MECANICA DE FLUIDOS E HIDRÁULICA II
(FLUID MECHANICS II)

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The following topics are covered:

→ External & internal flow
→ Pressure distributions & forces on the aircraft
→ Numerical simulation
→ Computational fluid dynamics

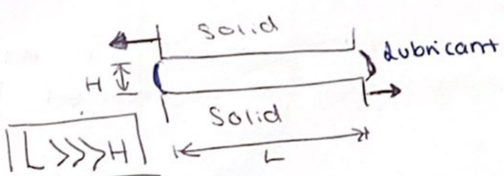
-- Pequeño resumen de qué vamos a dar:

$$u\left(\frac{\partial u}{\partial x}\right) + v\left(\frac{\partial u}{\partial y}\right) = \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

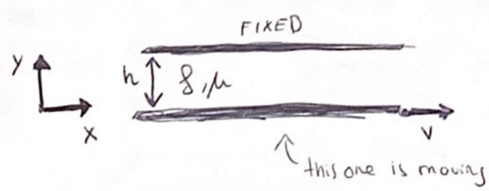
$$Re = \frac{\text{Inertial}}{\text{viscous}} \qquad \boxed{f(\text{viscous}) >> f(\text{inertial})}$$

$$Re <<< 1$$

when this happens it's called Stokes flow, and it could happen in a situation like this:



A point flow is when there is a fluid between two moving plates:



We assume the following:

1. Steady state
2. 1D
3. Newtonian
4. No gravity
5. incompressible
6. Pression gradient is zero

so we have the continuous equation for incompressible flow:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad ; \quad \text{y que el gradiente de presión es zero}$$

asique: $\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}}^{\,0} = 0$; además tenemos el momentum equation:

$$\cancel{\frac{\partial u}{\partial t}} + u\cancel{\frac{\partial u}{\partial x}} + \cancel{v}\frac{\partial u}{\partial y} = \cancel{\frac{1}{\rho}\frac{\partial P}{\partial x}} + \cancel{g_x} + \frac{\mu}{\rho}\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2}\right)$$

tachamos por lo que asumimos y tenemos la siguiente ecuación

$$\boxed{\frac{\mu}{\rho}\cdot\frac{\partial^2 u}{\partial y^2} = 0}$$

Another type of flow is when there is two fixed planes, called Poiseuille

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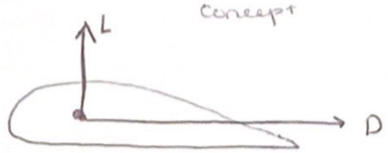
A $\underbrace{\text{potential fluid}}_{\text{concept}}$ for an airfoil has to assume
- Inviscid
- Incompressible
- Irrotational

Ideal fluid



→ If we want the airfoil to have drag, we can't use the potential fluid concept because it's only for ideal fluids, we need to use : Bernoulli

CONTENIDO :

- Fluid - dynamic lubrication
- Intro to the fluids in porous media
- Gas dynamics
- Liquids in ducts
- Laminar & turbulent boundary layer
- Application for the distribution of pressures & forces on the aircraft
- Computational fluid dynamics, practice advanced on fluid dynamics simulators

ASSESMENT :

- Final exam ....(min. 5/10)
- Project ....(minim. 5/10)
- Homework, lab reports....(minimum 5/10)

First exam period

In order to be evaluated you must have a minimum of 50% attendance

Exercises, tasks	20%
Lab & report	10%
Project	20%
Oral presentation	15%
Final exam	35%

Second exam period

Recall :

Continuum hypothesis: Materials and transport properties

- Newtonian fluids

  Relation Stress vs Rate of strain ; pressure & density variations

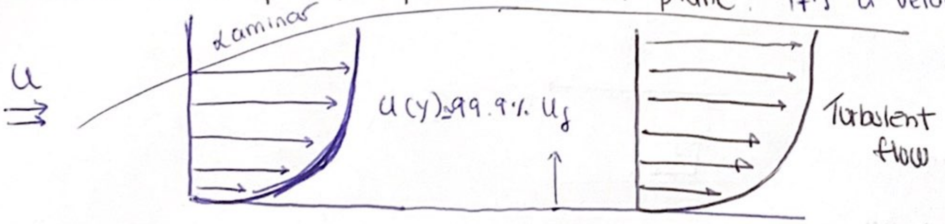
- Reynolds number , Navier - Stokes eqns - additional body forces
  interfacial tension : statics , interface deformation , gradients.

Types of effects on fluid :

- Von Karma vortex

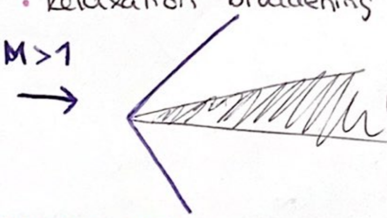
- Flat plate at zero incidence : the stream lineas near the plate are a bit curved due to the viscosity.
  We have the boundary layer, the momentum thickness & the displacement thickness.

- Blasius boundary - layer profile on a flat plane. It's a velocity profile for a laminar flow



- Leading - edge separation on a plate with laminar reattachment. A flat plate 2% thick which is incide 2.5° to the stream. La capa laminar límite se separa en la parte superior en el borde.

- Relaxation broadening of the shock wave from a wedge : It is a shock wave (onda de choque)

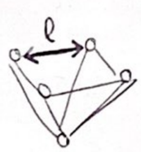


Elementory concepts :

Molecular Dynamics

- Knusend number (K) : $K = \dfrac{l \text{ (mean free path)}}{L \text{ (Characteristic lenght)}}$ if $K<1$: continuos bc $L>l$

  if $K>1$: $l>L$ Molecular dynamics



This has something to do with nanotubes.

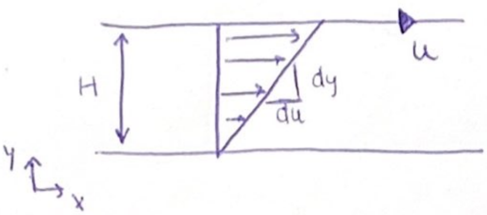
• Thin films · Experiments on shearing bt 2 molecularly smooth (mica) surfaces separated by thin films of organic liquids. Example: Thin-film photovoltaic cells.

3 types
- Films > 10 molecular diameters can be described in terms of bulk properties
- Thinner films: molecular ordering, quantization of some properties, "effective viscosities" > 10^5 bulk value
- Film w/ thickness less than 5 molecular diameters: "solid-like" response.

