19/1/16

AERODYNAMICS

- Fundamentos aerodinámicos - John Anderson
- Aerodynamics for eng. students. - E.L
- Intro. to structural Aerodynamics

Content

1. Basic concepts.
2. Aerodynamic moment and forces
3. L and D and field velocity.
4. 2D wing theory and finite wing theory.
5. Aeroelasticity static and dynamic
6. CFD

Bernoulli Equation

$$p + \frac{1}{2}\rho U^2 = p_\infty + \frac{1}{2}\rho U_\infty^2 = p_R$$

→ Presión de remanso

- Bernoulli equation can be applied in streamlines

$$P_\infty + \frac{1}{2}\rho V_\infty^2 = P + \frac{1}{2}\rho V^2$$

$\rho = \rho$ density.

$$V = \sqrt{\frac{2(P_\infty - P)}{\rho} + V_\infty^2}$$

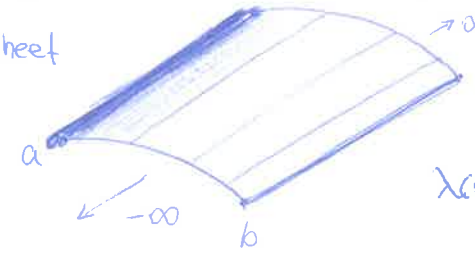
- Kutta Sow-Kawsky

$$\frac{L}{b} = \rho_\infty V_\infty \Gamma$$

- Source Panel Method

Non Lifting flows over arbitrary Bodies.

Source sheet



$$d\phi = \frac{\lambda\, ds}{2\pi} \cdot \ln r$$

$\lambda(s)$ = Volume flow rate / unit depth in z direction.

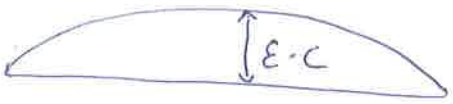
$$C_{p_i} = 1 - \left(\frac{V_i}{V_\infty}\right)^2$$

$$\phi_{(x,y)} = \int_J^d \frac{\lambda\, ds}{2\pi} \ln r$$

Exam (March)
Aerodynamics

(1) $z = c\varepsilon\left[1-\left(\frac{x}{c}\right)^2\right]$

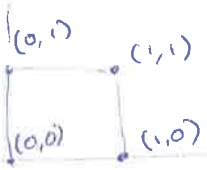
- Ideal α
- $\alpha_{L=0}$
- C_L C_m
- x_{cp}.



(2) Are similar the dynamic (conditions) for equal μ and a (speed of sound)?

Airfoil 1	Airfoil 2
$c = x$	$c = 2x$
$\rho = 1'23$	$\rho = 1'74$
$V = 100$	$V = 200$
$T = 200K$	$T = 800k$

(3) Circulation w/ velocity vectors $u = xy^2$ $V = xy$



(4) Obtain geometric α

- $V = k$
- A_0
- $\alpha_{L=0}$
- b
- Wing Area
- Weight.

2D. Degeneración corticobasal

- Ancianos con rigidez extrapiramidal, alteraciones motoras asimétricas y disfunción sensitiva cortical
- Se asocia al mismo haplotipo MALT que PSP

Degeneración corticobasal. AP

- Pérdida neuronal con gliosis y nª balonizadas en corteza premotora, motora y parietal anterior
- Pérdida de nª pigmentadas e inclusiones argirófilas en sustancia negra y locus ceruleus
- Inmunorreactividad frente a tau en
  - Astrocitos "astrocitos en ovillo"
  - Oligodendrocitos "cuerpos en bobina"

Coefficients

Lift Coefficient

- $C_L = C_n \cos\alpha - C_a \sin\alpha$

Drag Coef.

- $C_d = C_n \sin\alpha + C_a \cos\alpha$

$C_D = C_{D_\infty} + C_{DL}$ (induced drag from vortex.)

$$C_L = \frac{L}{q_\infty \cdot S} \qquad C_M = \frac{M}{q_\infty S l} \qquad C_D = \frac{D}{q_\infty S}$$

Same for N (normal) and A(xial) coefficient.

l = Reference length
S = Reference surface $\rightarrow S = c(l)$

Pressure Coef.

$$C_p = \frac{p - p_\infty}{q_\infty}$$

p_∞ - Freestream pressure

Skin Friction C.

$$c_f = \frac{\tau}{q_\infty}$$

Normal and Axial Coefficients:

$C_{P_L} = C_{P,L}$

- $$C_n = \frac{1}{c}\left[\int_0^c (C_{P_L} - C_{P_U})\,dx + \int_0^c \left(C_{f_U}\frac{dy_U}{dx} + C_{f_L}\frac{dy_L}{dx}\right)dx\right]$$

- $$C_a = \frac{1}{c}\left[\int_0^c \left(C_{P,U}\frac{dy_U}{dx} - C_{P,L}\frac{dy_L}{dx}\right)dx + \int_0^c (C_{f,U} + C_{f,L})\,dx\right]$$

Moment at Leading Edge Coefficient.

- $$C_{M_{LE}} = \frac{1}{c^2}\left[\int_0^c (C_{P,U} - C_{P,L})\,x\,dx - \int_0^c \left(C_{f,U}\frac{dy_U}{dx} + C_{f,L}\frac{dy_L}{dx}\right)x\,dx + \right.$$

$$\left. + \int_0^c \left(C_{P,U}\frac{dy_U}{dx} + C_{f,U}\right)y_U\,dx + \int_0^c \left(-C_{P,L}\frac{dy_L}{dx} + C_{f,L}\right)y_L\,dx\right]$$

Ecuation of State (p)

- $P = \rho R T$

Speed of Sound (a)

- $a = \sqrt{\gamma R T}$

γ = Specific heat ratio

$\gamma_{air} = 1'4$

R = Gas Constant =

$= 8'314\ J/mol \cdot K$

$R_{specific} = 287\ J/kg \cdot K$

Dynamic Pressure: $(q_\infty)[N/m^2]$

- $q_\infty = \frac{1}{2} \rho_\infty V_\infty^2$

Drag coefficient for entire a/c ($C_{D,T}$)

- $C_{D,T} = C_{D_0} + \frac{C_L^2}{\pi e AR}$

$= C_{D_0} + C_L^2 \left(\frac{1}{\pi e AR}\right)$

Oswald Factor (e)

- $e = 1'78(1 - 0'045\ AR^{0'68}) - 0'64$

$C_{D,0}$ → Zero-Lift C_D

$C_{D,0} = C_D - C_{D,i}$

$\frac{C_L}{C_D} = \frac{C_L}{C_{D_0} + \frac{C_L^2}{\pi e AR}}$

Aspect Ratio

$AR = \frac{b^2}{S}$

Fourier Coefficient

- $A_0 = \alpha - \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{dz}{dx}\, d\theta$

- $A_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{dz}{dx}\cos(n)\, d\theta$

$\frac{x}{c}$

- $C_{M_{LE}} = -\frac{C_L}{4} + \frac{\pi}{4}\left(A_2 - A_1\right)$

$C_L = 2\pi\left(\alpha - \alpha_{L=0}\right)$ $\quad C_L = a\left(\alpha - \alpha_{L=0}\right)$ $\quad a$ = Slope in C_L vs α

Incomp. $C_L = 2\pi\alpha$

$C_L = 2\pi\left(A_0 + \frac{A_1}{2}\right)$ $\quad$ In terms of Mach n°: $C_L = \frac{C_{Li}}{\sqrt{1-M_\infty^2}}$

In

In terms of Mach No:

$C_{P_{Corit.}} = \frac{C_{P_i}}{\sqrt{1-M_\infty^2}}$

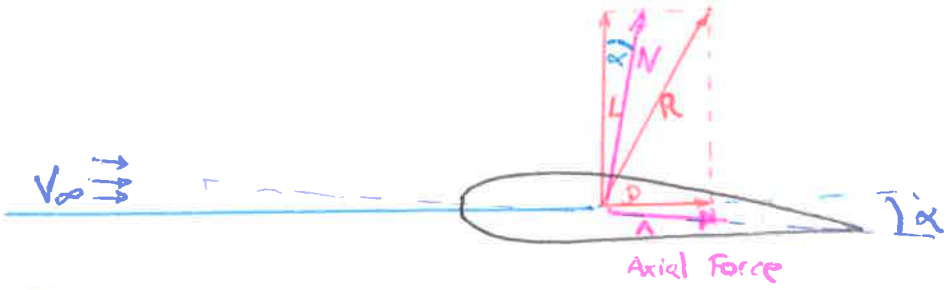
M_∞ = Mach No.

