Ejemplos tipo

①

Se plantean dos casos de carga (simétrica y antisimétrica) para un pórtico . Los objetivos son comparar los métodos de compatibilidad y equilibrio para el cálculo de reacciones.

Antes de resolver los casos, hay que tener presente los pasos para resolver pórticos por métodos analíticos:

① Separar en estado de cargas simétrica y antisimétrica.

② 2.1 Compatibilidad: Establecer el grado h para definir las condiciones de compatibilidad.

2.2 Equilibrio: Establecer el grado K para definir las ecuaciones de equilibrio

③ 3.1 Compatibilidad: Determinar reacciones (fuerzas externas desconocidas) (R, N, M, V)

3.2 Equilibrio: Determinar giros y desplazamientos → Cálculo reacciones (R)

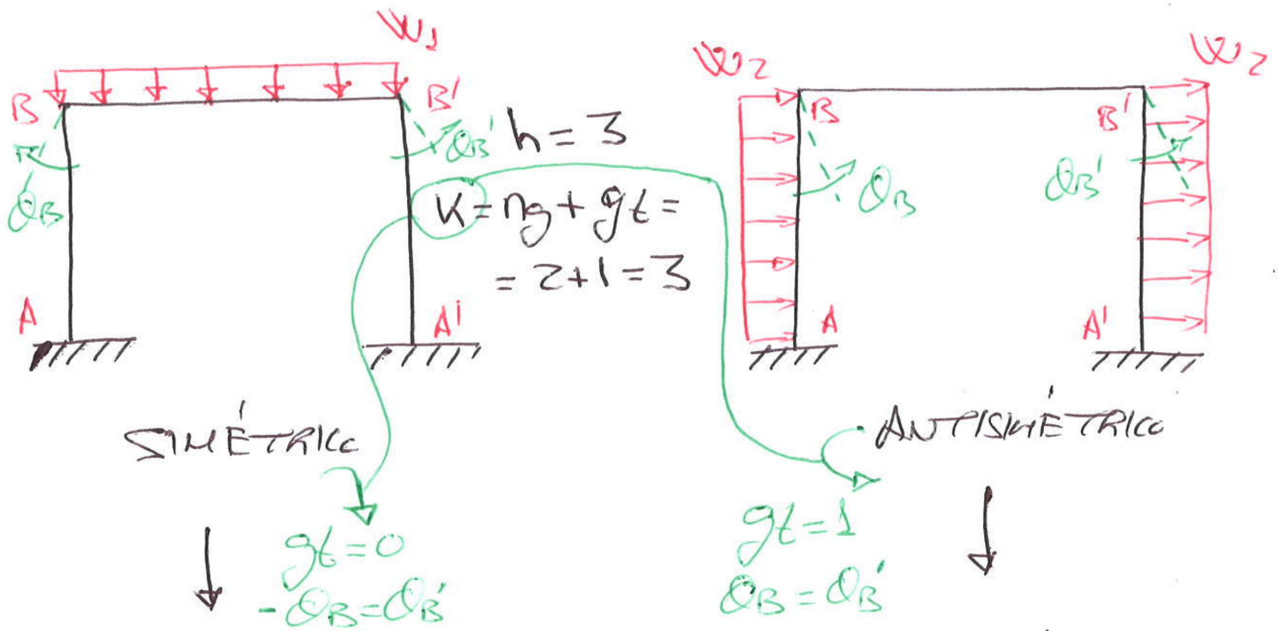
④ Con las reacciones se determinan (dibujan) los diagramas de M(x), V(x), N(x) de cada una de las barras.

* LAS REACCIONES TOTALES SE CORRESPONDEN CON LA SUMA DE LOS ESTADOS DE CARGA SIMÉTRICOS Y ANTISIMÉTRICOS. LOS DIAGRAMAS SE PUEDEN (DEBEN) DIBUJAR PARA EL ESTADO TOTAL DE CARGA (DE LA HIPÓTESIS CONSIDERADA).

⑤ Dibujar la elástica aproximada.

⊛ Obtener giros y desplazamientos en los nudos que falten. Notar que por el método de equilibrio esto se calcula en el paso ③.

Tener en cuenta los puntos de inflexión (cambios de signo en el momento flector.



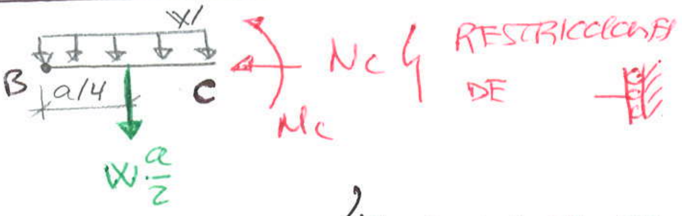
SIMPLIFICACIÓN (QUE SE PUEDE USAR O NO)

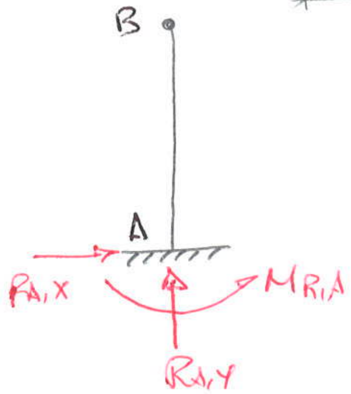
SIMPLIFICACIÓN (QUE SE PUEDE USAR O NO)

$h=2$ (REDUCIMOS A DOS ECUACIONES DE COMPATIBILIDAD)

$h=1$ (REDUCIMOS A UNA ECUACIÓN DE COMPATIBILIDAD)

SIMÉTRICO - COMPATIBILIDAD





Las condiciones de compatibilidad son:

$$\boxed{\theta_C = 0 \quad y \quad u_C = 0}$$

Las ecuaciones de compatibilidad que se pueden definir son:

$$\begin{cases} \theta_C = \theta_A + \displaystyle\int_{s=0}^{\boxed{s=\frac{3}{2}a}} \left(\frac{M(s)}{EI}\right) ds = 0 \\ u_C = u_A - \theta_A \left(Y_C - Y_A\right) - \displaystyle\int_A^{\boxed{C}} \left[\frac{M(s)}{EI}\left(Y_C - Y(s)\right)\right] ds = 0 \end{cases}$$

DOS FORMAS $C \equiv S = \frac{3}{2}a$

ECUACIONES DE NAVIER

Notar:

① $\boxed{\theta_A = u_A = Y_A = 0}$ → SIMPLIFICA LAS ECUACIONES

② El momento flector hay que definirlo a tramos.

$$AB \equiv S \in (0, a)$$
$$BC \equiv S \in \left(a, \tfrac{3}{2}a\right)$$

$$M(s) = \begin{cases} -\frac{w}{2}\left(\frac{a}{2}\right)^2 + M_C + N_C\,(a-s) & s \in (0, a) \\ -\frac{w}{2}\left(\frac{3}{2}a - s\right)^2 + M_C & s \in \left(a, \frac{3}{2}a\right) \end{cases}$$

EXPLICAR + DESPACIO EN PIZARRA. COMPRENSIÓN A DESPACIO!

$$\theta_C = \int_A^B \left[-\frac{w}{2}\left(\frac{a}{2}\right)^2 + M_C + N_C\,(a-s)\right]\frac{1}{EI}\,ds + \underbrace{\int_B^C \left[-\frac{w}{2}\left(\frac{3}{2}a - s\right)^2 + M_C\right]\frac{1}{EI}\,ds}$$

CAMBIO DE VARIABLES

$$t = \frac{3}{2}a - s \rightarrow dt = -ds$$

$$\theta_C = -\frac{w}{8}a^3 + M_C\cdot a + N_C\cdot a^2 - N_C\cdot\frac{a^2}{2} + \frac{w}{2}\frac{1}{3}\left(0-\frac{1}{8}a^3\right) - M_C\left(0-\frac{1}{2}a\right)$$

$$\boxed{3M_C + N_C\cdot a - \frac{7}{24}wa^2 = 0} \quad (1)$$

$$\mathcal{U}_C = 0 = \cancel{\mathcal{U}_A}^{0} - \cancel{\theta_A}^{0}\left(Y_C - Y_A\right) - \int_A^C \left[\frac{M(s)}{EI}\left(Y_C - Y(s)\right)\right] ds$$

$$Y_C = a \;;\; Y(s) = \begin{cases} s\;;\; s\in(0,a) \\ a\;;\; s\in\left(a,\frac{3}{2}a\right)\end{cases} \quad \text{TRAMO DE B A C} \quad Y_C - Y(s) = 0$$

$$t = a - s$$
$$\downarrow\; dt = -ds$$

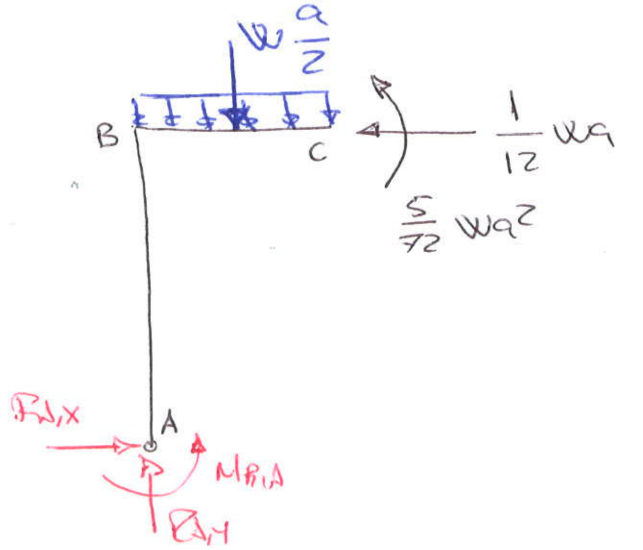
$$\mathcal{U}_C = \frac{1}{EI}\int_{s=0}^{s=a}\left[-\frac{w}{2}\left(\frac{a}{2}\right)^2 + M_C + N_C(a-s)\right](a-s)\,ds =$$

$$= -\left[-\frac{w}{2}\left(\frac{a}{2}\right)^2\frac{t^2}{2} + M_C\frac{t^2}{2} + N_C\frac{t^3}{3}\right]_{t=a}^{t=0} = -\frac{w}{2}\cdot\frac{a^4}{8} + M_C\frac{a^2}{2} + N_C\frac{a^3}{3}$$

$$\boxed{3M_C + 2N_C\cdot a - \frac{3}{8}wa^2} \quad (2)$$

⊛ (1) y (2) son las ecuaciones de compatibilidad.