• Viscosity & Newtonian Fluids

τ ≡ SHEAR STRESS (Force/Area)

unidades $\rightarrow \dfrac{kg}{m\,s^2} \cdot \dfrac{s}{m^2} \cdot \dfrac{m}{s \cdot m} = N \cdot \dfrac{s}{m^2} \cdot \dfrac{m}{s \cdot m} = \dfrac{N}{m^2} = Pa$; $N \cdot s$; $N = kg\,\dfrac{m}{s^2}$



$$\boxed{\underset{(Pa)}{\tau} = \mu \frac{U}{H}}$$

donde μ ≡ shear viscosity

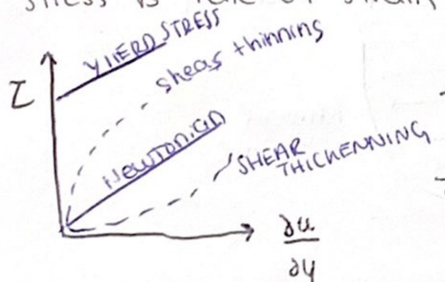
$\tau \propto \dfrac{\partial u}{\partial y}$; $\tau\,(\text{shear stress}) = \mu \dfrac{du}{dy}$ (Newtonian)

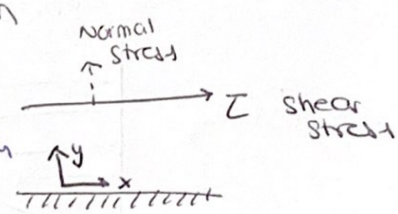
Proporcional

Por lo tanto u ≡ velocity $= y \cdot \dfrac{v}{h}$

• On to equations of motions

(a) stress vs rate of strain





(b) Navier-stokes equations

Assume that the material properties ρ & μ are constant

MASS BALANCE (continuity) $\nabla \cdot u = 0$

LINEAR MOMENTUM BALANCE
$$\underbrace{\rho\left(\frac{\partial u}{\partial t} + u \cdot \nabla \cdot u\right)}_{\text{ACCELERATION}} = \underbrace{-\nabla p + \mu \nabla^2 u}_{\text{SURFACE FORCES}} + \underbrace{\rho \cdot g}_{\text{BODY FORCES}}$$

Navier Stokes

$\Longrightarrow$ EACH TERM FORCE/VOLUME

(c) pressure changes accompanying flow

- inertially dominated : $\Delta p = O(\rho U^2)$
- viscously dominated : $\Delta p = O\left(\mu \cdot \dfrac{U}{\ell}\right)$

U ≡ typical velocity

ℓ ≡ characteristic lenght.

Euler equation : $\rho\left(\dfrac{\partial u}{\partial t} + u \cdot \nabla u\right) = \Delta p + \rho \cdot g$

$u \cdot \nabla u = \Delta p + g$

& Bernoulli : $\dfrac{P}{\rho} \cdot \dfrac{u^2}{2} + g z$ ≡ constant along the stream line.

+ on to equations of motions

(d) incompressibility ($\nabla \cdot u = 0$) $\quad \nu = \frac{1}{\rho}$

variation of density accompanying motion should be small ($\Delta\rho \ll \rho$)

$$\Delta\rho \approx \frac{\partial\rho}{\partial p} \cdot \Delta p \,, \quad c^2 = \left(\frac{\partial p}{\partial \rho}\right)_s \equiv c = \text{speed of sound}$$

$$\text{Mach} \equiv M = \frac{U}{c}$$

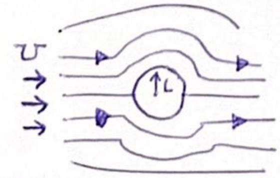
☆ Rembember, for ideal gas:

$$c = \sqrt{\gamma R T}$$

- Inertially dominated flows : $U/c \ll 1$
- Viscously dominated flows : $(U/c)^2 \ll \rho \cdot U \cdot \ell / \mu$

(e) Reynolds number

$$Re = \frac{\rho \cdot U \cdot \ell}{\mu} = \frac{U \cdot \ell}{\nu} \qquad \rho q \quad \nu = \frac{1}{\rho}$$



Low-Reynolds-number motions: lubrication, film coating, suspensions, MEMS, ... $\Rightarrow \quad 0 = -\nabla p + \mu \cdot \nabla^2 \cdot u$

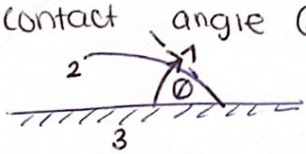
Es la relación entre las fuerzas inerciales y las fuerzas viscosas

- interfacial tension (Force / Lenght or energy / area)

(a) Statics

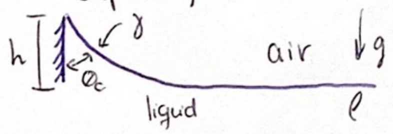
capillary lenght : $\ell_{cap} \left(\frac{\gamma}{\rho g}\right)^{1/2}$

contact angle ϕ_c

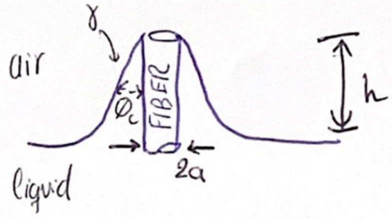


$$\gamma_{12} \cdot \cos \phi_c = \gamma_{13} - \gamma_{23}$$

capillary rise on vertical planes & fibers



$$h = f(\phi_c)\left(\frac{\gamma}{\rho g}\right)^{1/2}$$

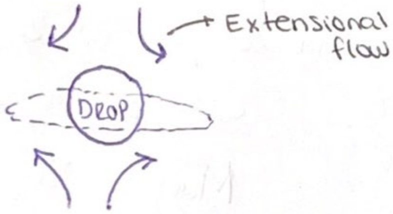


$$h = \left(\frac{\gamma}{\rho g}\right)^{1/2} \cdot f\left(\phi_c, \frac{\rho g a^2}{\gamma}\right)$$

Mirar

(b) dynamics

- drop deformation, formation of emulsions



$$\text{DEFORMATION} \propto f\left(\frac{\mu \cdot G a}{\gamma}\right)$$

$G \equiv$ shear rate
$a \equiv$ drop radius

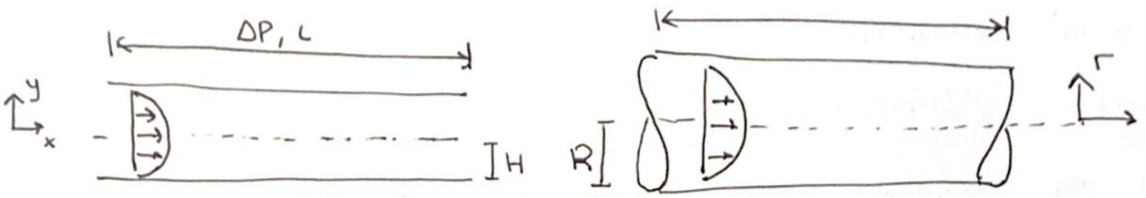
- drop spreading

PROTOTYPICAL FLOWS

Types → ① Steady pressure Driven Flow
→ ② Condition flow

1. Steady pressure Driven Flows

CHANNEL Y PIPES FLOWS



NO SLIP ON BOUNDARIES

$$u(y) = \frac{H^2}{2}\,\frac{\Delta P}{\mu L}\left[1 - \left(\frac{y}{H}\right)^2\right]$$

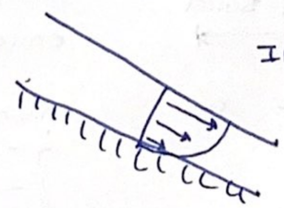
AVG. VELOCITY $\langle U\rangle = \frac{R^2}{8\mu}\cdot\frac{\Delta P}{L}$

$$u(r) = \frac{R^2}{4\mu}\cdot\frac{\Delta P}{L}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