AERODYNAMICS

$D = N \cdot \sin\alpha + A \cdot \cos\alpha$

$L = N \cos\alpha - A \sin\alpha$

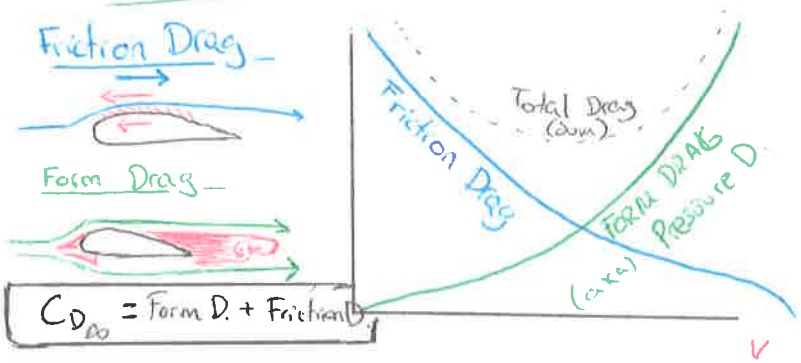
A' axial and N' normal forces PER UNIT SPAN



DRAG

Friction Drag

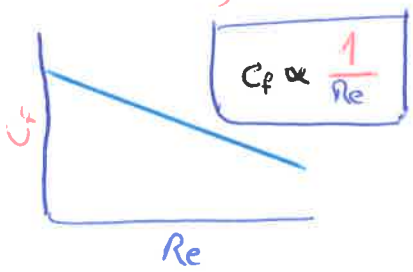
Form Drag



FORM DRAG = Pressure Drag + Skin Friction

≃

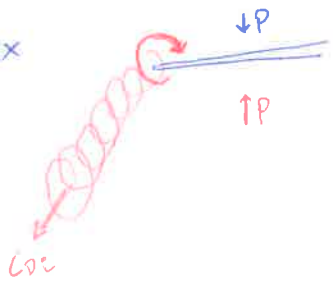
Skin Friction Coeff. vs Reynolds



C_f decreases with Re (and V), as

Induced Drag

Wingtip Vortex



$$C_{D_{wing}} = C_{D_\infty} + C_{D_i} \cdot f(C_L^2)$$

$$C_D' = \frac{D'}{\frac{1}{2}\rho V_\infty^2 \cdot \delta} = \frac{D'}{q_\infty \cdot C \cdot (1)}$$

↓ Dynamic pressure q_∞

$$M = \frac{V_\infty}{c}$$

$$M \cdot c = V_\infty$$

$$2 \cdot \sqrt{\gamma R T_\infty} = V_\infty$$

↑ 1'4 ↑ R = 287 $\frac{J}{kg \cdot K}$ ↑ 288 K

$V_\infty = 680'4\ m/s$

$M_\infty = 2$

$\alpha = 0°$

5° half angle wedge.

$P_\infty = 1\ Pa$

$M_\infty = 2$

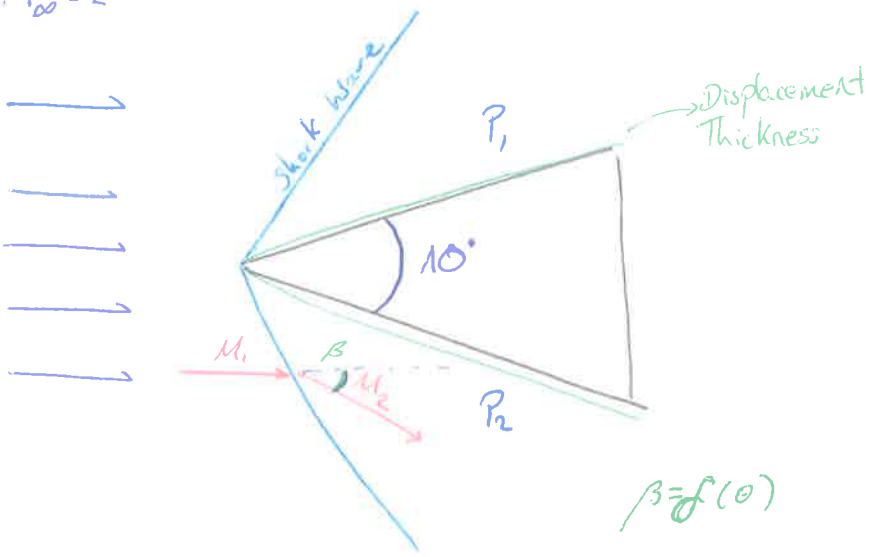
$\rho_\infty = 1'23\ kg/m^3$

$P_1 = 1.31 \cdot 10^5\ Pa$

$P_2 = P_\infty$

Shear $\tau_\infty = 431 \cdot 5^{-2}\ N/m^2$

$L = 2$

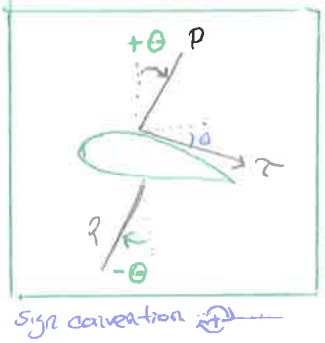


$\beta = f(\theta)$

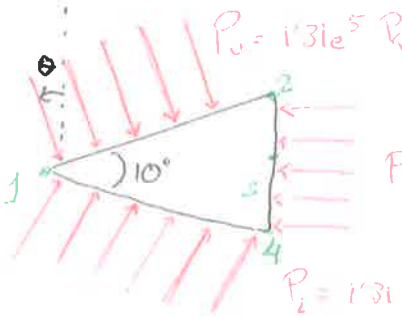
↳ weak shock wave. Flow after it, keeps supersonic regime.

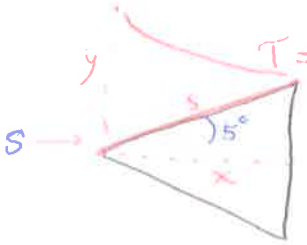
* Find C_D

What is θ ? + ?