$$\boxed{N_C = \frac{1}{12}wa} \;;\; \boxed{M_C = \frac{5}{72}wa^2}$$



$$\sum F_x = 0\;;\; \rightarrow \boxed{R_{A,X} = \frac{1}{12}wa}$$

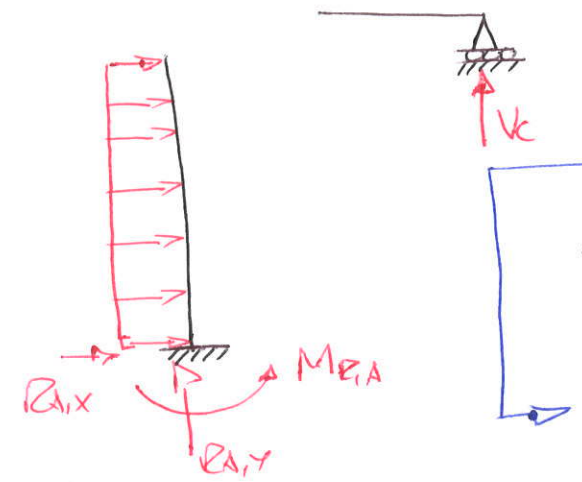
$$\sum F_y = 0\;;\; \rightarrow \boxed{R_{A,Y} = w\cdot\frac{a}{2}}$$

$$\sum M_A = 0\;;\; M_{R,A} = -\frac{5}{72}wa^2 + w\frac{a^2}{8} - \frac{1}{12}wa^2$$

$$\boxed{M_{R,A} = -\frac{1}{36}wa^2}$$

PARA COMPLETAR $M_{BB'} = -M_{B'B} = \frac{1}{18} w a^2$

ANTISIMÉTRICO – COMPATIBILIDAD



CONDICIÓN DE COMPATIBILIDAD $v_C = 0$

DOS TRAMOS DE $M(s)$

$$v_C = \cancel{v_A}^{\,0} + \cancel{\theta_A}^{\,0} (x_C - x_A) + \int_A^C \left[\frac{M(s)}{EI} \left(x_C - x_{(s)} \right) \right] ds$$

$$\begin{cases} s \in (0, a) \longrightarrow AB \\ s \in \left(a, \frac{3}{2}a\right) \longrightarrow BC \end{cases}$$

TRAMO AB $\quad M(s) = V_C \cdot \frac{a}{2} - w \cdot \frac{(a-s)^2}{2}$

TRAMO BC $\quad M(s) = V_C \cdot \left(\frac{3}{2}a - s \right)$

$$v_C = 0 = \underbrace{\int_{s=0}^{s=a} \left[V_C \cdot \frac{a}{2} - w \frac{(a-s)^2}{2} \right] \left(\frac{a}{2} - 0 \right) ds}_{①} + \underbrace{\int_{s=a}^{s=\frac{3}{2}a} V_C \cdot \left(\frac{3}{2}a - s \right) \overbrace{\left(\frac{a}{2} - s + a \right)}^{\frac{3}{2}a - s} ds}_{②}$$

① → CAMBIO VARIABLE $t = a - s \rightarrow dt = -ds$

② → CAMBIO DE VARIABLE $t = \frac{3}{2}a - s \rightarrow dt = -ds$

$$= \underbrace{-\frac{a}{2} \int_{t=a}^{t=0} \left(V_C \frac{a}{2} - \frac{w}{2} t^2 \right) dt}_{①} \underbrace{- \int_{t=\frac{1}{2}a}^{t=0} V_C t^2 dt}_{②} = \frac{a^3}{4} V_C - \frac{w}{12} a^4 + V_C \frac{a^3}{3 \cdot 8} = 0$$

$$\boxed{V_C = \frac{2}{7} w a}$$

$\boxed{R_{A,Y} = -\frac{2}{7} w}$ $\quad (\Sigma F_y = 0)$

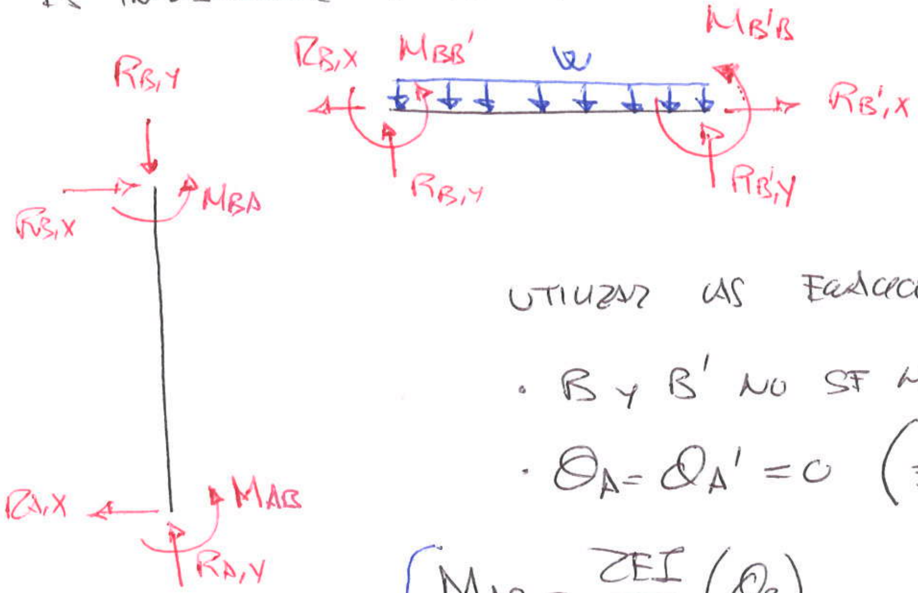
$\boxed{R_{A,X} = -wa}$ $\quad (\Sigma F_x = 0;)$

$\Sigma M_A = 0 \; ; \; V_C \frac{a}{2} + M_{R,A} = \frac{w a^2}{2}$

$$\boxed{M_{R,A} = \frac{5}{14} w a^2}$$

SIMÉTRICO – EQUILIBRIO

(*) NO HACE FALTA LA SIMPLIFICACIÓN POR SIMETRÍA. ES ÚTIL SABER QUE EL PÓRTICO ES INTRASLACIONAL → SOLO HAY GIROS.



UTILIZAR LAS ECUACIONES ELÁSTICAS SABIENDO QUE:

- B Y B' NO SE MUEVEN ; $\theta_B = -\theta_{B'}$
- $\theta_A = \theta_{A'} = 0$ (EMPOTRAMIENTO)

PLANTEAR ECUACIONES DE EQUILIBRIO (EN NODOS)

$$\begin{cases} M_{AB} = \dfrac{2EI}{a}(\theta_B) \\ M_{BA} = \dfrac{2EI}{a}(2\theta_B) \\ M_{BB'} = \dfrac{2EI}{a}(2\theta_B - \theta_B) + \dfrac{wa^2}{12} = \dfrac{2EI}{a}\theta_B + \dfrac{wa^2}{12} \\ M_{B'B} = -\dfrac{2EI}{a}\theta_B - \dfrac{wa^2}{12} \end{cases}$$

$$\sum M_B = 0 ; \rightarrow M_{BA} + M_{BB'} = 0 ; \quad \frac{2EI}{a}2\theta_B + \frac{2EI}{a}\theta_B + \frac{wa^2}{12} = 0 ;$$

$$\boxed{\theta_B = -\frac{w/a^3}{72EI}} \qquad \boxed{M_{AB} = M_{R,A} = \frac{2EI}{a}\left(-\frac{wa^3}{72EI}\right) = -\frac{1}{36}wa^2}$$

(*) IGUAL QUE EN EQUILIBRIO

(*) SE PUEDEN PLANTEAR ECUACIONES EN ELEMENTOS PARA CONOCER EL RESTO DE REACCIONES. ES IMPORTANTE CONOCER EL RESTO DE MOMENTOS EN LOS NODOS (AYUDA A DIBUJAR LOS DIAGRAMAS).

$\sum M_B = 0 ;$ (BARRA VERTICAL) $\quad M_{AB} + M_{BA} = R_{A,X} = 0 \rightarrow \boxed{R_{A,X} = \frac{1}{12} w/a}$

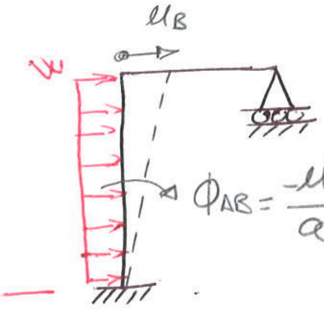
IGUAL QUE POR COMPATIBILIDAD

$\sum F_y = 0 ;$ (BARRA VERTICAL) $\quad R_{A,Y} = R_{B,Y}$

$\sum M_{B'} = 0 ;$ (BARRA HORIZONTAL) $\quad \underbrace{M_{BB'} + M_{B'B}}_{0} + \dfrac{wa^2}{2} = R_{B,Y} \cdot a ;$

$$\left\{ \boxed{R_{A,Y} = R_{B,Y} = \frac{w/a}{2}} \right.$$

– En este caso se puede resolver utilizando la estructura simplificada y sin simplificar. Se plantea la simplificación para ver la utilidad de las ecuaciones elásticas con empotrado–apoyado.