MASS FLOW RATE : $Q = \frac{\pi}{8}\frac{\rho}{\mu}\cdot R^4\cdot\frac{\Delta P}{L}$

PARABOLLIC (POUSILLE) VELOCITY PROFILE

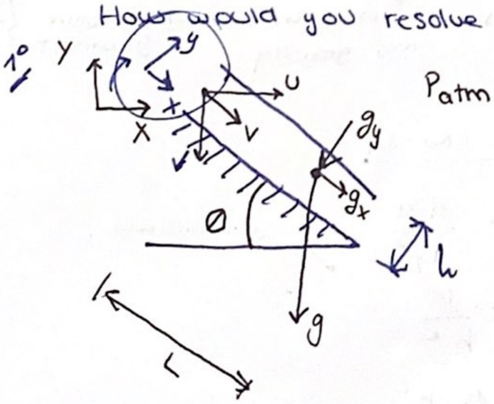
Applications :
- Blood flow
- Pipe flow
- Mems
- Film flows :



Inclined film

How would you resolve this?

1º



P_{atm}

2º Assuming :
1) 1 Dimension
2) Steady Rate
3) Incompressible
4) $\Delta P / \Delta L = 0$

3º We write down the Governing equation:

→ Continuity eq : $\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}}^{0} = 0$

→ Momentum eq (N-S) :

$$\rho\left[\cancel{\frac{\partial u}{\partial t}}^{0} + u\cdot\cancel{\frac{\partial u}{\partial x}}^{0} + v\cdot\cancel{\frac{\partial u}{\partial y}}^{0}\right] = \cancel{\frac{-\partial p}{\partial x}}^{0} + \rho\cdot g_x + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}}^{0} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$0 = g\cdot\sin\theta + \frac{\mu}{\rho}\cdot\frac{\partial^2 u}{\partial y^2}$$

Como $\frac{\mu}{\rho} = \nu$, entonces,

↳ $0 = g\cdot\sin\theta + \nu\cdot\frac{\partial^2 u}{\partial y^2}$, aplicamos las boundary conditions $\begin{cases} y=0 & u=0 \\ y=h & u=u_{max} \end{cases}$

Buscamos $u(y)$, que ya sabemos que será parabólico

External flow :

$Re = 5 \cdot 10^5$ Laminar

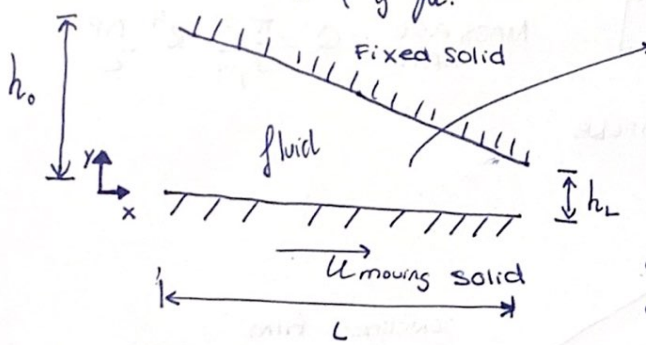
Transtitiona

$Re \geq 3.6 \cdot 10^6$ Turbulent

New concept : $\boxed{\text{Lubrication}}$

Invented by Reynolds (1886), pero ya se utilizaba en práctica por los egipcios, ya que echaban agua para que las rocas resbalaran y pudieran moverlas mejor.

Ejemplo : Tenemos 2 solidos, con uno en movimiento de característic-as ρ y μ.

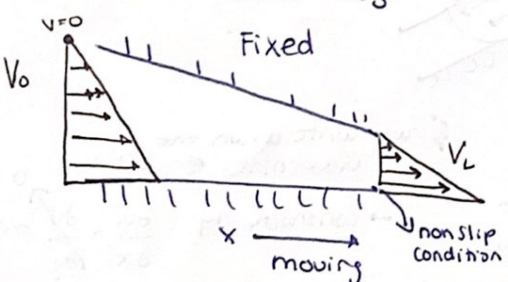


Donde $L >>> h(x)$

Este es un $\boxed{\text{covette flow}}$ porque una pared se está moviendo

Entonces, sabemos que en la pared de abajo aplicaremos non-slip condition y en la de arriba, el fluido tenará $u=0$.

Graficamente será algo así :



Asumimos :

- 1. Dimension
- S.S.
- Incompresible
- No gravity
- No pressure gradient

x - component (N-S)

$$\rho \cdot u \cdot \frac{du}{dx} = \mu \cdot \frac{d^2u}{dy^2}$$ cond. iniciales.

$$\rho \cdot \cancel{U} \cdot \frac{U}{L} = \mu \cdot \frac{\cancel{U}}{h^2}$$

$$\frac{\rho \cdot U}{L} = \mu \cdot h^2$$

$$\frac{\rho \cdot u}{L} = \mu \cdot h^2$$

$$\underbrace{\frac{\rho \cdot u}{\mu} \cdot \frac{L}{L}}_{Re} \cdot \frac{1}{L} = \frac{1}{h^2}$$

entonces. $\boxed{Re \cdot \frac{h^2}{L^2} = 1} \rightarrow \boxed{\frac{Re\, h^2}{L^2} <<< 1} \longrightarrow$ STOKES FLOW.

Dandole unidades :

SAE 50

$\nu = 7 \cdot 10^4\ m^2/s$

$L = 4cm$

$h = 0.1mm$

$u = 10 m/s$

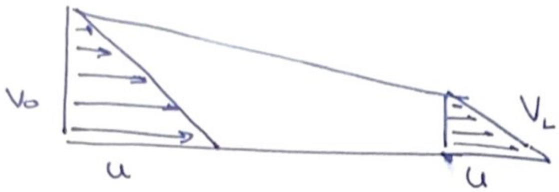
$$Re = \frac{10 \cdot 0.04}{7 \cdot 10^{-4}} = 570$$

y resolviendo $\frac{Re \cdot h^2}{L^2} = \frac{570(0.0001)^2}{(0.04)^2} = 0.004 <<< 1$ ✓

And It satisfies the stokes flow

¿Que problema hay con el dibujo?