Sign convention





$\alpha = 0$

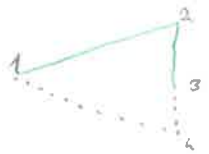
$A' = D'$

$$A' = \int_{LE}^{TE} \left(-P_u \cdot \sin\theta + \tau_u \cdot \cos\theta\right) ds_u + \int_{LE}^{TE} \left(P_L \cdot \sin\theta + \tau_u \cos\theta\right) ds_L$$

(Page 3)

$$\int_{LE}^{TE} -P_u \cdot \sin\theta \, ds_u = \int_{S_1}^{S_2} P_u \sin(\theta)\, ds_u + \int_{S_2}^{S_3} P \cdot \sin\theta \, ds_u$$

(Ktte) (Ktte)



$$= 1'31e^5 \frac{N}{m^2} \cdot \sin(-5)\,(S_2 - S_1) + 1'01 \times 10^5 \frac{N}{m^2} \cdot \sin(90)\,(S_3 - S_2)$$

$\left(\frac{c}{\cos 5}\right)$ $\left(< \tan(5)\right)$

$= 5260N$

$\tan = \frac{S_2 - S_1}{c}$

continuation:

$$\int_{LE}^{TC} P_L \cdot \sin\theta \, ds_L = \int_{S_1}^{S_4} P_L \cdot \sin(5) \, ds_L + \int_{S_4}^{S_3} P \cdot \sin(90) \, ds_L$$

$$= 5260\,N$$

$$\int_{LE}^{TE} T_u \cdot \cos\theta \, ds_u = \int_{S_1}^{S_2} 431 \cdot s^{-0'2} \cdot \cos(-5) \, ds_u$$

$$= 431 \cdot \frac{s^{0'8}}{0'8} \Big]_{S_1}^{S_2} \cdot \cos(-5)$$

$$= 429 \left(S_2^{0'8} - S_1^{0'8}\right) \frac{1}{0'8} = 936'5\,N$$

$$\int_{LE}^{TE} T_L \cdot \cos\theta \, ds_L = 936'5\,N$$

~(85%) Pressure Drag ~(15%) Shear force (friction Drag) ($\propto v$)

$$D' = A' = 5260\,N + 5260\,N + 936'5N + 936'5\,N = 12\,393\,N$$

As it's supersonic, shock wave drag must be

Pg 1 $\longrightarrow C_D'$ $M = \frac{V_\infty}{L}$

$$C_D' = \frac{D'}{\frac{1}{2}\rho V_\infty^2 \cdot S}$$

$$C_D' = \frac{12.393\,KN}{\frac{1}{2} \cdot 1'23 \cdot (680'4)^2 \cdot 2}$$

END

Pitching Moment

Upper

$$dM'_u = (P_u \cdot \cos\theta + \tau_u \cdot \sin\theta) \times ds_u + (-P_u \cdot \sin\theta + \tau_u \cdot \cos\theta) y\, ds_u$$

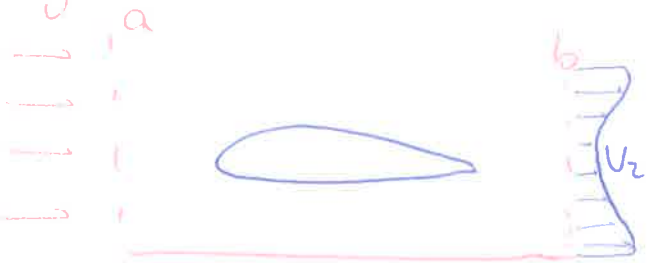
Lower

$$dM'_L = (-P_L \cos\theta + \tau_L \cdot \sin\theta) \times ds_L + (P_L \cdot \sin\theta + \underbrace{\tau_L \cdot \cos\theta}_{\text{Distribution of shear stress at u or L surface}}) y\, ds_L$$

Moment @ LE

$$M'_{LE} = \int_{LE}^{TE} (dM'_u + dM'_L)\, ds$$

$$M'_{LE} = -x_{cp} L' = -\frac{c}{4} L' + M_{c/4}$$



$$D' = \iint P_{ss}$$

$$D' = \rho \int_h^b u_2 (U_1 - u_2)\, dy$$

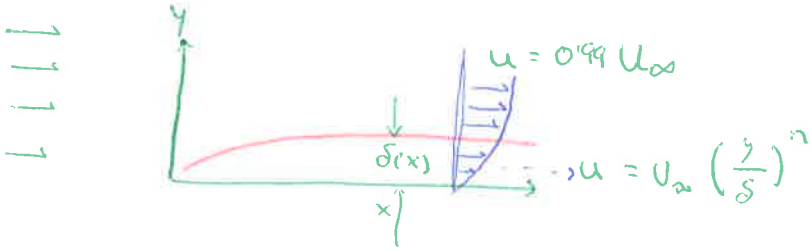
i.e:

Consider an incompressible flow laminar BL growing along the surface of a flat plate. chord = c. BL thickness at TE → $\delta = \frac{5.C}{\sqrt{Re}}$ → $\frac{1'328}{\sqrt{Re_c}}$

Skin friction coeff. for flat plate laminar → $C_f = \frac{D'}{q_\infty C(1)}$ (unit span)

Assuming velocity profile through the BL given by power law. $u = U_\infty \left(\frac{y}{\delta}\right)^n$

Calculate the value of n to satisfy



Assumptions

- Ideal Gas
- Steady State
- Laminar BL
- Incompressible
- No heat transfer
- Viscous flow.

Solution

$$C_f = \frac{D'}{q_\infty C} = \rho_\infty \int_0^\delta \frac{u_2 (u_1 - u_2)\, dy}{q_\infty \cdot C}$$

$\left(q_\infty = \frac{1}{2} u^2 \cdot \rho\right)$

$$= \frac{\rho_\infty \cdot 2}{u_\infty^2 \rho_\infty \cdot C} \int_0^\delta \frac{u_2 (u_1 - u_2)\, dy}{}$$

$u_1 = u_\infty$

$$= \frac{2}{C} \int_0^\delta \frac{u_2}{u_\infty} \left(\frac{u_1 - u_2}{u_\infty}\right) dy$$

Change of variable

$$C_f = 2 \int_0^{(\delta/c)} \frac{u}{u_\infty} \cdot \left(1 - \frac{u}{u_\infty}\right) d(y/c)$$

$\delta = f(x)$

$\delta = \frac{5C}{\sqrt{Re}}$

$\frac{\delta}{C} = \frac{5}{\sqrt{Re}}$

$$C_f = 2 \int_0^{(\delta/c)} \frac{U_\infty (y/\delta)^n}{U_\infty} \left(1 - \frac{u_\infty}{U_\infty} (y/\delta)^n\right) d(y/c)$$

$$C_f = 2 \int_0^{\delta} \frac{(y/c)^n}{(y/c)^n} \left(1 - \frac{(y/c)^n}{(\delta/c)^n}\right) d(y/c)$$

$$C_f = 2 \int_0^{\delta/c} \left[\left(\frac{y/c}{\delta/c}\right)^n - \left(\frac{y/c}{\delta/c}\right)^{2n}\right] d(y/c)$$

Replace

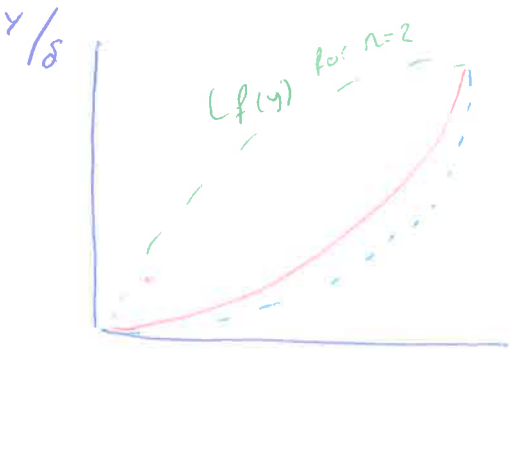
$$C_f = \frac{2}{n+1}\left(\frac{\delta}{c}\right) - \frac{2}{2n+1}\left(\frac{\delta}{c}\right) = \frac{1'328}{\sqrt{Re_c}}$$

$$= \frac{2}{n+1}\left(\frac{5 \cdot c}{\sqrt{Re}}\,\frac{1}{c}\right) - \frac{2}{2n+1}\left(\frac{5 \cdot c}{\sqrt{Re} \cdot c}\right) = \frac{1'328}{\sqrt{Re_c}}$$

$$\Rightarrow \frac{1}{n+1} - \frac{1}{2n+1} = \frac{1'328}{10}$$

$$0'2656\, n^2 - 0'6016\, n + 0'1328 = 0$$

$$\boxed{n = 2 \quad \text{or} \quad n = 0'25}$$



$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^n$$

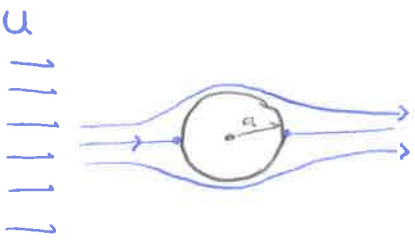
Potential Flow

- Inviscid
- Incompressible
- Irrotational

Potential Calculation:

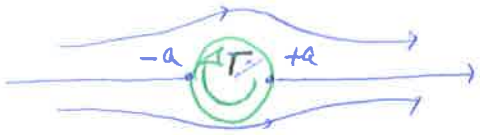
$$w(z) = f(z) + \overline{f\left(\frac{a^2}{z}\right)}$$ → conjugated

$$\left[w(z) = Uz + \frac{Ua^2}{z} \right]$$ Real Part, No circulation.



With Circulation

$$\left[w(z) = U \cdot z + U\frac{a^2}{z} - \frac{i\Gamma}{2\pi} \log z \right]$$



Stream function ψ

$$\psi = U \cdot y \left(1 - \frac{a^2}{x^2+y^2}\right) - \frac{\Gamma}{2\pi} \cdot \log(x^2+y^2)$$

$z^2 = x^2 + y^2$ As is a circle $a^2 = x^2 + y$.

Streamline → $\psi = -\frac{\Gamma}{2\pi} \log(a)$

Any stagnation points in the flow must satisfy (2D-Flow)

$$\left[u - iv = 0 = \frac{dw}{dz} = U - \frac{Ua^2}{z^2} - \frac{i\Gamma}{2\pi z} \right]$$

$$0 = z^2 U - \frac{i\Gamma z}{2\pi} - Ua^2$$

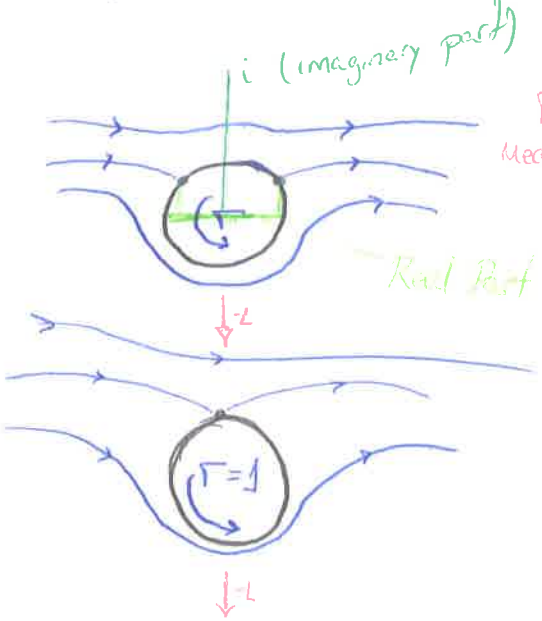
(1) No circulation → $\Gamma = 0$

$$\frac{z}{a} = \pm 1 \rightarrow \underline{z = \pm a}$$

$$\left(\frac{z}{a}\right) = \frac{i\Gamma}{2\pi a} \pm \sqrt{1 - \Gamma^2/8\pi^2 U^2 a^2}$$

(2) Increasing circulation (anticlockwise) acw

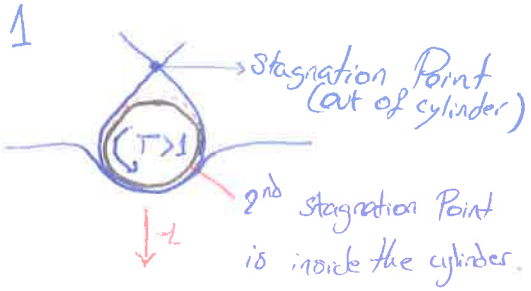
(2.1) when $\frac{\Gamma}{4\pi U a} = 1$



Γ = 1

(2.2) when $\frac{\Gamma}{4\pi U a} > 1$

$\Gamma > 1$



$$\left(F_x - iF_y\right) = i\frac{\rho}{2}\oint \left(\frac{\partial w}{\partial t}\right)^2 dz$$

Solution of the integral:

$$\oint_{|z|=a} \left(u - \frac{Ua^2}{z^2} - \frac{i\Gamma}{2\pi z}\right)^2 dz$$

radius

by Cauchy Residue:

$$\oint_z f(z)\,dz = 2\pi i \sum_{K=1}^{n} \text{Residue}\left(f(z)\cdot z\right)$$

Laurent Series $(-\infty, \infty)$

$$f(z) = \ldots + \frac{a_{-2}}{(z-z_0)} + \frac{a_{-1}}{(z-z_0)} + a_0 + a_1(z-z_0)^1 \ldots$$

Cauchy Residue

a coefficient should be calculated.

Pole $\rightarrow z_0 = 0$

aka Singularity

$$\text{Res}\left(f(z); z_0\right) = a_1$$

For single pole $\frac{f_0(z)}{z-z_0} =$

$$= \frac{a_{-1}}{(z-z_0)} + a_0 + a_1(z-z_0) + a_2(z-z_0)^2$$

$$(z-z_0)\cdot f_0(z) = a_1 + a_0(z-z_0) + a_1(z-z_0)^2$$

$\rightarrow$ Drop cut

$$\lim_{z\to z_0} (z-z_0)\, f(z) = a_1 = \text{Res}\left(f(z); z_0\right)$$

$$a_k = \frac{1}{2\pi i}\oint_c \frac{f(z)}{(z-z_0)^{k+1}}\,dz$$

$$a_{-1} = \frac{1}{2\pi i}\oint_c f(z)\,dz = \text{Res}\left(f(z); z_0\right)$$

$$\oint f(z)\,dz = 2\pi i\, \text{Res}\left(f(z); z_0\right)$$

Example

$$f(z) = \frac{z+3}{z+2} \qquad \text{Singularity } z_0 = -2$$

$$\oint_C \frac{z+3}{z+2}\,dz = 2\pi i \,\text{Res}\,(f(z); -2)$$

$$\text{Res}\,(f(z); -2) = \lim_{z \to -2} \frac{z+3}{z+2}(z - z_0)^{-2} = 1$$

Solution:

$$\oint \frac{z+3}{z+2}\,dz = 2\pi i\,(1) = 2\pi i$$

For $z_0 = 0$

$$\text{Res}\,(f(z); 0) = -\frac{i\Gamma}{\pi}$$

$$\oint_{|z|=a} \left(\frac{dw}{dz}\right)^2 dz = 2\pi i\left(-i\frac{\Gamma}{\pi}\right) = 2\Gamma$$

$$= \frac{i\rho}{2}\int \left(\frac{dw}{dz}\right)^2 dz = \frac{i\rho}{2}\,2\Gamma U = \rho U \Gamma i$$

$$F_x = 0 \longrightarrow \text{Drag} = 0$$

$$F_y = -\rho U \Gamma = \boxed{\text{LIFT}} \uparrow \qquad \Gamma < 0 \longrightarrow +(\text{Positive})\ \text{LIFT} \uparrow$$