$\phi_{AB} = \frac{-u_B}{a}$ (u_B y ϕ_{AB} tienen sentidos contrarios)

$$M_{AB} = \frac{2EI}{a}\theta_B + \frac{6EI}{a}\frac{u_B}{a} + \frac{wa^2}{12}$$

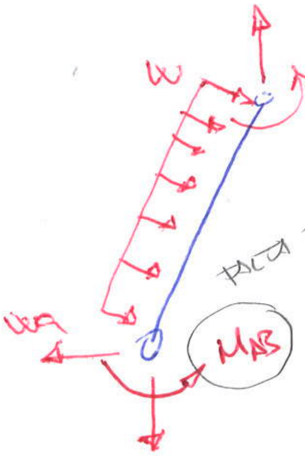
$$M_{BC} = \frac{3EI}{(a/2)}\theta_B = \frac{6EI}{a}\theta_B$$

↳ $K=2$ ($gt=1$ porque el pórtico es traslacional)

↳ $K = \underbrace{ng}_{\hookrightarrow\ \theta_B} + \underbrace{gt}_{\rightarrow\ u_B\ (\text{o } \phi_{AB})}$

* La primera ecuación de equilibrio es: $M_{BA} + M_{BC} = 0$

$$\boxed{\frac{10EI}{a}\theta_B + \frac{6EI}{a^2}u_B - \frac{wa^2}{12} = 0} \quad (1)$$



⊛ Linealidad geométrica

↳ Ecuaciones de equilibrio de la estructura sin deformar.

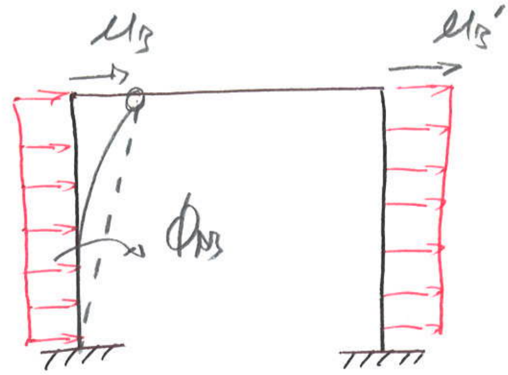
$$\Sigma M_A = 0;\ M_{AB} + M_{BA} - \frac{wa^2}{2} = 0$$

$$\boxed{\frac{6EI}{a}\theta_B + \frac{12EI}{a^2}u_B - \frac{wa^2}{2} = 0} \quad (2)$$

$(1)-(2) \rightarrow \begin{cases} \boxed{\theta_B = -\frac{wa^3}{42EI}} \\ \boxed{u_B = \frac{3wa^4}{56EI}} \end{cases} \longrightarrow M_{AB} = -\frac{2}{42}wa^2 + \frac{9}{28}wa^2 + \frac{1}{12}wa^2 = \boxed{\frac{5}{14}wa^2}$

⊛ Igual que por método de compatibilidad.

(*) LA OPCIÓN SIN SIMPLIFICAR TAMBIÉN ES VIABLE



$$\theta_B = \theta_B' \quad ; \quad \mathcal{U}_B = \mathcal{U}_B' \qquad \boxed{K=2}$$

$$M_{AB} = \frac{2EI}{a}\theta_B + \frac{6EI}{a}\frac{\mathcal{U}_B}{a} + \frac{wa^2}{12}$$

$$M_{BA} = \frac{2EI}{a}(2\theta_B) + \frac{6EI}{a}\frac{\mathcal{U}_B}{a} - \frac{wa^2}{12}$$

$$M_{BB'} = \frac{2EI}{a}(3\theta_B)$$

$$\Sigma M_B = 0 \; ; \; \boxed{\frac{10EI}{a}\theta_B + \frac{6EI}{a^2}\mathcal{U}_B - \frac{wa^2}{12} = 0} \rightarrow$$ IGUAL QUE SIN LA SIMPLIFICACIÓN

RESULTADOS

SIMÉTRICO

$R_{A,X} = \frac{1}{12} wa$

$R_{A,Y} = \frac{1}{2} wa$

$M_{R,A} = -\frac{1}{36} wa^2$

$\theta_B = -\frac{wa^3}{72EI}$

ANTISIMÉTRICO

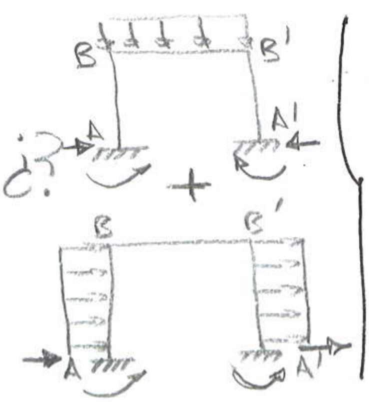
$R_{A,X} = -wa \; ; \; R_{A',X} = -wa$

$R_{A,Y} = -\frac{2}{7} wa \quad R_{A',Y} = \frac{2}{7} wa$

$M_{R,A} = \frac{5}{14} wa^2 \quad M_{R,A'} = \frac{5}{14} wa^2$

$\theta_B = -\frac{wa^3}{42EI}$

$\mathcal{U}_B = \frac{3wa^4}{56EI}$



$R_{A,X} = \frac{1}{12} wa - wa = -\frac{11}{12} wa \; ; \; R_{A',X} = \frac{13}{12} wa$

$(\rightarrow) \qquad (\leftarrow)$

$R_{A,Y} = \frac{1}{2} wa - \frac{2}{7} w = \frac{-3}{14} wa \; ; \; R_{A',Y} = -\frac{3}{14} wa \quad \frac{11}{14} wa$

+ OJO

$(\uparrow) \qquad (\uparrow)$

$M_{R,A} = \frac{-1}{36} wa^2 + \frac{5}{14} wa^2 = 0{,}33\, wa^2 \; ; \; M_{R,A'} = 0{,}38\, wa^2$

$(\circlearrowleft) \qquad (\circlearrowleft)$

RESULTADOS REACCIONES (CORRECCIÓN)