Que no satisface la ec. de continuidad



$$V_0 = \left(u - \frac{u}{h_0} y\right)$$

$$V_L = \left(u - \frac{u}{h_L} \cdot y\right)$$

$$\dot{m}_0 = \dot{m}_L = \rho_0 \cdot V_{Avg_0} \cdot A_0 = \rho_L \cdot V_{Avg\,L} \cdot A_L$$

Ec. continuidad

Hallamos Avg. velocity:

- $V_{Avg_0} = 1/A_{cs} \int_A V_0 \cdot dA = \frac{1}{h_0 \times 1} \int_0^{h_0} \left(u - \frac{u y}{h_0}\right) dy$

$$\cancel{V_{Avg\,L} = \frac{1}{h_L}} = \frac{1}{h_0}\left[\left(uy - \frac{uy^2}{h_0 2}\right)\right]_0^{h_0} = \frac{1}{\cancel{h_0}}\left(u\cancel{h_0} - \frac{u\cancel{h_0^2}}{\cancel{h_0 2}}\right)$$

$$\boxed{V_{Avg_0} = \frac{u}{2}}$$

- $V_{Avg\,L} = \frac{1}{h_L}\int_0^{h_L} \left(u - \frac{uy}{h_L}\right) dy = \frac{u}{2}$

- $\cancel{\rho_0} \frac{\cancel{u}}{2} \cdot h_0 = \cancel{\rho_L} \cdot \frac{\cancel{u}}{2} \cdot h_L$

$$h_0 \neq h_L$$

x - component (NS)

$$\rho \cdot \frac{\partial u}{\partial t} + \cancel{u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y}} = -\frac{\partial P}{\partial x} + \rho g_x + \mu\left(\frac{\partial u^2}{\partial x^2} + \frac{\partial u^2}{\partial y^2}\right)$$

$$\boxed{\frac{dP}{dx} = \mu \frac{d^2 u}{dy^2} \equiv cte}$$

Boundary cond:
$y = 0 \quad u = u$
$y = h(x) \quad u = 0$

$$\boxed{u(y) = \frac{1}{2}\frac{1}{\mu}\frac{dP}{dx}\, y(y-h) + u\left(1 - \frac{y}{h(x)}\right)}$$

→ velocity profile

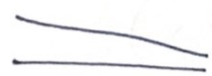
Ahora tenemos que encontrar la distribución de presión $\left(\frac{dP}{dx}\right)$, aplicando la ecuación de continuidad:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\int_0^h \frac{\partial u}{\partial x} \cdot dy + \int_0^h \frac{\partial v}{\partial y} = 0 \;;\; \int_0^h \frac{du}{dx} dy = -\int_0^h \frac{\partial v}{\partial y} \cdot dy \;\; // \int_0^h \frac{\partial u}{\partial x} dy = 0 \;\; // \int_0^h \frac{\partial v}{\partial y} dy = V(h) - V(0)$$

$$\left[\ \frac{\partial}{\partial x}\left[\frac{\partial P}{\partial x}\cdot h^3(x)\right] = 6\cdot\mu\cdot U\cdot\frac{dh(x)}{dx}\ \right]$$

$$P(0) = P_\infty = P(L)$$



$$\left[\ h(x) = h_0 + (h_0 - h_L)\cdot\frac{x}{L}\ \right]$$

Se acaba la clase, pero la solución es:

$$\left[\ \frac{P(x) - P_\infty}{\mu\cdot\frac{U\cdot L}{h_0^2}} = \frac{6\left(x/L\right)\left(1-\frac{x}{L}\right)\left(1-\frac{h_L}{h_0}\right)}{\left(1+\frac{h_L}{h_0}\right)\left(1-\left(1-\frac{h_L}{h_0}\right)\frac{x}{L}\right)^2}\ \right]$$

LUBRICATION 25/10

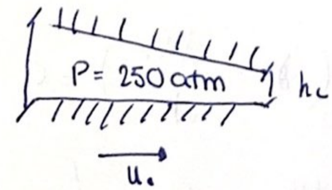
* Velocity and pressure profile

Example

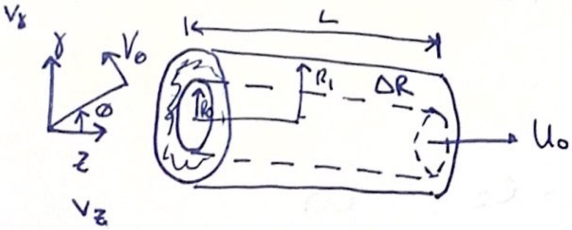
SAE = 50 $\qquad U = 10\ m/s \qquad L = 4\ cm \qquad h_0 = 0.1\ mm$

Resolución:
como ya conocemos h_0, primero hallamos:

$$\mu \frac{UL}{h_0^2} = (2.5 \cdot 10^7\ Pa) = 250\ atm$$



Ahora lo resolvemos para un cilindro 3D:



Suponemos que el cilindro interior se mueve con velocidad U_0, y el fluido está entre el interior y el exterior.

$V: (V_r; V_\theta; V_z)$

$V: (0; 0; V_z)$

$\Delta R = R_1 - R_0 <<< L$

Asumimos:
1) Steady state
2) 1D in z-direction
3) Incompressible
4) No gravity
5) Pressure & temperature gradient are neglected

Governing equation:

· Continuity:

$$\frac{1}{r}\cancel{\frac{\partial(r V_r)}{\partial r}}_{0} + \frac{1}{r}\cancel{\frac{\partial V_\theta}{\partial \theta}}_{0} + \frac{dV_z}{\partial z} = 0 ;$$

$$\frac{\partial V_z}{\partial z} = 0 \Rightarrow V_z(r) = f(r)$$

· Momentum (NS)

z-component (Navier Stokes)

$$\rho\Big(\underbrace{\cancel{\frac{\partial V_z}{\partial t}}}_{0\ \text{steady state}} + \underbrace{\cancel{V_r \cdot \frac{\partial V_z}{\partial r}}}_{0} + \underbrace{\cancel{\frac{V_\theta}{r}\cdot\frac{\partial V_z}{\partial \theta}}}_{0} + \underbrace{\cancel{V_z \cdot \frac{\partial V_z}{\partial z}}}_{0}\Big) = \overbrace{\cancel{-\frac{\partial p}{\partial z}}}^{\text{no hay gradiente} \to 0} + \overbrace{\cancel{\rho \cdot g_z}}^{\text{no hay gravedad} \to 0} + \mu\left[\frac{1}{r}\cdot\frac{\partial\left(r \cdot \frac{\partial V_z}{\partial r}\right)}{\partial r}\right] +$$

$$+ \mu\left[\underbrace{\cancel{\frac{1}{r^2}\cdot\frac{\partial^2 V_z}{\partial \theta^2}}}_{0\ \text{no depende de}\ \theta} + \underbrace{\cancel{\frac{\partial^2 V_z}{\partial z^2}}}_{\text{pq}\ \frac{\partial V_z}{\partial z} = 0}\right]$$

$$0 = \frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial\left(r \cdot \frac{\partial V_z}{\partial r}\right)}{\partial r}\right]$$

como sabemos que $\frac{\mu}{\rho} \neq 0$ (sino, no habría fluido),

entonces $\frac{1}{r}\left[\frac{\partial}{\partial r}\left(\frac{r \cdot \partial V_r}{\partial r}\right)\right] = 0$

. . .