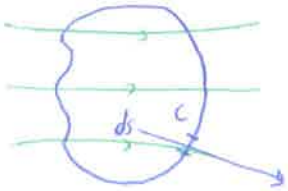
CIRCULATION [Γ]

$$\Gamma = -\oint \vec{V} \cdot ds$$

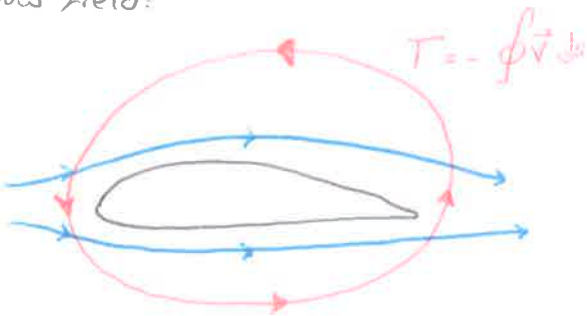
(is a finite value)

If $\Gamma = \infty \rightarrow \Gamma = 0$

Consider a close curve (c) in a flow field:



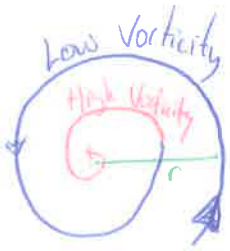
$\vec{V}$ is the velocity of ds (a segment) of the total shape



Kutta - Joukoski Theorem

$F_y = -\rho U \Gamma \rightarrow$ Lift

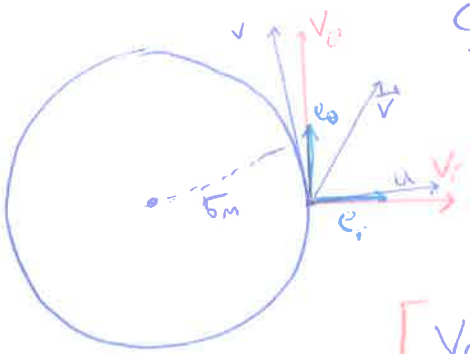
$$\Gamma = -\oint \vec{V}\, ds = \iint_S (\vec{\nabla} \times \vec{V})\, ds$$



$r \propto \frac{1}{\nabla V}$

Vorticity

eg: $V\left(\frac{y}{x^2+y^2}\,;\,\frac{-x}{x^2+y^2}\right)$ Calculate the circulation around a circular path of radius 5m.

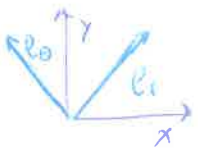


Cylindrical coords !

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$r^2 = x^2 + y^2$$

$$\begin{bmatrix} V_r = u\cos\theta + v\sin\theta \\ V_\theta = -u\sin\theta + v\cos\theta \end{bmatrix}$$

e_θ & e_r is a transformation of axis



$$u = \frac{y}{x^2+y^2} = \frac{r\cdot\sin\theta}{r^2} = \frac{\sin\theta}{r}$$

$$v = \frac{-x}{x^2+y^2} = -\frac{r\cdot\cos\theta}{r^2} = -\left(\frac{\cos\theta}{r}\right)$$

Replacing into V_r and V_θ:

$$V_r = \frac{\sin\theta}{r}\cdot\cos\theta + \left(-\frac{\cos\theta}{r}\right)\cdot\sin\theta = 0$$

$$V_\theta = -\frac{\sin\theta}{r}\cdot\sin\theta + \left(-\frac{\cos\theta}{r}\right)\cdot\cos\theta = -\frac{1}{r}$$

$$\vec{V}\,ds = (V_r + V_\theta)(\vec{e}_\theta + \vec{e}_r)(dr\,\vec{e}_r + r\,d\theta\,\vec{e}_\theta)$$

$$= (V_r e_r + V_\theta e_\theta)(dr\cdot\vec{e}_r + r\,d\theta\,\vec{e}_\theta)$$

$$= \vec{V}_r\,dr + r\vec{V}_\theta\,d\theta$$

$$= 0\,dr + r\left(\frac{-1}{r}\right)d\theta \longrightarrow \vec{V}\,ds = -d\theta$$

$$\Gamma = -\oint \vec{V}\,ds = -\int_0^{2\pi} -d\theta = +2\pi\left(\frac{m^2}{s}\right)$$

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Anderson 3ED

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$V = 10\ m/s$

$\Gamma = 20\ m^2/s$

C_p V_B Θ

$$C_p = 1 - 4\sin^2\Theta\left(1 + \frac{\Gamma}{4\pi a V_\infty \sin\Theta}\right)^2$$

$$= \frac{P - P_\infty}{\frac{1}{2}\rho V_\infty^2}$$

Statements of Kutta condition.

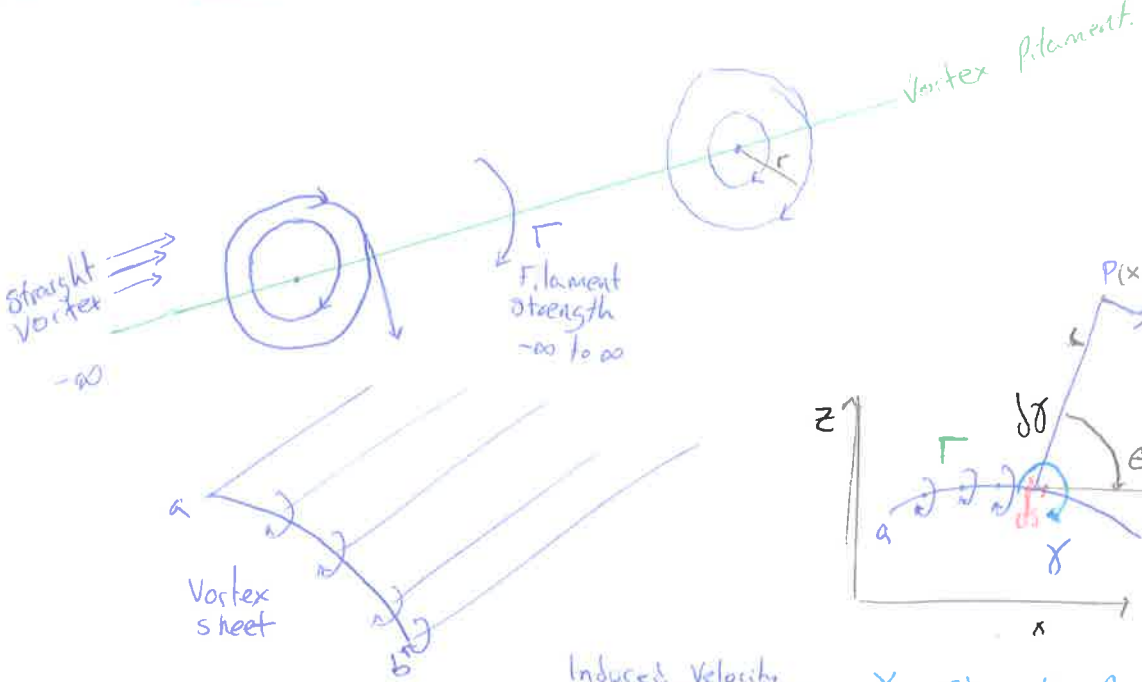
1°/ For any given Γ is such that the flow leaves the T.E. smoothly.