SIMÉTRICO

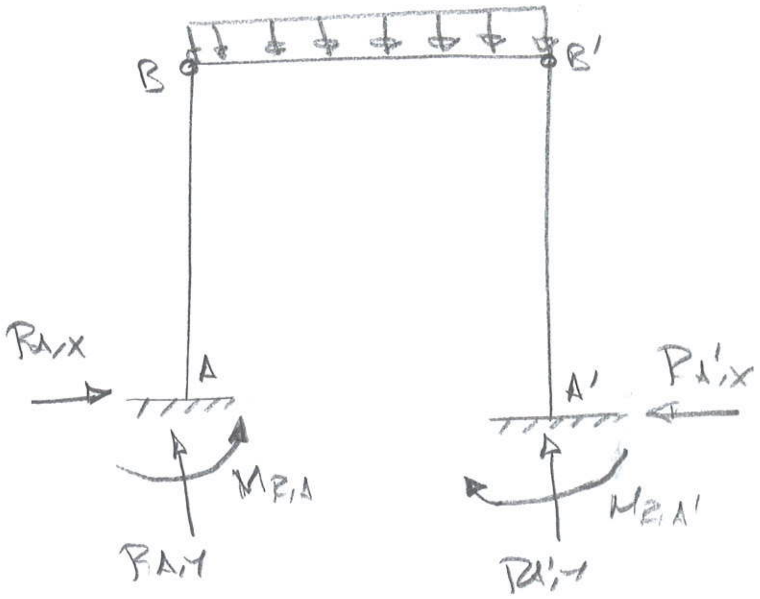
DATOS) $W = 9 N/m$; $a = 4 m$

$$R_{A,x} = \frac{1}{12} W a = 3N$$

$$R_{A,Y} = \frac{1}{2} W a = 18N$$

$$M_{R,A} = -\frac{1}{56} W a^2 = 1 N \cdot m$$

$$\theta_B = -\frac{W a^3}{72 EI} = \frac{8}{EI}$$



ANTISIMÉTRICA

DATOS $W = 7 N/m$ $a = 4 m$

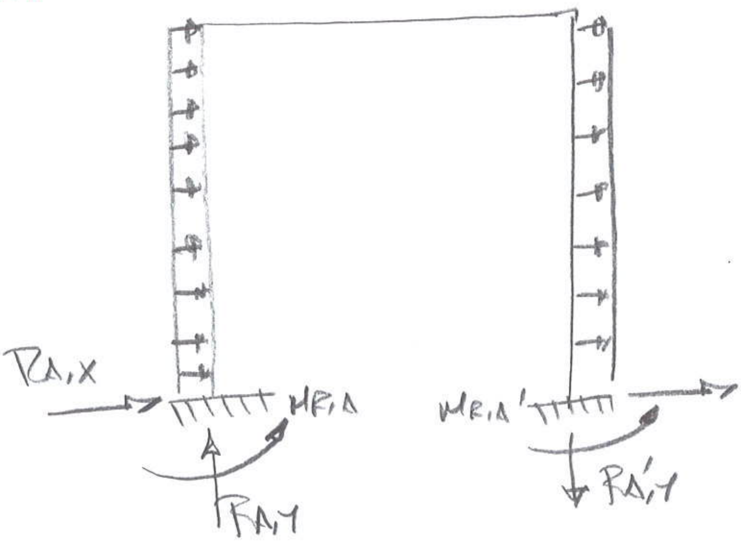
$$R_{A,x} = -W a = -28N$$

$$R_{A,Y} = -\frac{2}{7} W a = -8N$$

$$M_{R,A} = \frac{5}{14} W a^2 = 40 N \cdot m$$

$$\theta_B = -\frac{W a^3}{42 EI} = -\frac{2 \cdot 16}{3EI}$$

$$u_B = \frac{3 W a^4}{56 EI} = \frac{3 \cdot 2 \cdot 16}{EI}$$



SUMA DE LOS DOS

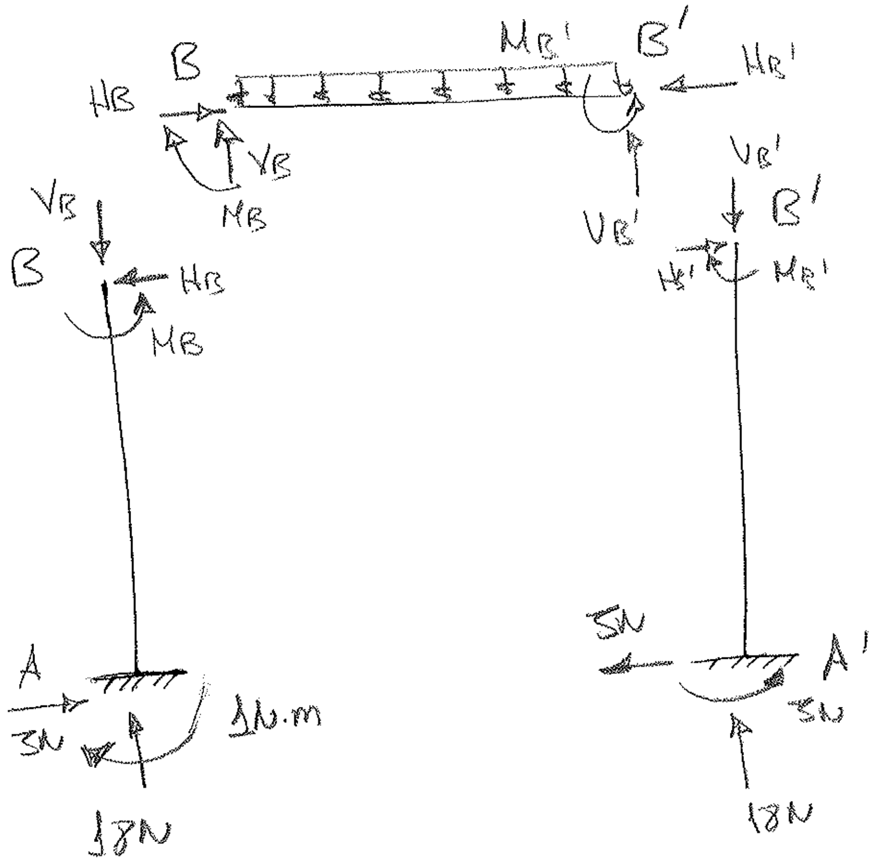
$$R_{A,x} = \frac{1}{12} W a - W a = -\frac{11}{12} W a \; (\rightarrow) \quad ; \quad R'_{A,x} = \frac{1}{12} W a + W a = \frac{13}{12} W a \; (\leftarrow)$$

$$R_{A,Y} = \frac{1}{2} W a - \frac{2}{7} W a = \frac{3}{14} W a \; (\uparrow) \quad ; \quad R'_{A,Y} = \frac{1}{2} W a + \frac{2}{7} W a = \frac{11}{14} W a \; (\uparrow)$$

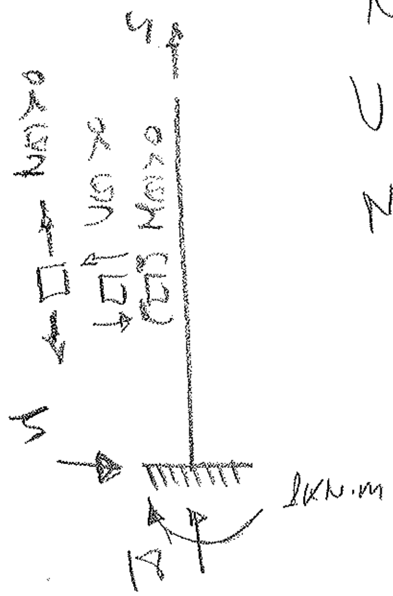
$$M_{R,A} = -\frac{1}{56} W a^2 + \frac{5}{14} W a^2 = 0{,}33 W a^2 \; (\circlearrowleft) \quad ; \quad M_{R,A'} = \frac{1}{56} W a^2 + \frac{5}{14} W a^2 = 0{,}38 W a^2 \; (\circlearrowleft)$$

DIAGRAMAS.

SIMÉTRICO

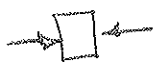


BARRA AB



$N(s) = -18\ N$ (Trabaja a compresión)

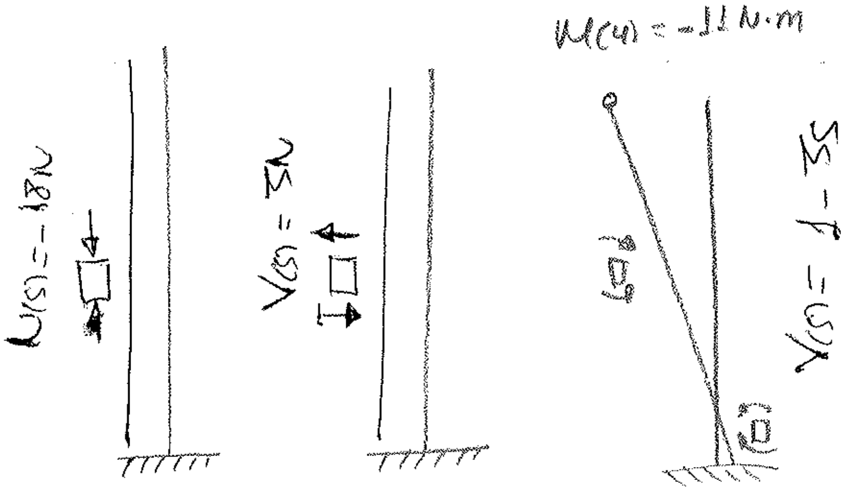
18N



$V(s) = 3N$

$M(s) = 1 - 3s$

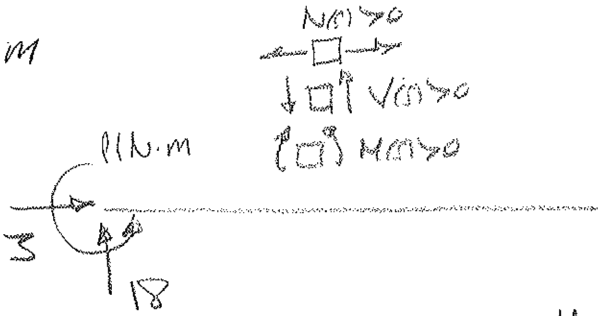
AXIL

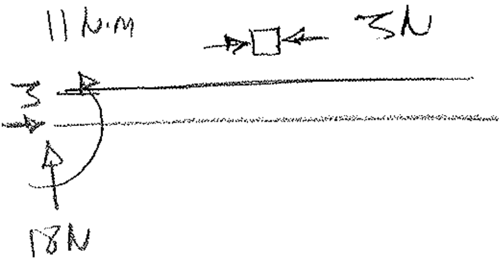


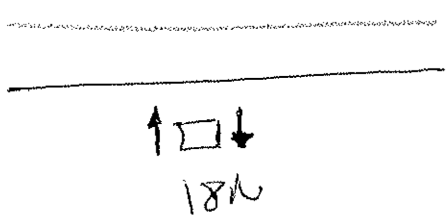
$M_B = 1 - 3 \cdot 4 = -11 \, N \cdot m$

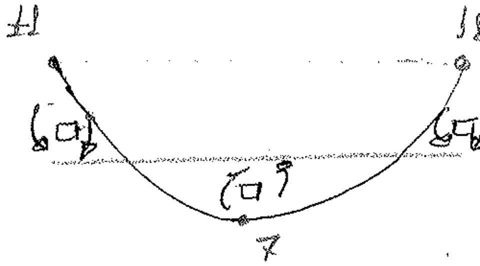
$V_B = 18N$

$H_B = 3N$







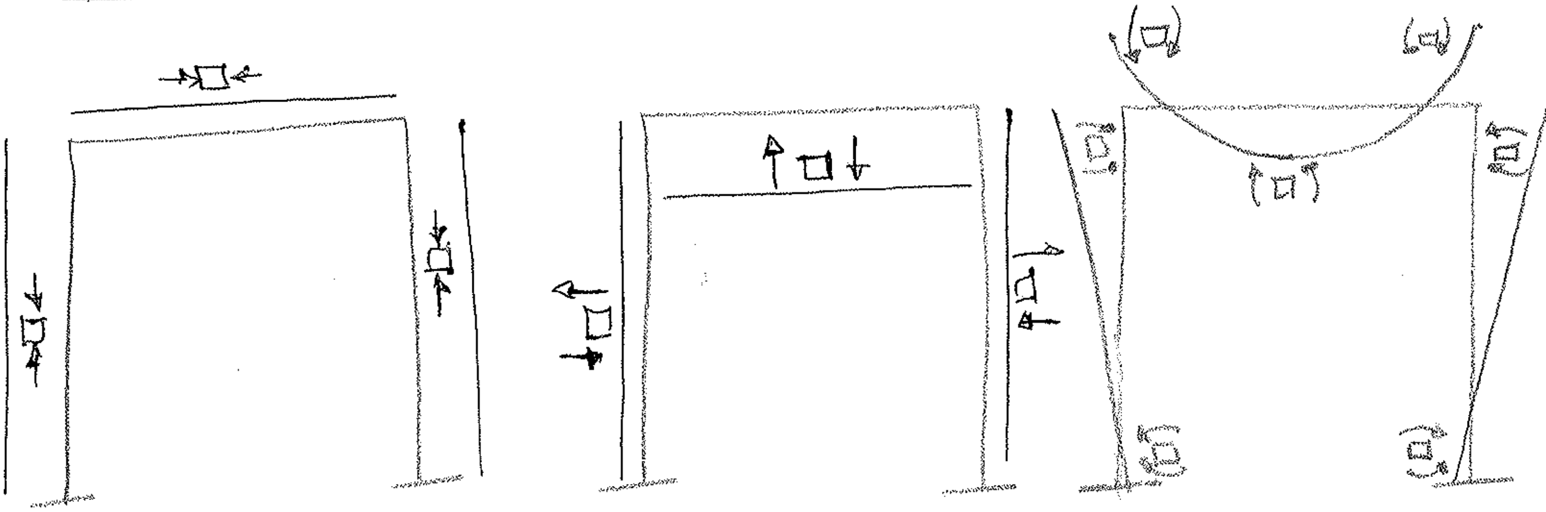


$$\begin{cases} M(s) = -11 + 18 \cdot \hat{S} - w \dfrac{\hat{S}^2}{2} \\ \hat{S} = S - 4 \end{cases}$$