Boundary conditions

$r(R_o) = U_o$

$r(R_1) = 0$

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \quad \frac{1}{\gamma}\left[\frac{\partial}{\partial r}\left(\frac{\gamma \cdot \partial V_r}{\partial \gamma}\right)\right] = 0$$

Solving the diferential eq by separation of variables

$$\frac{d}{dr}\left(\gamma \cdot \frac{dV_r}{d\gamma}\right) = 0$$

$$\gamma \cdot \frac{dV_r}{d\gamma} = C_1 \qquad V_\gamma = C_1 \cdot Ln(\gamma) + C_2$$

$$\frac{dV_r}{d\gamma} = \frac{C_1}{\gamma}$$

$$V_z(R_o) = U_o = C_1 \cdot Ln(R_o) + C_2 \qquad (1)$$

$$V_z(R_1) = 0 = C_1 \cdot Ln(R_1) + C_2 \qquad (2)$$

$$C_1 - C_2 :> \quad U_o - 0 = C_1 \cdot Ln(R_o) + \cancel{C_2} - \left[C_1 \cdot Ln(R_1) + \cancel{C_2}\right]$$

$$U_o - 0 = C_1\left[\ln(R_o) - Ln(R_1)\right]$$

$$\left[\, C_1 = \frac{U_o}{Ln(R_o/R_1)} \,\right]$$

$$C_2 = -C_1 \cdot Ln(R_1)$$ ← Reeplazo C1 en ecuación 2.

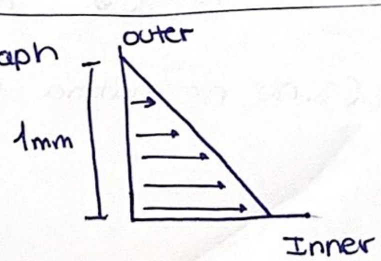
$$V_z(\gamma) = \frac{U_o}{Ln(R_o/R_1)}\left[\ln(\gamma) - Ln(R_1)\right]$$

$$\boxed{V_z(\gamma) = \frac{U_o \cdot Ln(\gamma/R_1)}{Ln(R_o/R_1)}}$$

Shear stress

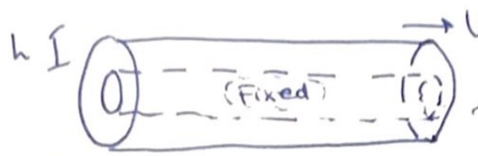
tau ↙ $$\boxed{\tau = \mu \cdot \frac{dV_z}{d\gamma} = \mu \cdot \frac{U_o}{\gamma \cdot Ln(R_o/R_1)}}$$

Graph



★ + velocity & pressure profile

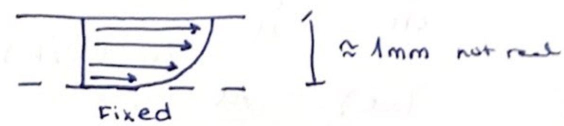
Example 2



El cilindro exterior se mueve y el interior está quieto, al contrario que en el ejemplo 1.
Así que ya sabemos V_z:

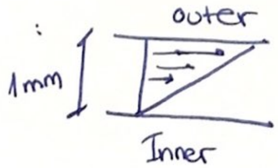
$$\left[V_z = \frac{U_0 \cdot Ln\,(\gamma/R_0)}{Ln\,(R_1/R_0)} \right]$$

Si hacemos zoom en el fluido

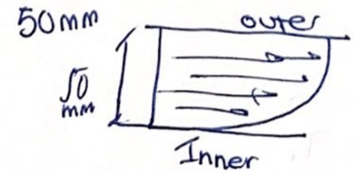


Si el hueco h es muy pequeño, o más grande

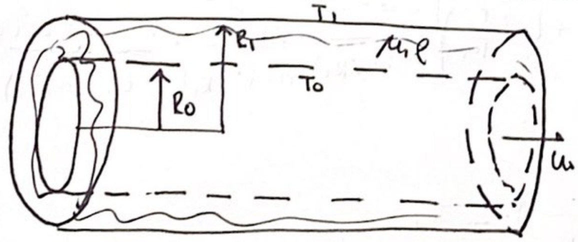
1 mm :



↔



★ Heat transfer between cocentric cylinders (same example as before)



El fluido se encuentra entre ambos cilindros, el interior se mueve.
Ya conocemos:

$$V_z = \frac{U_0\, Ln\,(\gamma/R_1)}{Ln\,(R_0/R_1)}$$

Necesitamos the <u>Energy equation</u>:

$$\rho \cdot C_p \Big(\underbrace{\cancel{\frac{\partial T}{\partial t}}}_{0} + \underbrace{\cancel{V_r \cdot \frac{\partial T}{\partial r}}}_{0} + \underbrace{\cancel{\frac{V_\theta}{r} \cdot \frac{\partial T}{\partial \theta}}}_{0} + \underbrace{V_z \cdot \frac{\partial T}{\partial z}}_{\neq 0\ \to 0} \Big)_{\text{convection}} = -\overset{\text{cte}}{K} \overbrace{\left[\frac{1}{r} \cdot \frac{\partial (r \cdot \frac{\partial T}{\partial r})}{\partial r} + \underbrace{\cancel{\frac{1}{r^2} \cdot \frac{\partial^2 T}{\partial \theta^2}}}_{0} + \underbrace{\cancel{\frac{\partial^2 T}{\partial z^2}}}_{0} \right]}^{\text{conduction}} + \Phi$$

Φ → Due to friction: VISCOUS DISSIPATION

La dirección de heat transfer is predomently ↗ in the r direction, así que $V_z \cdot \frac{dT}{\partial z} \sim 0$, en otras y $T(r)$.

Viscous dissipation:

$$\Phi = \mu \Bigg[2 \left(\left(\cancel{\frac{\partial V_r}{\partial r}} \right)^2 + \left(\frac{1}{r} \cdot \cancel{\frac{\partial V_\theta}{\partial \theta}} + \cancel{\frac{V_r}{r}} \right)^2 + \left(\frac{\partial V_z}{\partial z} \right)^2 \right) + \left(\frac{1}{r} \cdot \cancel{\frac{\partial V_z}{\partial \theta}} + \cancel{\frac{\partial V_\theta}{\partial z}} \right)^2 +$$
$$+ \left(r \cdot \frac{\partial}{\partial r} \left(\cancel{\frac{V_\theta}{r}} \right) + \frac{1}{r} \cancel{\frac{\partial V_r}{\partial \theta}} \right)^2 + \left(\cancel{\frac{\partial V_r}{\partial z}} + \frac{\partial V_z}{dr} \right)^2 \Bigg]$$

Differential Equation

$$\frac{1}{\gamma} \cdot \frac{d}{dr} \left(r \cdot \frac{dT}{dr} \right) = \frac{\mu}{K}$$

separation of variables!!

↓

...