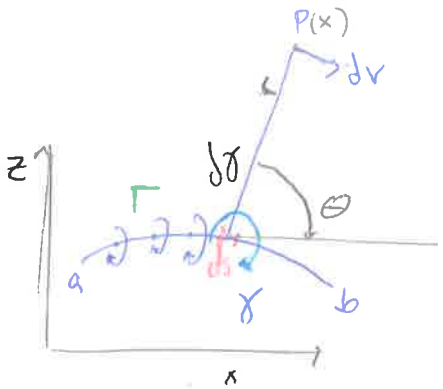
2°/ In the T.E. must exist an stagnation point.

3°/ Cusped t.E. and exit velocities must be equal in magnitude and direction.

4.4 Theorical solutions for low speed flow over airfoils: is the vortex sheet.

Pg 285
3rd





Induced Velocity

$$d\vec{V} = \frac{-\gamma ds}{2\pi r}$$

γ: Strength of the vortex sheet.

$\gamma = \gamma(s)$ [Per Unit length along s]

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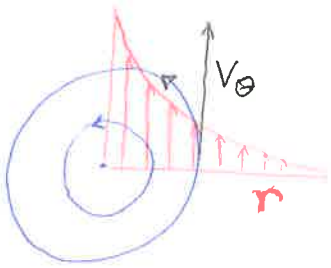
3ED

Vortex Flow



$V_\theta = \frac{c}{r}$. (Determine constant c)

$$\Gamma = -\oint \vec{V}\, ds$$

$$= \iint_s (\vec{\nabla} \times \vec{V})\, ds \quad \longleftarrow \text{[Escalar]}$$

$\vec{\nabla} \times \vec{V}$ = Vorticity.

$$\Gamma = -\int_0^{2\pi} \vec{V}_\theta \cdot r\, d\theta$$

$$= -V_\theta\, 2\pi r$$

$$V_\theta = \frac{-\Gamma}{2\pi r} = \frac{c}{r} \longrightarrow c = \frac{-\Gamma}{2\pi} \longrightarrow \boxed{\Gamma = -c\, 2\pi}$$

what is ϕ?

Velocity potential of the vortex.

$$V_r = \frac{d\phi}{dr} = 0$$

$$V_\theta = \frac{1}{\cancel{r}} \frac{d\phi}{d\theta} = -\Gamma / 2\pi \cancel{r}$$

$$\cancel{d\theta}\; d\phi = \frac{-\Gamma}{2\pi}\, d\theta$$

ϕ Velocity potential $\rightarrow$ $\boxed{\phi = \frac{-\Gamma}{2\pi}}$ $\qquad \Gamma = \gamma\, ds$

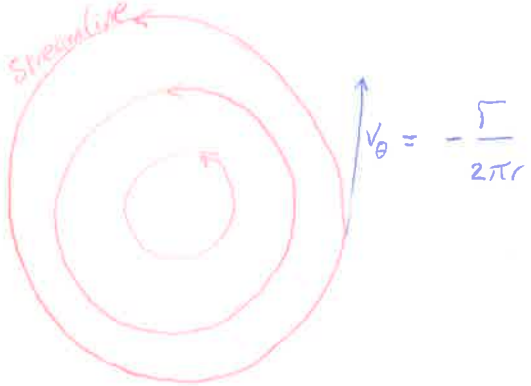
$$d\phi = (-\gamma\, ds / 2\pi)\, \theta$$

$$\phi(x,z) = \left(\int_a^b \frac{-\gamma\, \theta\, ds}{2\pi} \right)$$

This is important for numerical vortex panel method.
Pg 247 / 3ed.

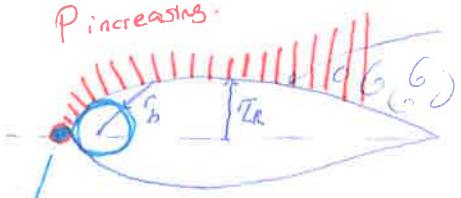
$$\Gamma = \int_a^b \gamma\, ds$$

3.14 Vortex Flow



Airfoil Defined geometrically by:

- THICNESS τ_R
- CURVATURE ε = Espesor relativo ε = = Espesor max / c
- α



r_b = Radio de curvatura

P=∞ → Pico de succión de borde de ataque (Normal a la superficie)

(Al haber presión baja)

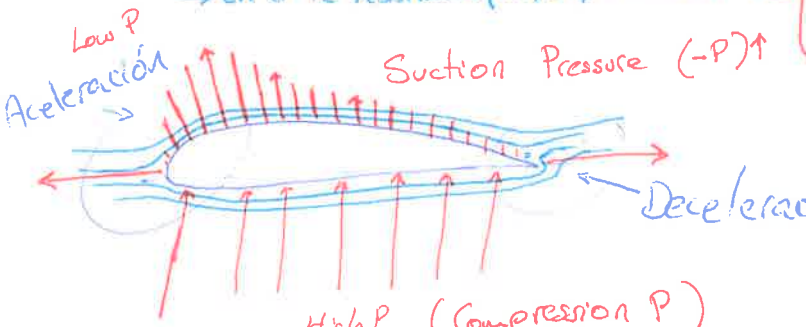
Inverso al radio de curvatura.

← En el LE las presiones generan tracción

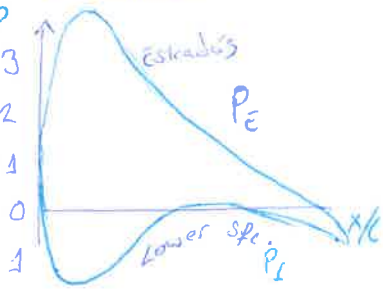
→ En el TE resistencia (Mayor que la tracción LE)

Diseño de planos:

- *Borde de salida (TE)
- *Borde de ataque (LE)



Kutta Th. → Refleja la importancia del TE para la generación de sustentación



$L = P_E - P_I$ (Ambas P van en la misma dirección)

3.16 pg 244 Bed.

Kutta theorem:

La circulación al rededor de un perfil ha de ser la apropiada para que el punto de remanso posterior no esté en el estradós ni en el interior, sino en el TE (o que desaparezca).

Una vez las corrientes confluyen, lo hacen con la misma P.

Kutta ideal circulation

Como la P_R (P de remanso) es uniforme, por la eq. de Bernoulli se deduce que igualdad de P requiere igualdad de V.

Bernoulli

$$p + \frac{1}{2}\rho U^2 = p_\infty + \frac{1}{2}\rho U_\infty^2 = P_R$$

Circulation

$$\Gamma = \oint_C V \cdot ds$$

Lift/unit span

$$L' = \rho_\infty V_\infty \Gamma$$

* $\oint_C$ → Line integral $\int_C$
  Normaly is anticlockwise.
  In aerodyn. better clockwise

Se habla de circulación para referirse a que la line integral es finita.