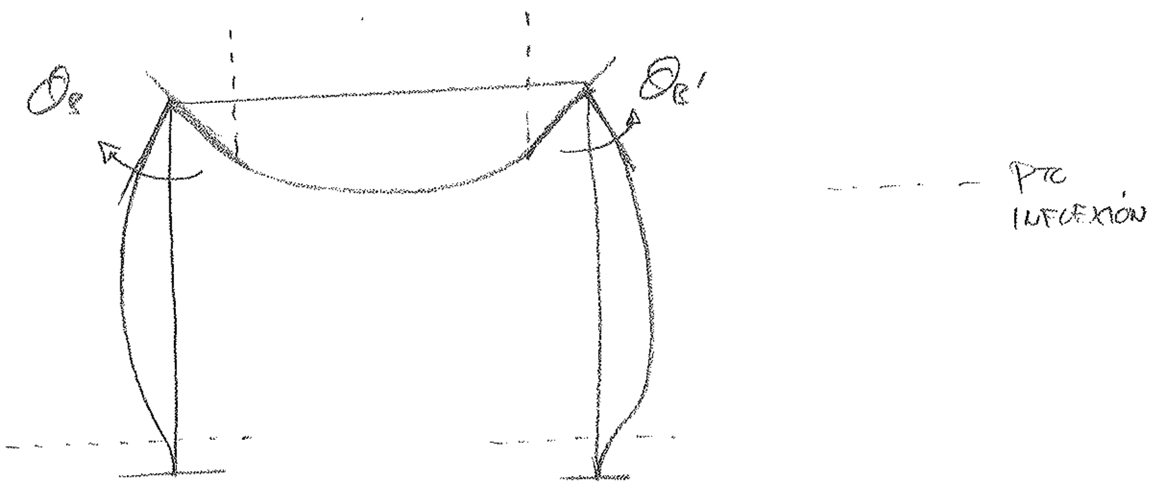
$M(s) =$

C

PÓRTICO SIMÉTRICO

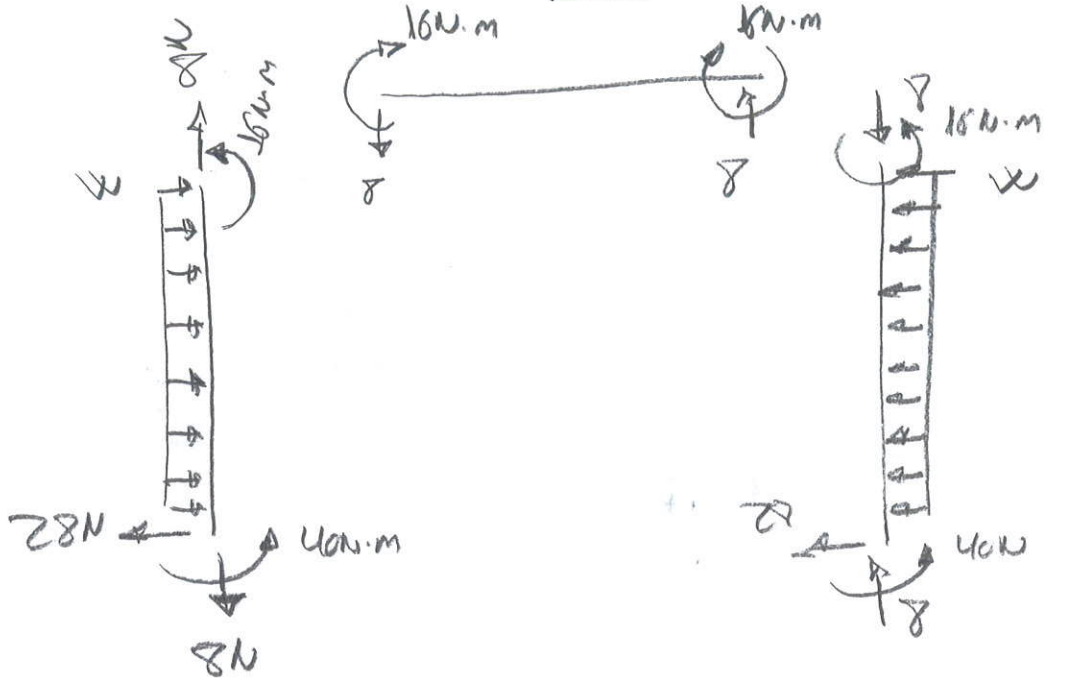


AXIL CORTANTE FLECTOR

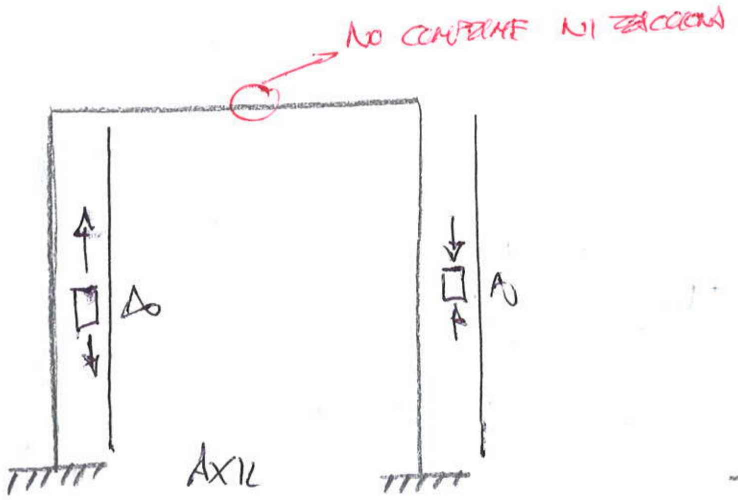


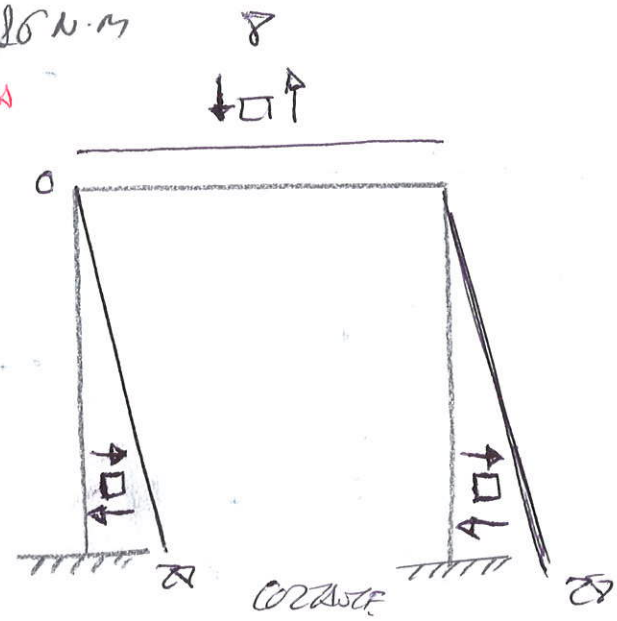
ELÁSTICA APROXIMADA

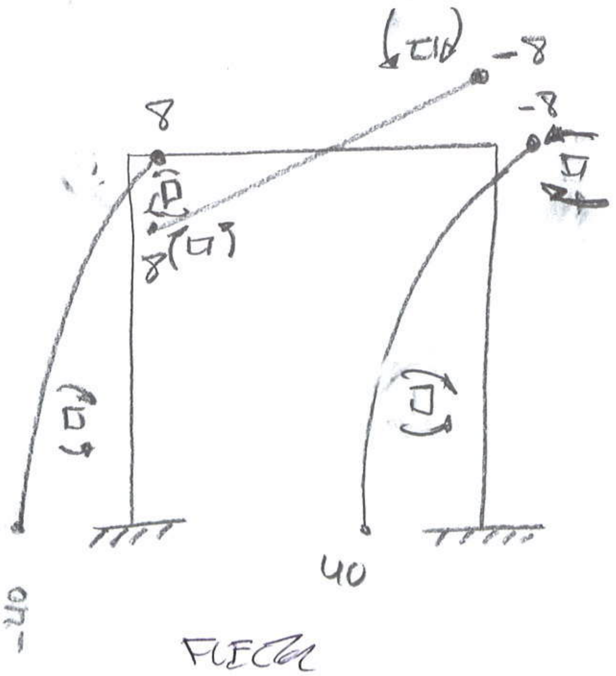
PÓRTICO ANTISIMÉTRICO.