Separation of variable

$T(r)$

$$r \cdot \frac{dT}{dr} = \frac{\mu}{k} \cdot \frac{U_0^2}{Ln^2(R_0/R_1)} \cdot \ln r + C_1$$

$$\frac{dT}{dr} = \frac{\mu}{k} \cdot \frac{U_0^2}{Ln^2(R_0/R_1)} \cdot \frac{Ln\, r}{r} + \frac{C_1}{r}$$

$$T(r) = \frac{\mu}{k} \cdot \frac{U_0^2}{Ln^2(R_0/R_1)} \cdot \frac{Ln^2 r}{2} + C_1 \cdot \ln r + C_2$$

BC'S

$T(R_0) = T_0$

$T(R_1) = T_1$

$$T(r) = \frac{\mu \cdot U_0^2}{2k \cdot Ln^2(R_0/R_1)} \cdot \left[\ln^2(r) - Ln^2(R_1)\right] + \ln\left(\frac{r}{R_1}\right)\left[\frac{(T_0 - T_1)}{Ln(R_0/R_1)} - \frac{\mu U_0^2}{Ln^2(R_0/R_1)} \cdot \frac{Ln^2(R_0) - Ln^2(R_1)}{Ln(R_0/R_1)}\right] + T_1$$

Only by conduction

$$\boxed{T(r) = \frac{T_0 - T_1}{Ln(R_0/R_1)} \cdot \ln\left(\frac{r}{R_1}\right) + T_1}$$

by conduction

$$T(R_0) = T_0 = T_0 - \cancel{T_1} \cdot \cancel{\frac{Ln(R_0/R_1)}{Ln(R_0/R_1)}} + \cancel{T_1} \checkmark$$

$$T(R_1) = T_1 = (T_0 - T_1) \frac{\cancel{Ln(R_1/R_1)} \to 0}{Ln(R_0/R_1)} + T_1 = T_1 \checkmark$$

Heat transfer on the creeping

ENERGY EQUATION

$$\rho C_p \left(\frac{\partial T}{dt} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = K \nabla^2 T + \Phi$$

Assumptions

1) Steady state flow
2) Temperature gradient is on x-axis
3) 1D flow u-component
4) $\Phi = 0$

$$\left[\rho C_p U \frac{\partial T}{\partial x} = K \nabla^2 T \right]$$

Dimensionless Analysis

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial v^*}{dy^*} = \frac{1}{Re} \nabla^{*2} u^*$$

$Re >>> 1$

$Re <<< 1$

$$X^* = \frac{x}{L} \qquad u^* = \frac{u}{U}$$

$$T^* = \frac{T}{T_1 - T_0}$$

$$\left[u = \frac{\partial T}{\partial x} = \frac{K}{\rho C_p} \nabla^2 T \right]$$

$$u^* U \frac{\partial T^*}{\partial x^* L} (T_1 - T_0) = \frac{K}{\rho C_p} \frac{\nabla^{*2} T^* (T_1 - T_0)}{L^2}$$

$$\boxed{u^* \frac{\partial T^*}{\partial x^*} = \underbrace{\frac{K \cdot \cancel{L}}{\rho \cdot C_p \cdot L^{\cancel{2}} U}}_{\text{adimensional}} \cdot \nabla^{*2} T^*}$$

$$\frac{K}{\rho C_p \cdot U \cdot L} = \frac{1}{Pe}$$

Peclet Number

$$\boxed{Pe = \frac{\rho \cdot C_p \cdot U \cdot L}{K}}$$

★ Juguemos con Peclet number:

$$Pe = \frac{\rho \cdot C_p \cdot U \cdot L}{K} \times \frac{\mu}{\mu} = Re \times Pr$$

where: $Pr \equiv$ Prandtl number $= \dfrac{C_p \cdot \mu \cdot \rho}{K \cdot \rho} = \dfrac{\nu}{\alpha} = \dfrac{\text{Viscous dissipation } \cancel{\text{rate}}}{\text{Thermal diffusion rate}}$

$$Pr = \frac{\text{Kimatic viscosity}}{\text{Thermal diffusivity}}$$

If the Prandtl number:

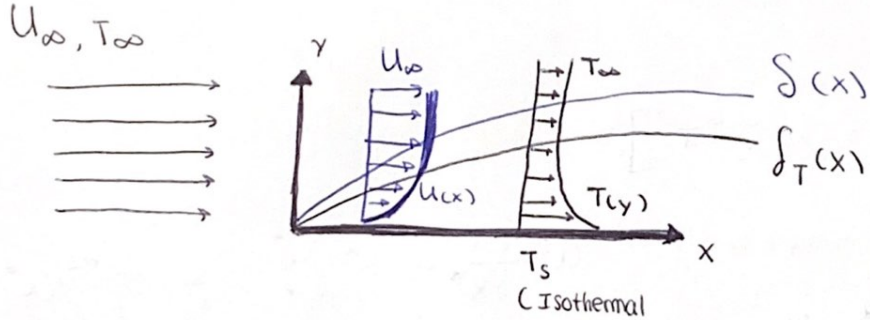
$Pr \downarrow\downarrow\downarrow$ (low) $\Longrightarrow$ Conductive Transfer is Strong $\uparrow\uparrow\uparrow$

(Liquid metals)

$Pr \uparrow\uparrow\uparrow$ (high) $\Longrightarrow$ Convective Transfer is strong $\uparrow\uparrow\uparrow$

(water, oil)

Thermal Boundary Layer



$$\frac{\delta}{\delta_T} = Pr^n$$

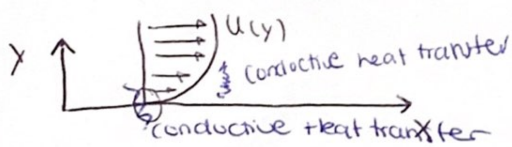
(n = positive number)

For liquid metals $\delta \ll \delta_T$

For oil, water $\delta \gg \delta_T$

For gases $\delta \cong \delta_T$

$$\frac{T(y) - T_\infty}{T_s - T_\infty} = 0.99 \Rightarrow \delta_T$$



$U(0) = 0$ no-slip condition

$$q_s'' = -k \left.\frac{\partial T}{\partial y}\right|_{y=0} \quad \text{Fourier's law}$$

$$q_s'' = h\,(T_s - T_\infty) \quad \text{Newton's cooling law}$$

$$h \text{ (heat thermal conductivity)} = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$$

Experimental results for the local heat transfer coefficient for flow over a flat surface

$$h_x(x) = a\,x^{-0.1}$$

where a is a coefficient $\left(\frac{W}{m^{1.9}-K}\right)$ and x (m) distance from the L.E.

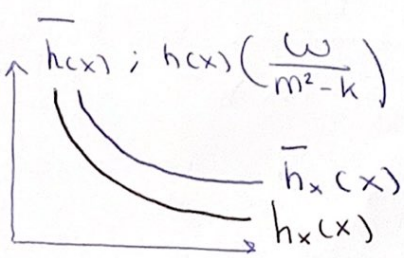
Determine the ratio of the avergge $\overline{h_x}(x)$ to the local h_x heat transfer coefficient.

Solution:

$$\overline{h_x(x)} = \frac{1}{X}\int_0^x h_x(x)\,dx$$

$$\overline{h_x(x)} = \frac{1}{X}\int_0^x a\,x^{-0.1}\,dx \;;\quad \overline{h}_x(x) = \frac{a}{X}\left(\frac{X^{0.9}}{0.9}\right) = 1.11\,a\,x^{-0.1}$$

$$\frac{\overline{h_x(x)}}{h_x(x)} = 1.11$$



Enunciado:

* $U = 10\,m/s$; the chord lenght is 1 m and the flow (a) air and (b) water, at sea level conditions.

