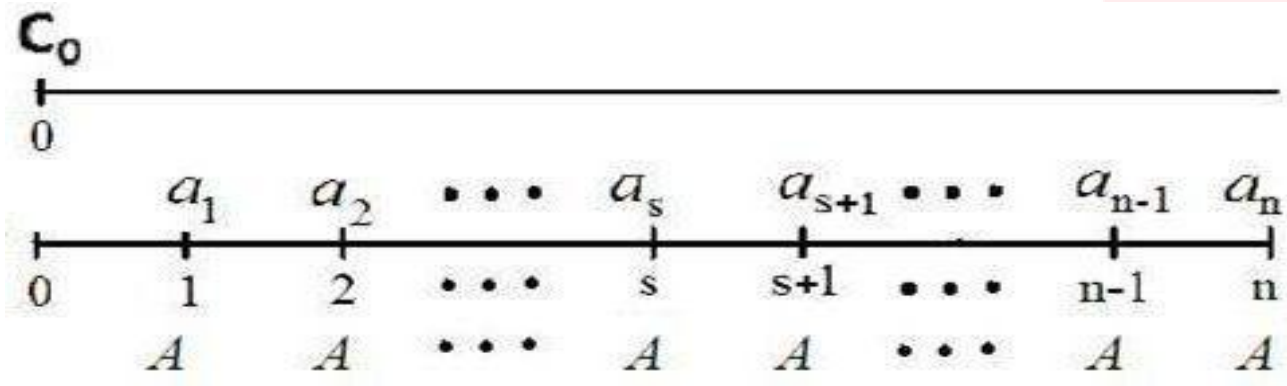
Italian Loan

The Italian loan is characterized by the following :

All **principal loan repayments are periodic and constant**

$$A = A_1 = A_2 = \cdots = A_n$$

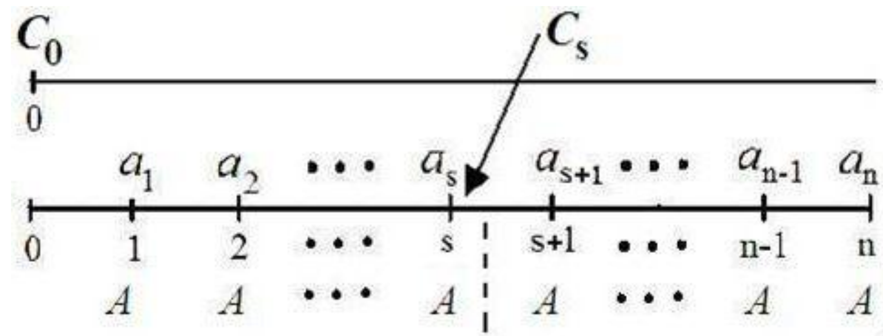
This means that the borrower will repay the same amount of money at the end of every period to pay off the debt.



Due to the fact that ,the sum of all the principal repayments coincides with the loan principal.

$$C_0 = \sum_{s=1}^{n} A = n \cdot A \Longrightarrow A = \frac{C_0}{n}$$

Italian Loan



In every moment the decrease of the **outstanding capital will be constant**. that's why we can determine the outstanding capital at the end of any period using any of these three methods.

By means of previous principal loan repayments (*Retrospective Method*)

$$C_s = C_0 - s \cdot A$$

By means of the next principal loan repayments. (*Prospective* Method)

$$C_s = (n-s)A$$

By means of the outstanding capital at the end of the previous period (*Recurrent Method*)

$$C_s = C_{s-1} - A$$

Italian Loan

THE VARIATION OF THE INTEREST AND THE PAYMENTS

Using the Recurrent Method and supposing that the interest rate is constant

$$C_s = C_{s-1} - A \Rightarrow i \cdot C_s = i \cdot (C_{s-1} - A) \Rightarrow i \cdot C_s = i \cdot C_{s-1} - i \cdot A$$