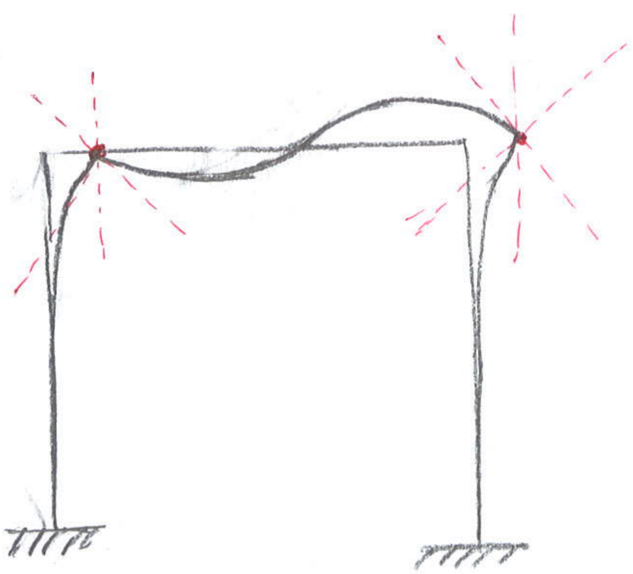


$$M_B = 28\cdot 4 - 40 - 7\cdot 8 = 16\ \text{N·m}$$



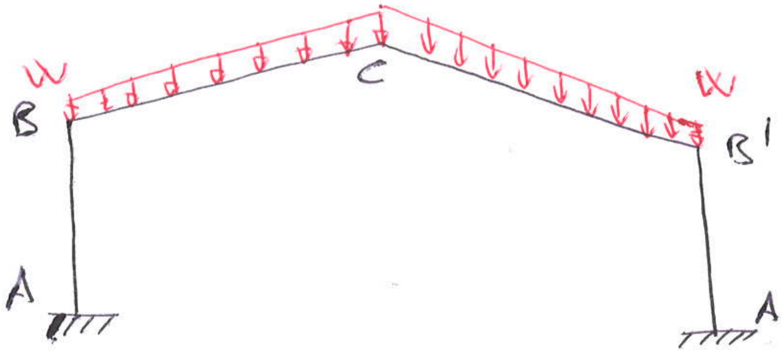




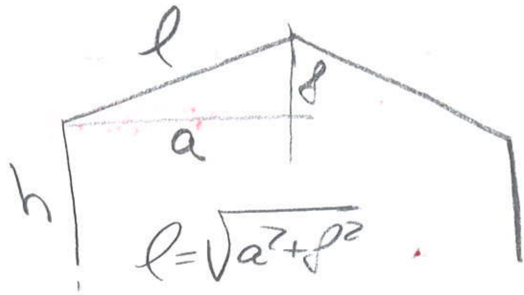


ELÁSTICA APROXIMADA

EJEMPLO 2 PÓRTICO A DOS AGUAS CON CARGA SIMÉTRICA.

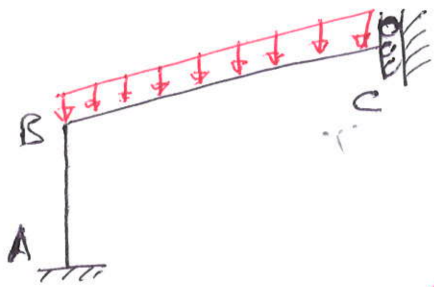


NOMENCLATURA.

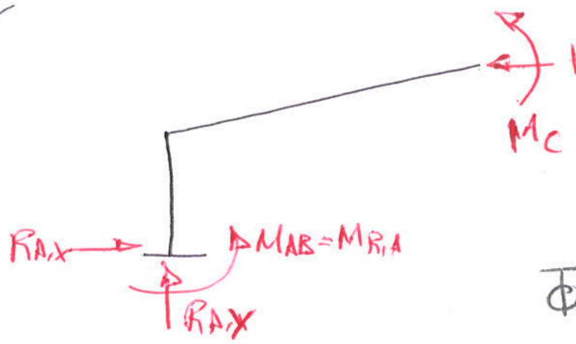


$$\ell = \sqrt{a^2 + f^2}$$

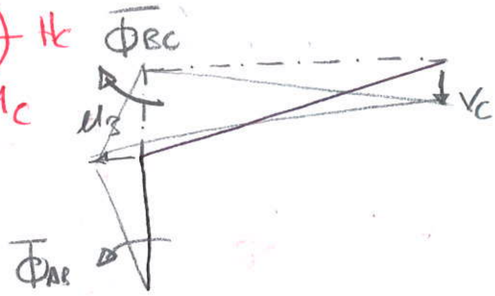
LA SIMPLIFICACIÓN POR SIMETRÍA ES: PGS 194-196 y 254-257 CERVERA.



COMPATIBILIDAD



EQUILIBRIO



5 INCÓGNITAS CON 3 ECUACIONES

↓

$h = 2 \rightarrow$ 2 ECUACIONES DE COMPATIBILIDAD

↓

$$\boxed{\theta_C = u_C = 0}$$

$K = n_g + g_t = 1 + 1 = 2$

$n_g = 1 \rightarrow \theta_B$

$g_t = 1 \rightarrow \phi_{AB}$

(*) $\bar{\phi}_{BC}$, M_B, V_B SE PUEDE PONER COMO FUNCIÓN DE ϕ_{AB}

o

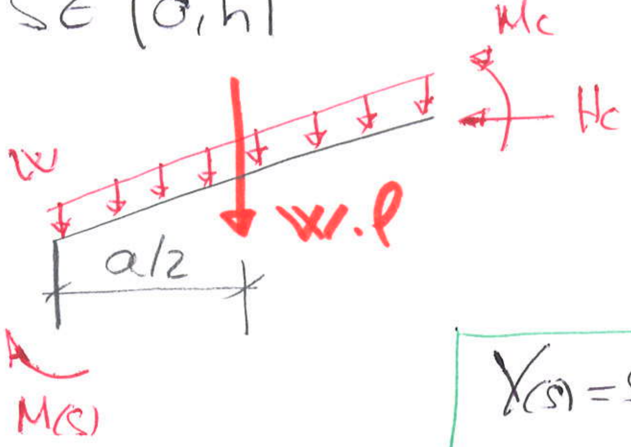
COMPATIBILIDAD

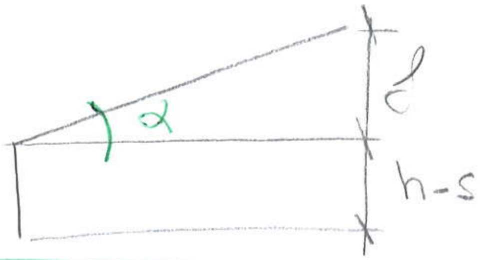
$$\theta_C = \cancel{\theta_A}^{\,0} + \int_{s=0}^{s=h+\ell} \frac{1}{EI} M(s)\, ds = 0$$

$$u_C = \cancel{u_A}^{\,0} - \cancel{\theta_A}^{\,0} (Y_C - Y_A) - \int_{s=0}^{s=h+\ell} \frac{1}{EI} M(s)(Y_C - Y(s))\, ds = 0$$

↳ DEFINIR LAS FUNCIONES $M(s)$ Y $Y(s)$ EN LOS DOS TRAMOS DEL PÓRTICO $\rightarrow$ $s \in (0, h]$ y $s \in (h, h+\ell]$

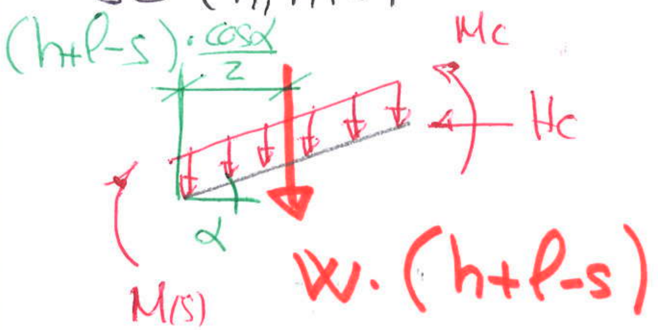
$s \in (0, h)$



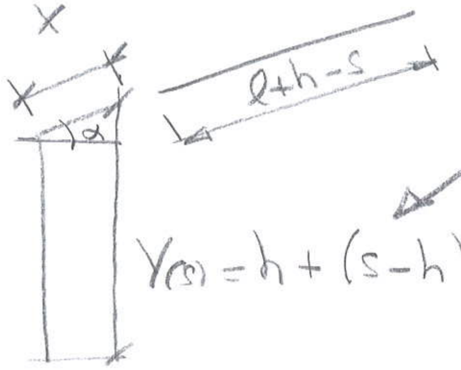


$$\boxed{\begin{array}{l} Y(s) = s \\ M(s) = M_c + H_c(f+h-s) - \frac{w\,l\,a}{2} \end{array}}$$

$s \in (h, h+l)$



$h + x + l + h - s = l + h \rightarrow \boxed{x = s-h}$



$Y(s) = h + (s-h)\operatorname{sen}\alpha$

$$\boxed{\begin{array}{l} Y(s) = h + (s-h)\operatorname{sen}\alpha \\ M(s) = M_c + H_c\,(h+l-s)\operatorname{sen}\alpha - \frac{w}{2}\cos\alpha \cdot (h+l-s)^2 \end{array}}$$

$$EI\theta_c = \underbrace{\int_{s=0}^{s=h} \left(M_c + V_c\,[f+h-s] - \frac{w\,l\,a}{2} \right) ds}_{1} \; +$$

$$\underbrace{\int_{s=h}^{s=h+l} \left(M_c + V_c\,[h+l-s]\operatorname{sen}\alpha - \frac{w}{2}\cos\alpha\,[h+l-s]^2 \right) ds}_{2}$$

RESOLVER 1 Y 2 POR CAMBIO DE VARIABLES

1 CAMBIO DE VARIABLE $t = f + h - s \rightarrow dt = -ds$

$s = 0 \rightarrow t = f + h \; ; \; s = h \rightarrow t = f$

$$-\int_{t=f+h}^{t=f} \left(M_c + V_c \cdot t - w\,\ell \frac{a}{2} \right) dt = M_c \cdot h + \frac{V_c}{2}\left(h^2 + 2h f\right) - \frac{w\,\ell\, a\, h}{2}$$

2 CAMBIO DE VARIABLE $t = h + \ell - s \rightarrow dt = -ds$

$s = h \rightarrow t = \ell \; ; \; s = h + \ell \rightarrow t = 0$

$$-\int_{t=\ell}^{t=0} \left(M_c + V_c \cdot t \,\text{sen}\,\alpha - \frac{w}{2} \cos\alpha \, t^2 \right) dt = M_c \ell + \frac{V_c}{2} \text{sen}\,\alpha\, \ell^2 - \frac{w}{6} \cos\alpha\, \ell^3$$