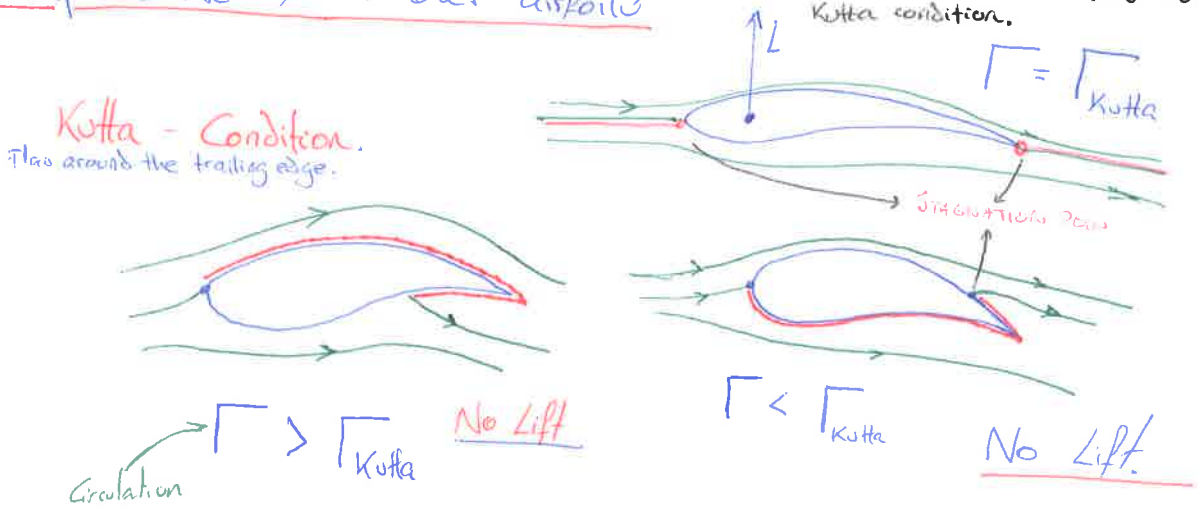
ds → Means we are moving along a closed curve "C"
(Instead of x, y axis)

«Circulation is an ABSTRACT, mathematical concept»

19/1/16

Incompressible flow over airfoils

(1) circulation that satisfies the Kutta condition.



Theory of Cambered Airfoil

Symmetric Airfoil

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)\,d\xi}{x-\xi} = V_\infty\left(\alpha - \frac{dz}{dx}\right)$$

Transformation $\xi = \frac{c}{2}(1-\cos\theta)$

$d\xi = \frac{c}{2}\sin d\theta$

$x = \frac{c}{2}(1-\cos\theta_0)$

$$\frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)\cdot\sin\theta\,d\theta}{\cos\theta-\cos\theta_0} = V_\infty\left(\alpha - \frac{dz}{dx}\right)$$

Mean Camber $\rightarrow$ $2V_\infty\sum_{n=1}^{\infty} A_n \sin(n\theta)$

$$\gamma(\theta) = 2V_\infty\left[A_0\,\frac{(1+\cos\theta)}{\sin\theta} + \sum_{n=1}^{\infty} A_n\cdot\sin(n\theta)\right]$$

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos nx + B_n \sin(nx)$$

Fourier series
↓ -Obtain the coefficient.

$P = 2\pi$

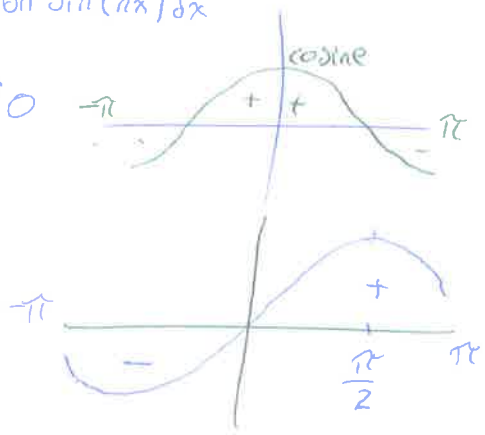
$$\int_{-\pi}^{\pi} f(x)\,dx = \int_{-\pi}^{\pi} A_0\,dx + \int_{-\pi}^{\pi} A_n \cdot \cos nx + B_n \sin(nx)\,dx$$

$\int_{-\pi}^{\pi} A_n \cos nx + B_n \sin(nx)\,dx = 0$

$\underbrace{X + \pi - (-\pi)}$

$$\int_{-\pi}^{\pi} f(x)\,dx = 2\pi A_0$$

$$A_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$



To obtain A_n multiply by $\cos mx$

$$\int_{-\pi}^{\pi} f(x)\cos mx\,dx = \int_{-\pi}^{\pi} A_0 \cos mx\,dx + \sum_{n=1}^{\infty}\left[\int_{-\pi}^{\pi} A_n \cos nx \cdot \cos mx + \int_{-\pi}^{\pi} B_m \sin(nx)\cos mx\,dx\right]$$

($\int f(x)\cos mx\,dx = 0$; $\int A_n \cos nx \cdot \cos mx = A_n$; $\int B_m \sin(nx)\cos mx\,dx = 0$)

$$\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(mx) = A_n$$

$B_n \sin(mx)$

$$\int_{-\pi}^{\pi} dx \sin(mx)\,dx = \int_{-\pi}^{\pi} A_0 \sin(mx)\,dx + \sum_{n=1}^{\infty}\int_{-\pi}^{\pi} A_n \cos nx \sin(mx) + B_n \sin(nx)\cdot\sin(mx)\,dx$$

($\int A_0 \sin(mx)\,dx = 0$; $\int A_n \cos nx \sin(mx) = 0$)

Period = 2 L

$$B_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(mx)\,dx$$

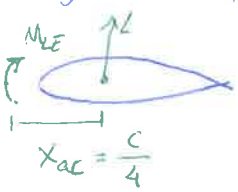
$$A_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx \qquad A_n = \frac{1}{L}\int_{-L}^{L} f(x)\cdot\cos\left(\frac{(n\pi)x}{L}\right)dx$$

AERODYNAMIC CENTER

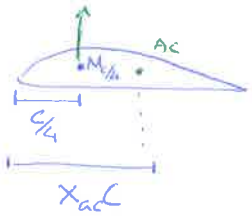
C_{Mac} = Moment Coef. at A.C.

X_{ac} = X% C

Symmetric a/f.



Cambered A/f.



$$C_{mac} = C_L (X_{ac} - 0'25) + C_{mc/4}$$

i.e:

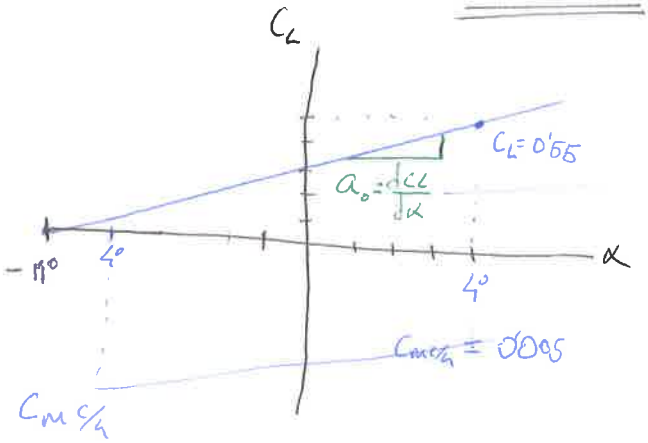
NACA 23012

For $\alpha = 4° \rightarrow C_L = 0'55 \rightarrow C_{mc/4} = -0'005$

For α_0 = zero-Lift = 11°(-1) $\rightarrow \alpha_0 = -11°$

For $\alpha = -4° \rightarrow C_{mc/4} = -0'0125$.

Obtain AC. (X_{ac})



a_o = slope of $\frac{dC_L}{d\alpha}$

$$\frac{dC_L}{\alpha} = \frac{0'55 - 0}{4 - 11} \qquad \frac{dC_L}{\alpha} = 0'1078 \text{ per Degree.}$$

$$\frac{dC_{m/4}}{d\alpha} = m_o = \frac{-0'005 - (-0'0125)}{4 - (-4)}$$

$$m_o = 9'375 \times 10^{-5} \text{ per degree.}$$

$$X_{ac} = -\frac{m_o}{a_o} + 0'25$$

Pregunta de Examen

Example Mean Cambered.