Determine thermal boundary layer

Answer:

$$Re = \frac{\rho \cdot U L}{\mu} = \frac{1{,}21 \times 10 \times 1}{1{,}789 \cdot 10^{-5}} = 676355{,}51$$

In case air (a)

Asumimos que es laminar flow

* The Prandtl # for air is $Pr \simeq 0.7$

$$\frac{\delta}{\delta_T} = Pr^{1/3} \;;\quad \delta = \frac{5 \times X}{\sqrt{Re}} = \frac{5 \times 1}{\sqrt{676355}} = 6.07 \cdot 10^{-3}\ m$$

$$\delta_T = \frac{\delta}{(0.7)^{1/3}} = 6{,}89 \cdot 10^{-3}\,m$$

Assuming turbulent flow:

$$\delta = \frac{0.38 \times X}{(Re)^{1/5}} = 0.025\ m$$

$$\delta_T = \frac{\delta}{Pr^{1/3}} = 0.029\,m$$

* if it's a compressible flow $\delta_T = \delta / Pr^{2/3}$

(b) for water

$$Re = \frac{1000 \cdot 10 \cdot 1}{0.001} = \frac{\rho \cdot U \cdot L}{\mu} = 1.0 \cdot 10^{7}$$ → sería turbulento

entonces $\delta = \dfrac{0.38 \times X}{(Re)^{1/5}} = 0.0151 \text{ m}$; $\delta_T = \dfrac{\delta}{(P_r)^{1/3}} = 8.05 \cdot 10^{-3} \text{ m}$

it's very low, so heat transfer will be

* For water $P_r \cong 6{,}6$

Remember! You can calculate the Prandtl number, if you need to:

$$\boxed{P_r = \frac{C_p \cdot \mu}{K}}$$

NUSSELT NUMBER

$$\overline{N_{um}} = \frac{\overline{q_w} \cdot L}{K (T_w - T_\infty)} = \frac{\bar{h} \cdot L}{K} = \frac{\text{convection}}{\text{conduction}}$$

$q'' = h\,(T_s - T_\infty)$ convection heat transfer

$$h = \frac{q}{(T_s - T_\infty)}$$

- caso (esfera). Longuitud característica (L=

high order terms

$$N_{um} = 2.0 + 0.5\, P_r \cdot Re + O(P_r^2 \cdot Re^2) + \dots\dots$$

- caso (circular cylinder). Longritud característica : (L = 2R)

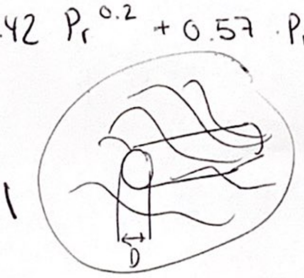
$$N_{um} = B - P_r^2 \cdot Re^2 (16 + B^2)$$

where $B = \dfrac{2}{Ln\left(\frac{8}{P_r \cdot Re}\right) - \gamma}$ donde γ = gamma = 0.577

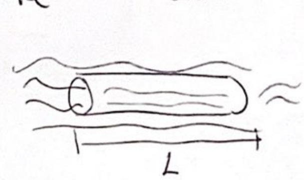
otra forma de aproximarse

$$N_{um} = 0.42\, P_r^{0.2} + 0.57 \cdot P_r^{1/3} \cdot Re^{1/2}$$ cuando $0.1 < Re_D < 10^4$

¿Qué es Re_D?



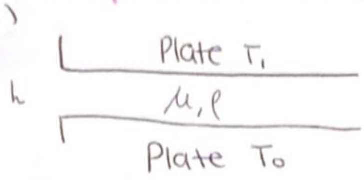
y



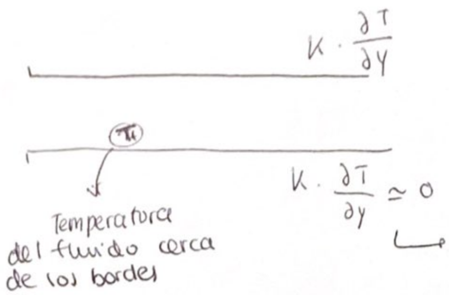
Temperature distribution

(diferentes casos)
para resolver con cada boundary conditi

(1)

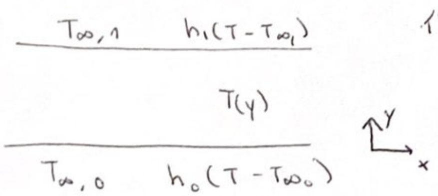


(2)



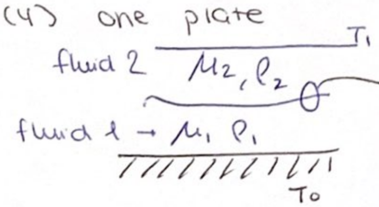
↳ when this is equal to 0, the process es adiabatico

(3)



This is convection

(4) one plate



cuando 2 fluidos están en contacto,
→ $T_{f_1} = T_{f_2}$ donde se tocan,
no hay gradiente de temperatura.

EJERCICIO

Lubricating oil at 20°c with $\rho = 890\ kg/m^3$; $\mu = 0.8\ Pa \cdot S$
$K = 0.15 \frac{W}{m-K}$ y $C_p = 1800 \frac{J}{kg-K}$ is to be cooled by flowing at an average velocity of 2m/s through a 3cm diameter of a circular cylinder the walls are at 10°c
Estimate the heat los $\left(\frac{W}{m^2}\right)$ at $x = 10$ cm

$$\overline{Nu}_m = \frac{\bar{q}_w \cdot L}{K_f (T_w - T_\infty)}$$

$$\overline{Nu}_m = 0.42\ Pr^{0.2} + 0.57 \cdot Pr^{1/3} \cdot Re_D^{1/2}$$

donde $Re_D = \frac{\rho \cdot u \cdot D}{\mu} = \frac{890 \frac{kg}{m^3} \times 2 \frac{m}{s} \times 0.03\,m}{0.8\ Pa \cdot s} = 66.75$

$0.1 < Re_D < 10^4$

$$P_r = \frac{C_p \cdot \mu}{K} = \frac{(1800\ J/kg \cdot \cancel{K}) \times (0.8\ Pa \cdot s)}{0.15 \frac{W}{m \cdot \cancel{K}}}$$

Resultado :

$\overline{Nu}_m = 101.5637$

$$\bar{q}_w = \frac{\overline{Nu}_m \times K (T_w - T_\infty)}{L} = \frac{(101,5637) \times 0.15 \frac{W}{m-K} \times (283-293) K}{0.1\,m}$$

$$\boxed{\bar{q}_w = 1523.4 \frac{W}{m^2}}$$

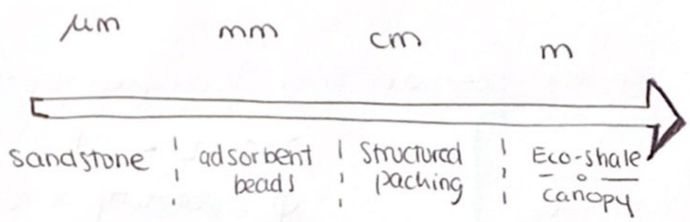
Heat loss

DYNAMIC FLUIDS IN POROUS MEDIA

POROUS MEDIA: it can be defined as anything that is comprised of a solid matrix / voids or simpler a material that contains pores.