$\Downarrow$

$$\boxed{\overbrace{M_c (h+\ell)}^{a_{11}} + \overbrace{\frac{V_c}{2}\left(\ell^2 \text{sen}\,\alpha + h^2 + 2hb\right)}^{a_{12}} = \overbrace{\frac{w\,\ell}{2}\left(a h + \frac{1}{3}\cos\alpha\, \ell^2\right)}^{a_{13}}}$$

1ª ECUACIÓN DE COMPATIBILIDAD $\left(\theta_c = 0 \rightarrow (1) + (2) = 0 \right.$

$$\mathcal{M}_c \cdot EI = \underbrace{\int_{s=0}^{s=h+\ell} \left(M_c + V_c \left[f + h - s \right] - \frac{w\,\ell\, a}{2} \right) \left(h + b - s \right) ds}_{(1)} -$$

$$\frac{f}{\ell}(h + \ell - s)$$

$\uparrow$

$$\underbrace{\int_{s=h}^{s=h+\ell} \left(M_c + V_c [h + \ell - s] \,\text{sen}\,\alpha - \frac{w}{2} \cos\alpha \left[h + \ell - s \right]^2 \right) \overbrace{\left(h + f - \underbrace{\left[h + \frac{f}{\ell} s - \frac{h f}{\ell} \right]}_{} \right)}^{} ds}_{(2)}$$

$$Y(s) = h + (s - h) \cdot \underbrace{\frac{f}{\ell}}_{\text{sen}\,\alpha}$$

RESOLVER (1) Y (2) POR CAMBIO DE VARIABLES

1 $t = \delta + h - s \rightarrow dt = -ds$; $s = 0 \rightarrow t = \delta + h$

$s = h \rightarrow t = \delta$

$$\int_{t=\delta+h}^{t=\delta} \left(M_c \cdot t + V_c t^2 - \frac{W l_a}{2} t \right) dt =$$

$$= \frac{-M_c}{2}\left(h^2 + 2\delta h\right) - \frac{V_c}{3}\left(h^3 + 3\delta h^2 + 3\delta^2 h\right) + \frac{W l_a}{4}\left(h^2 + 2\delta h\right)$$

2 $t = h + l - s \quad dt = -ds$

$$\frac{\delta}{l}\int_{t=l}^{t=0}\left(M_c t + V_c t^2 \operatorname{sen}\alpha - \frac{W}{2}\cos\alpha\, t^3\right) dt = \frac{\delta}{l}\left[-\frac{M_c}{2} l^2 - \frac{V_c}{3} l^3 \operatorname{sen}\alpha + \frac{W}{8}\cos\alpha\, l^4\right]$$

$1 + 2 = 0$

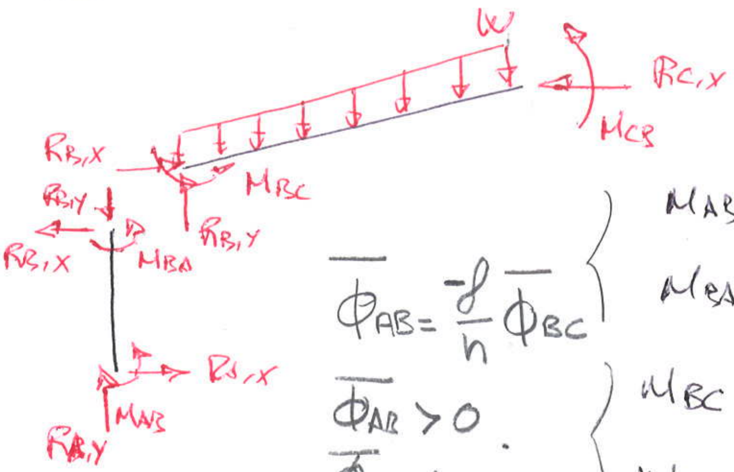
$$+ \underbrace{\frac{M_c}{2}\left[h^2 + 2\delta h + \delta l\right]}_{a_{21}} + \underbrace{\frac{V_c}{3}\left[h^3 + 3\delta h^2 + 3\delta^2 h + \delta l^2 \operatorname{sen}\alpha\right]}_{a_{22}} =$$

$$= \underbrace{\frac{W l_a}{4}\left(h^2 + 2\delta h\right) + \frac{\delta W}{8 l}\cos\alpha\, l^4}_{a_{23}}$$

$$M_c = \frac{\begin{vmatrix} a_{13} & a_{12} \\ a_{23} & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \qquad V_c = \frac{\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$M_{AB} = M_{B,A} = W l \frac{a}{2} - V_c(h + \delta) - M_c = \underbrace{-241{,}05\ \text{kN}\cdot\text{m}}_{\text{DATOS CERVEZA}}$$

EQUILIBRIO



LAS ECUACIONES ELÁSTICAS DE LAS DOS BARRAS SON.

$$\overline{\Phi}_{AB} = \frac{-f}{h}\overline{\Phi}_{BC} \qquad \overline{\Phi}_{AB} > 0 \qquad \overline{\Phi}_{BC} < 0$$

$$\left.\begin{aligned} M_{AB} &= 2K_{AB}\theta_B - 6K_{AB}\overline{\Phi}_{AB} \\ M_{BA} &= 4K_{AB}\theta_B - 6K_{AB}\overline{\Phi}_{AB} \end{aligned}\right\} K_{AB} = \frac{EI}{h}$$

$$\left.\begin{aligned} M_{BC} &= 4K_{BC}\theta_B - 6K_{BC}\overline{\Phi}_{BC} + \frac{wl^2}{12}\cos\alpha \\ M_{CB} &= 2K_{BC}\theta_B - 6K_{BC}\overline{\Phi}_{BC} - \frac{wl^2}{12}\cos\alpha \end{aligned}\right\} K_{BC} = \frac{EI}{l}$$

$$\cos\alpha = \frac{a}{l}$$

COMO $gl + ng = K - 1 + 1 = 2 \rightarrow$ 2 ECUACIONES DE EQUILIBRIO.

1 $\quad \Sigma M_B = 0$

$$M_{BA} + M_{BC} \rightarrow \underbrace{4\theta_B(K_{AB}+K_{BC})}_{a_{11}} + \underbrace{6\overline{\Phi}_{BC}\left(+\frac{f}{h}K_{AB} - K_{BC}\right)}_{a_{12}} + \underbrace{\frac{wla}{12}}_{-a_{13}} = 0$$

2 LA SEGUNDA ECUACIÓN DE EQUILIBRIO SERÁ $R^{AB}_{B,X} = R^{BC}_{B,X}$

$$AB \rightarrow \Sigma M_A = 0;\quad M_{AB} + M_{BA} + R_{B,X}\cdot h = 0;\quad R_{B,X} = -\frac{M_{AB}+M_{BA}}{h}$$

$$BC \rightarrow \Sigma M_C = 0;\quad M_{BC} + M_{CB} + R_{B,X}\cdot f - wl\left(a - \frac{a}{2}\right) = 0;$$

$$\rightarrow \Sigma F_y = 0 \rightarrow R_{B,Y} = w\cdot l$$

$$R_{B,X} = \frac{\frac{wla}{2} - M_{BC} - M_{CB}}{f}$$

IGUALANDO

$$\underbrace{6\theta_B(fK_{AB} - hK_{BC})}_{a_{21}} + \underbrace{12\overline{\Phi}_{BC}\left(K_{AB}\frac{f^2}{h} + hK_{BC}\right)}_{a_{22}} - \underbrace{\frac{hwla}{2}}_{-a_{23}} = 0$$

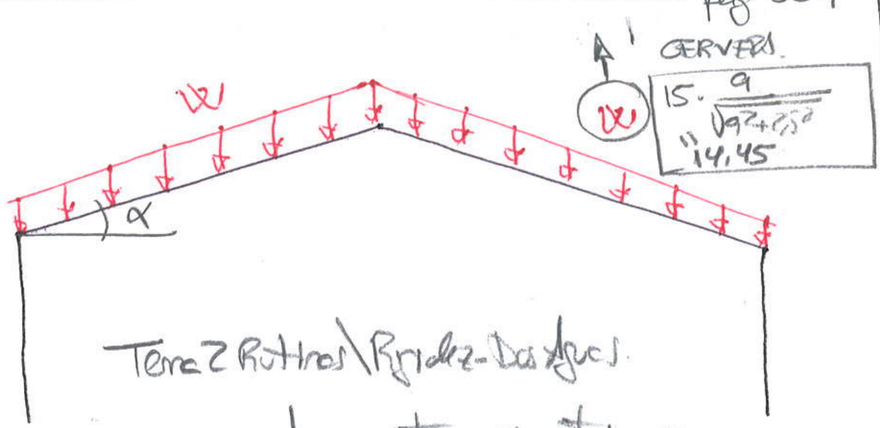
$$\theta_B = \frac{\begin{vmatrix} a_{13} & a_{12} \\ a_{23} & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \qquad \overline{\Phi}_{BC} = \frac{\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \Longrightarrow M_{AB},\ M_{BA},\ M_{BC},\ M_{BA}$$

$\downarrow$

RESTO DE REACCIONES.