$$I_{s+1} = I_s - i \cdot A$$

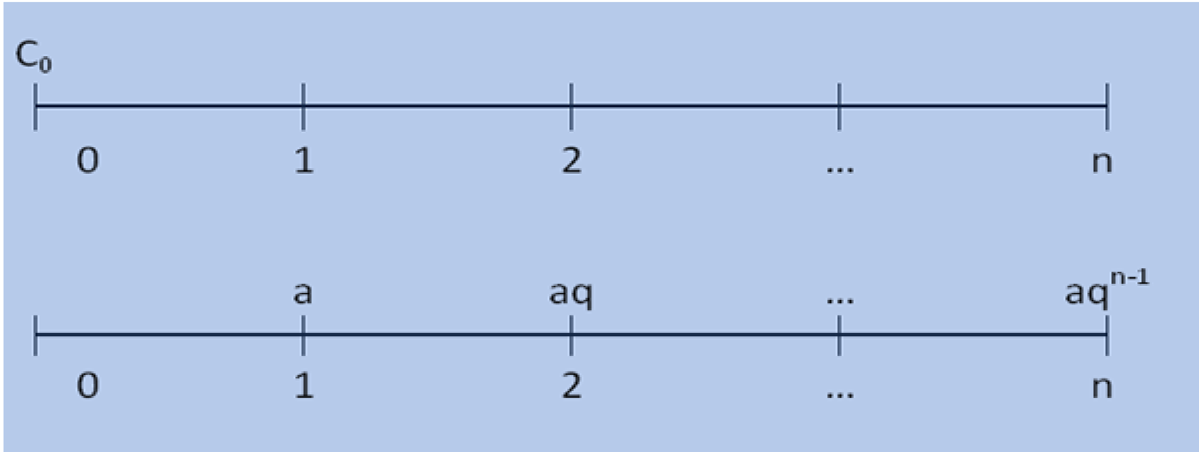
The interest decreases arithmetically in $-iA$

$$I_{s+1} = I_s - i \cdot A \Rightarrow A + I_{s+1} = A + I_s - i \cdot A$$

$$a_{s+1} = a_s - i \cdot A$$

The payments are decreasing arithmetically in $-iA$

Geometric Loan



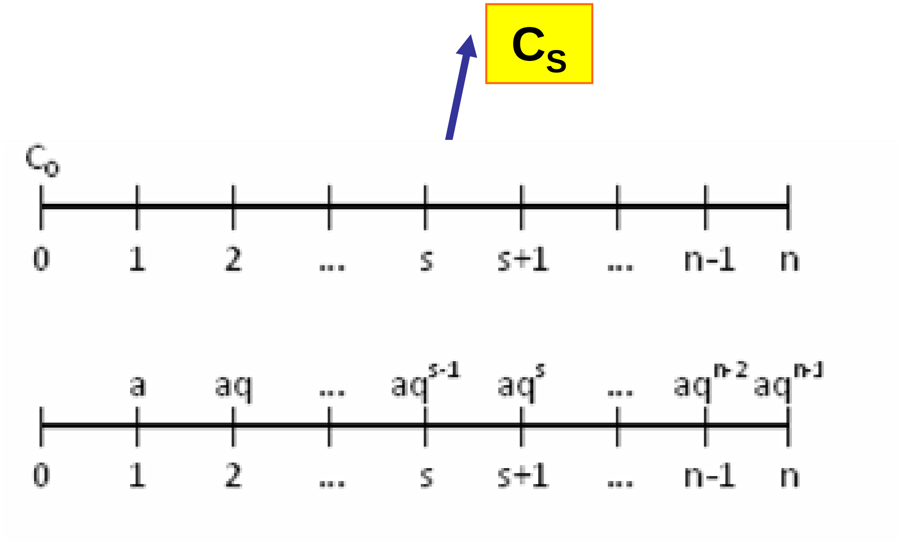
The financial value of the parties (lender and borrower) involved in a loan is equal at any point in time. So if we calculate the financial equivalence at time point 0 (t=0) we will get the following .

$$C_0 = A(a;q)_{\overline{n}|i} \Rightarrow \text{Payments}$$

$$a = \frac{C_0}{A(1;q)_{\overline{n}|i}}$$

Geometric Loan

Outstanding capital or mathematical reserve:



Retrospective M. → $$C_s = C_0(1+i)^s - S(a;q)_{\overline{s}|i}$$

Prospective M. → $$C_s = A(a_{s+1};q)_{\overline{n-s}|i} = A(a \cdot q^s;q)_{\overline{n-s}|i}$$

Recurrent M. → $$C_s = C_{s-1}(1+i) - a \cdot q^{s-1}$$