It is a body composed of a persistent solid part, called solid matrix, and the remaining void space (or pore space) that can be filled with one or more fluids (e.g. : water, oil, gas ...)

- Applications. oil exploration, ground-water flow, heat pipes, filtration, composites processing, wicking & bio heat & Mass transfer.
- Unidades :

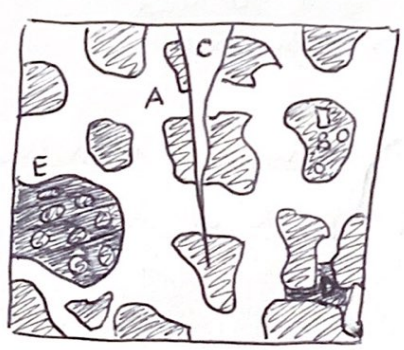


Phase is defined in a chemically homogeneous portion of a system under consideration that is separated from other such portions by a definite physical boundary.

→ Single phase: the void space of the porous medium is filled by a single fluid.

→ Multiphase system: the void is filled by 2 or + fluids that are immiscible w/ each other (they maintain a distinct boundary between them).

Void spaces: **TYPES**



A) bt the particles that comprise the matrix (intergranular)

B) within the particles themselves (intergranural)

C) due to secondary feautures such as fractures or root holes.

D) Soil particles may be cemented to one another

E) Particles may also be bound in aggregates.

A porous medium can be defined as anything that is comprised of a a solid matrix & voids.

In order to derive mathematical models for fluid flows in porous media some restrictions are placed upon the geometry of the porous medium:

- (P1) The void space of the porous medium is interconnected
- (P2) The dimensions of the void space must be large compared to the mean free path length of the fluid moleculer. ← hypotetical continuum
- (P3) The dimensions of the void space must be small enough so that the fluid flow is controlled by adhesive forcer at fluid-solid interfacer & cohesive forcer at fluid-fluid interfacer (multiphase systems)

↳ excluder caser like a network of piper from the definition of a porous medium.

POROSITY $n\,[L^3 L^{-3}]$

Is the volumetric fraction of the medium that is occupied by the voids:

$$\left[\phi = n = \frac{V_v}{V_T} = \frac{V_v}{V_s + V_v}\right]$$

$0 < n < 1$

ϕ = porosity $\equiv n$ (misma cosa)

where V_v, V_s, & V_T are the volumer of the voids, matrix solids, and total medium, respectively.

Porosity also definer the avg. fraction of a cross section through a medium that is occupied by voids.
The fraction of the area occupied by voids is = to the porosity.

En $[m^3 m^{-3}]$ para:

- gravel (0.25 - 0.4) ; • sand (0.25 - 0.50)
- silt (0.35 - 0.5) ; • clay (0.40 - 0.70)

- Primary porosity : occurs between the solid particler
- Secondary porosity : caused by fractoring & dissolution.

EFFECTIVE POROSITY $n_e\,[L^3 L^{-3}]$

Describer only that porosity that is connected, allowing for fluid flow through it :

$$\left[n_e = \frac{V_{vc}}{V_T}\right]$$ donde V_{vc} = volume of connected voids.

VOID RATIO $e\,[L^3 L^{-3}]$

$$e = \frac{V_v}{V_s}$$

Unlike porosity, void ratio can be greater than one. This parameter is used more often in engineering applications.

Tortuosity t [$L L^{-1}$]

Is the ratio of the avg. travel path to the distance of separation.

The ↑↑↑ t the more complicated path, ⇒ less well-connected pore space

Specific surface area [$L^2 L^{-3}$]

Describes the area of the interface bt the matrix solids & the void space per unit volume of porous medium.

↑↑↑ specific surface area ↑↑ Roughness of the surface of the particles ↓↓ Size of particles.

Water saturation S_w

The fraction of pore spaces that are filled w/ water & it is defined as the ratio of the volumetric water content to the porosity

$$S_w = \frac{\theta}{n}$$

n → porosity

Gravimetric water content θ_g [$M M^{-1}$]

of a medium is = to the ratio of the mass of water in a sample M_w to the mass of oven-dried soil, M_s:

$$\theta_g = \frac{M_w}{M_s}$$

Bulk density ρ_b [$M L^{-3}$]

It's equal to the ratio of the total mass of a sample of its total volume.
If a medium only has water & aire in the pores, ρ_b is = to:

$$\rho_b = \frac{M_T}{V_T} = (1-n)\rho_s + \theta \rho_w + (n-\theta)\rho_a$$

Compressibility, α [$L^2 M^{-1} L^{-1} T^2$]

It's defined as the change in volume of a unit volume of medium under a unit applied pressure.

[↑ compress a medium ↓ volume ↑ density ↓ porosity ↑ bulk density]

Three fluid properties are of primary importance in the study of subsurface hydrogeology.

- ~~Fluid density~~ $\rho_f\ [M\ L^{-3}]$ (ρ_w for water density)

Is the ratio of the mass of a sample of the fluid to the sample volume

- ~~Dynamic viscosity~~ $\mu\ [M\ L^{-1}\ T^{-1}]$

describes the resistance to flow presented by a flowing fluid

$$\mu = \frac{\sigma}{dv/dy}$$

donde { σ = shear stress applied to the fluid
y = distance perpendicular to solid surface
v = velocity parallel to surface.

- ~~Fluid compressibility~~ $\beta\ [L^2 M^{-1} L^{-1} T^2]$

Describes the resistance of the fluid to changing its volume in response to a change in the applied pressure per unit volume of fluid

$$\beta = -\frac{1}{V_0} \cdot \frac{dV}{dP}$$

{ V_0 = sample volume before compression
Fluido incompresible : $\beta = 0$

$$PV = RT \quad ; \quad V = \frac{RT}{P} \quad ; \quad \frac{P}{RT} = \frac{1}{V_0}$$

$$\beta \quad -\frac{1}{V_0} \cdot \frac{dv}{dP} = -\frac{\cancel{R}T}{P^2} - \frac{P}{\cancel{RT}} \quad ; \quad \beta = \frac{1}{P}$$

* Permeability $K\ [L^2]$

Describes the ability of the medium to transmit a fluid under an applied potential gradient.

Fluid going through a porous medium can be visualized most simply as a fluid flowing through a collection of interconnected tubes.
At the walls of each tube, fluid velocity = 0.

[↑ velocity ↑ further from tube walls bc of "drag" of walls exerts less influence on fluid.]

- [↓ smaller tube : ↑ more fluid in contact w/ walls : ↓ Avg. velocity is lower.]

The permiability $[L^2]$ should be directly related to the avg. particle diameter $[L]$

Relationship: $K = Cd^2$ { d = avg. grain diameter
C = adimensional, cte.

therefore : $$\boxed{K = \frac{n^3}{(1-n)^2} \cdot \frac{dm^2}{180}}$$ { dm = can be determined from a particle size distribution