COMPROBACIÓN POR MÉTODO DE RIGIDEZ

pág 554 CERVERA



Tema 2 Pórticos\Rigidez-Dos Aguas

↳ portico_simetrico.m

↳ IPE 80

NOTA

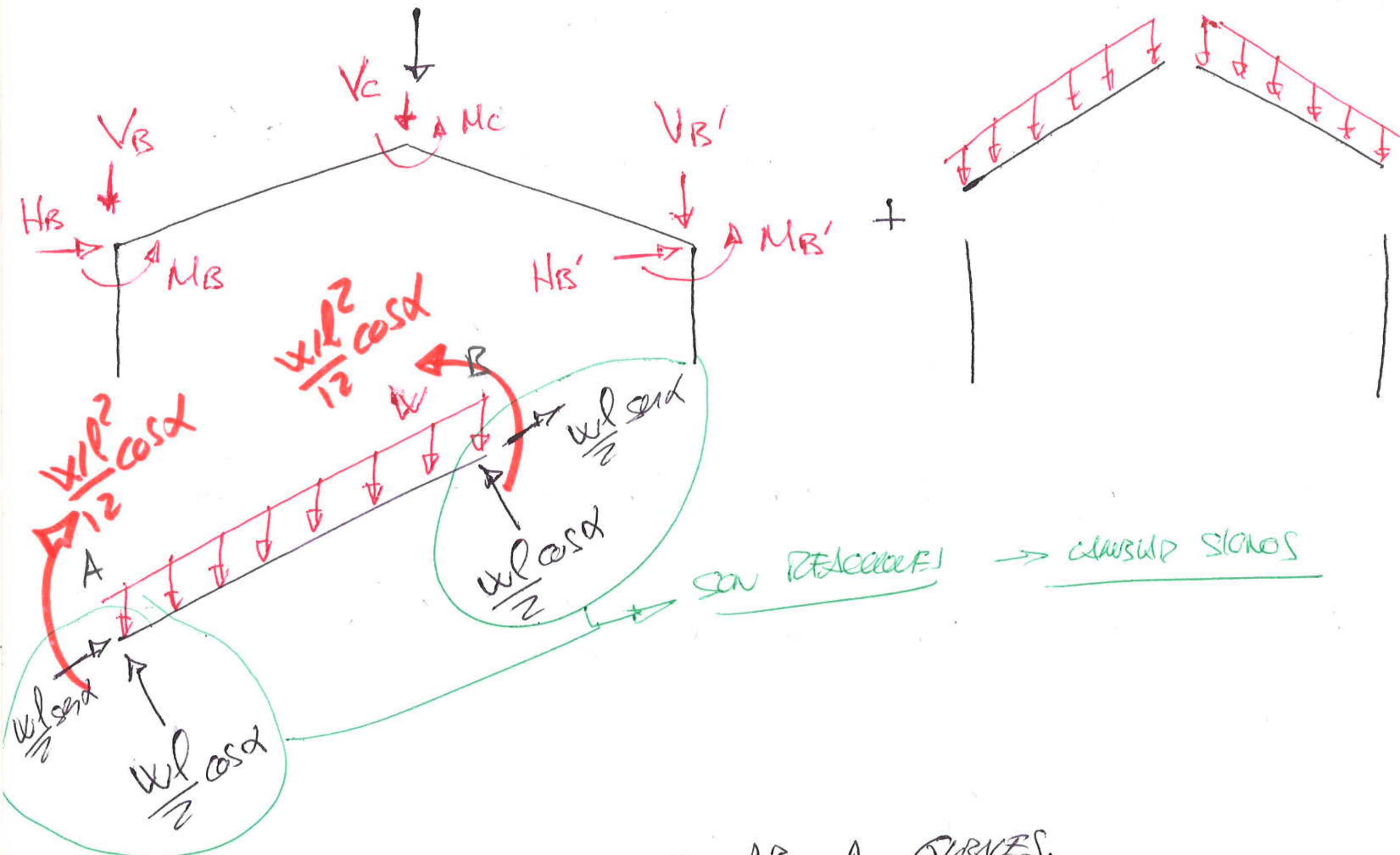
DIFERENCIA CON MÉTODOS.

RIGIDEZ → DIFERENCIA ENTRE IPE 80 E IPE 600

241,4 — 231,8 ZAPATA REACCIÓN Mz.

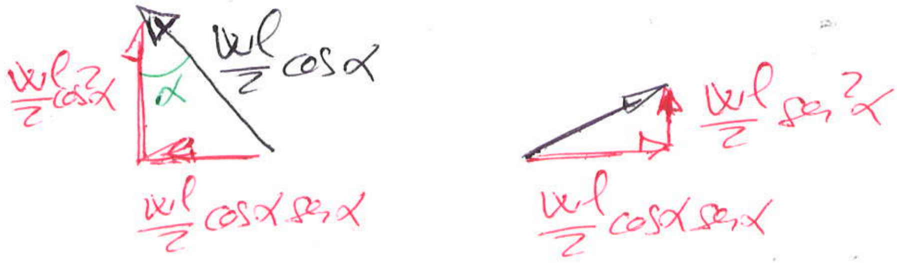
EQUILIBRIO Y COMPATIBILIDAD NO DEPENDE DE EI

241,7



PASAR DE COMPONENTES LOCALES DE AB A GLOBALES.

$$H_B = \frac{wl}{2}\cos\alpha\operatorname{sen}\alpha - \frac{wl}{2}\cos\alpha\operatorname{sen}\alpha = 0$$

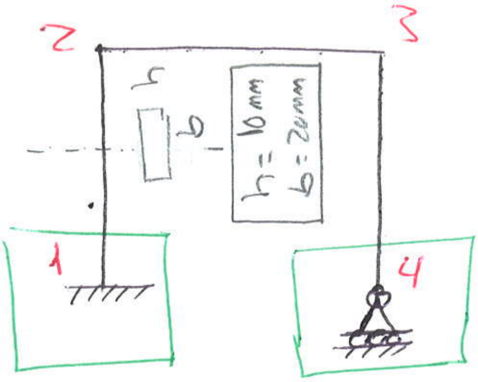


$$V_B = \frac{wl}{2}(\cos^2\alpha + \operatorname{sen}^2\alpha) = \frac{wl}{2}$$

$$M_B = -\frac{wl^2}{12}\cos\alpha$$

$M_C = 0 \quad V_C = 2V_B \quad V_{B'} = V_B \quad M_{B'} = -M_B$

ACUMULACIÓN RIGIDEZ



$$\begin{bmatrix}F_1\\F_2\\F_3\\F_4\end{bmatrix} = \begin{bmatrix} (K_{aa})_{12} & (K_{ab})_{12} & 0 & 0 \\ (K_{ba})_{12} & (K_{bb})_{12}+(K_{aa})_{23} & (K_{ab})_{23} & 0 \\ 0 & (K_{ba})_{23} & (K_{bb})_{23}+(K_{aa})_{34} & (K_{ab})_{34} \\ & & (K_{ba})_{34} & (K_{bb})_{34} \end{bmatrix} \begin{bmatrix}D_1\\D_2\\D_3\\D_4\end{bmatrix}$$

↳ ELIMINAR FILA / COLUMNA $3 \times 3 + 2 = 11$

└→ ELIMINAR LAS 3 PRIMERAS FILAS Y COLUMNAS.

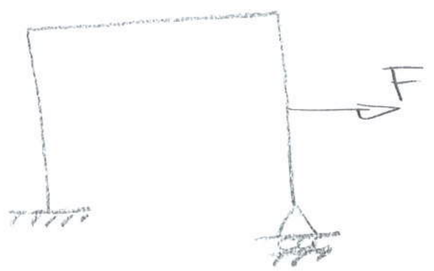
1 → $M_1 = 0$, $V_1 = 0$, $\theta_1 = 0$

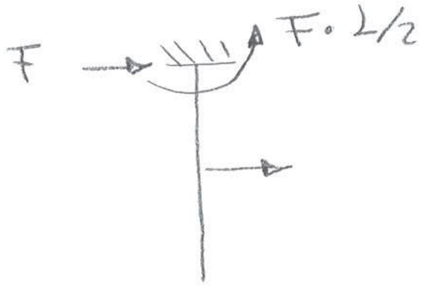
SE ELIMINAN LAS TRES

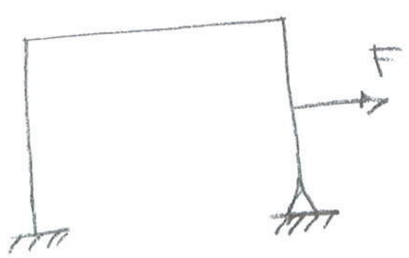
2 → $M_4 \neq 0$, $V_4 = 0$, $\theta_4 \neq 0$

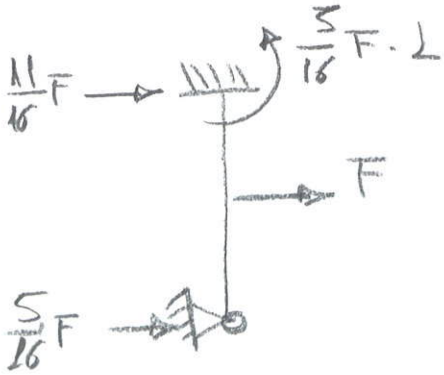
SE ELIMINA UNA.

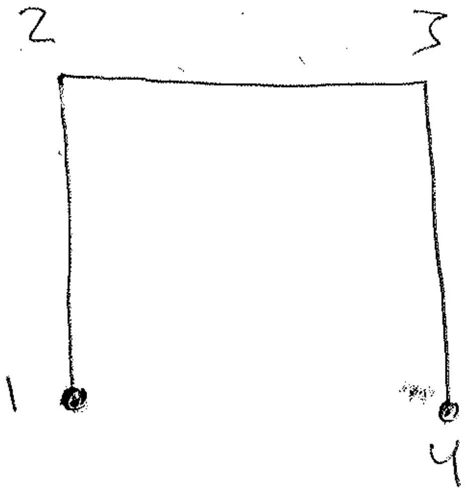
✱ ÚTIL PARA RESOLVER PRÁCTICA DE LABORATORIO.











K ES UNA MATRIZ DE 12 X 12

1 y 4 EMPOTRADO

K_HAT = K([4:9],[4:9])

1 EMP
4 APOYADO SIN DESPLA.

K_HAT = K([4:9,12],[4:9,12])

1 EMP
4 APOYADO CON DESPLAZ.

K_HAT = K([4:10,12],[4:10,12])

NOMENCLATURA MATLAB.

K_HAT ES $\hat{K}$