Mean Camber:

$$\frac{z}{c} = 3'212\left[\frac{x^3}{c^3} - 0'83\frac{x^2}{c^2} + 0'187\frac{x}{c}\right]$$

$\alpha_0 = \alpha_{L=0}$
α ideal
C_L
C_M
$C_{M,ac}$
?

for $0 \le \frac{x}{c} \le 0'21$

$$\frac{z}{c} = 0'043\left(1 - \frac{x}{c}\right) \qquad x = \frac{c}{2}(1-\cos\theta)$$

for $0'21 \le \frac{x}{c} \le 1$

$$A_0 = \alpha - \frac{1}{2\pi}\int_0^{\pi} \frac{dz}{dx}\, d\theta$$

$$A_n = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos n\theta\, d\theta$$

Transformar $\frac{z}{c}$ en función de θ con:

$x = \frac{c}{2}(1-\cos\theta)$

$dx = \frac{c}{2}\sin\theta\, d\theta$; $d\theta = \frac{2\,dx}{\sin\theta}$

$$\frac{dz}{dx} = 3'212\left[\frac{3x^2}{c^2} - 1'66\frac{2x}{c} + 0'187\right] \qquad \frac{dz}{dx} = -0'043$$

$$A_0 = \alpha - \frac{1}{\pi}\left[\int_0^{\beta} 3'212\left(\frac{3x^2}{c^2} - 1'66\frac{2x}{c} + 0'187\right)d\theta + \int_{\beta}^{\pi} -0'043\, d\theta\right] =$$

$$= \alpha - \frac{1}{\pi}\left[\int_0^{\beta} \frac{3}{4}(1-\cos\theta)^2 - 1'66(1-\cos\theta) + 0'187\, d\theta + \int_{\beta}^{\pi} -0'043\, d\theta\right] =$$

$$= \alpha - \frac{1}{\pi}\left[\int_0^{\beta} \frac{3}{4} + \frac{3}{4}\cos^2\theta - \frac{3}{2}\cos\theta - 1'66 + 1'66\cos\theta + 0'187\, d\theta + \int_{\beta}^{\pi} -0'043\, d\theta\right] =$$

$$= \alpha - \frac{1}{\pi}\left[\int_0^{\beta} -0'723 + \frac{3}{4}\cos^2\theta + 0'16\cos\theta\, d\theta + \int_{\beta}^{\pi} -0'043\, d\theta\right] =$$

Cont. Example Mean Camber.

$$\int \cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$$

$$A_0 = \alpha - \frac{1}{\pi}\left[-0'723\,\theta\right]_0^{\beta} + \frac{3}{4}\left[\frac{1}{2} + \frac{1}{2}\cos 2\theta\right]_0^{\beta} + \left[0'16\sin\theta\right]_0^{\beta} -$$

$$- \left[0'043\,\theta\right]_{\beta}^{\pi} = \int \cos 2\theta\, d\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$$

$$= \alpha - \frac{1}{\pi}\left[-0'723 \cdot 2'55\right.$$

~~$\beta = 73'14 \cdot \frac{2\pi}{180}$~~ $x = \frac{c}{2}(1-\cos\theta)$

$$\frac{0'21}{c} = \frac{c}{2}(1-\cos\theta)$$

$$0'21 - \frac{1}{2} = -\cos\theta\,\frac{1}{2}$$

$$\cos\theta = 0'5 \cdot 0'21 \cdot$$

$$\beta = 54$$

Aerodynamics anderson solution

Biot-Savarts

$$dw = \frac{\Gamma}{4\pi} \frac{\cos\beta \, ds}{r^2}$$

Relationship between the velocity induced by a vortex tube and the strength (circulation)

$$C_L = C_n \cos\alpha - C_a \sin\alpha$$

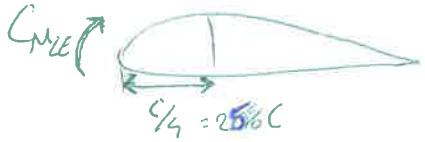
$$C_{M_{LE}} = -\frac{C_L}{4}$$

Moment C. in Leading Edge

$$C_n = \frac{1}{c}\int_0^c (C_{p_L} - C_{p_u})\,dx + \int_0^c \left(C_{f_u}\frac{dy_u}{dx} + C_{f_L}\frac{dy_L}{dx}\right)dx$$

$$C_a = \frac{1}{c}\int_0^c \left(C_{p_u}\frac{dy_u}{dx} - C_{p_L}\frac{dy_L}{dx}\right)dx + \int_0^c (C_{f_u} + C_{f_L})\,dx$$

$$C_{M_{c/4}} = \frac{\pi}{4}(A_2 - A_1)$$



$$C_{M_{ac}} = \frac{\pi}{4}\left(\frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos 2\theta\, d\theta - \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos\theta\, d\theta\right)$$

Moment at Aerodynamic cent.

At Aerod. Center the momen remains constant regardless α

$$C_{M_{ac}} = \frac{1}{2}\int_0^\pi \frac{dz}{dx}(\cos 2\theta - \cos\theta)\,d\theta \longrightarrow C_{M_{ac}} = -\frac{2M\pi}{c}$$

$$C_{M_{LE}} = -\frac{\pi c}{2}\left(A_0 + A_1 - A_2\right)$$

$$X_{cp} = \frac{c}{4}\left(1 + \frac{\pi}{C_L}(A_1 - A_2)\right)$$

Net Skin friction: (D_f) Laminar B.L.

$$C_f = \frac{D_f}{q_\infty S}$$

Lasius solution.

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = V_\infty \frac{\partial f'}{\partial y}$$

$$C_f = \frac{2\mu V_\infty}{\rho_\infty V_\infty^2}\sqrt{\frac{V_\infty}{\nu x}}\, f''(0)$$

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = V_\infty \sqrt{\frac{V_\infty}{2\nu x}}\, \delta''(0)$$

$$C_f = 2\sqrt{\frac{\mu}{V_\infty \rho x}}\, \delta''(0)$$

$$D_f = C_f \cdot q_\infty \cdot C$$

Ex. NACA 2412

$Re = 3'1 \times 10^6$

a) BL. Thickness at TE for $C = 1'5m$

b) Net Laminar Skin-friction coeff.

Solution

a) $\delta = \frac{5 \cdot C}{\sqrt{3'1 \times 10^6}} = 0'00426\,m$

b) $C_{fs} = \frac{1'328}{\sqrt{Re}} = 7'54 \times 10^{-4}$

Net $C_f = 2 \times 7'5 \times 10^{-4} = 0'0015$

$D_f = C_f \cdot q_\infty \cdot C$

$3'1 \times 10^6 = \frac{V_\infty \cdot 1'5m}{1'789 \times 10^{-5}\ m^2/s}$

$V_\infty = \left(3'1 \times 10^6 \times 1'789 \times 10^{-5}\right) / 1'5$

$D_f = 1'89\ N \cdot m^2 \rightarrow \frac{D_f}{A} = 1'89\ N$

$$D_f = q_\infty x_{cr} (C_{f,s})_L + \frac{q_\infty C}{q_\infty C}$$

$$C_f = \frac{x_{cr}}{C} (C_f)_L + (C_{f,C})_T - \frac{x_{cr}}{C} (C_{f,1})_T$$

Skin Friction in the Laminar BL. region

$$(C_f)_* = \frac{1'328}{\sqrt{Re_{cr}}}$$

(C_f → C_{f_x})

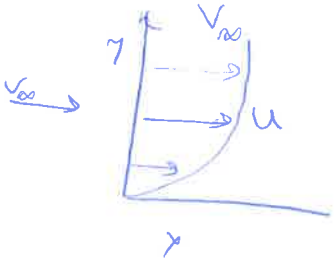
Turbulent

$$(C_f)_T = \frac{0'074}{Re_c^{1/5}}$$

$Re_{cr} = Re_x$

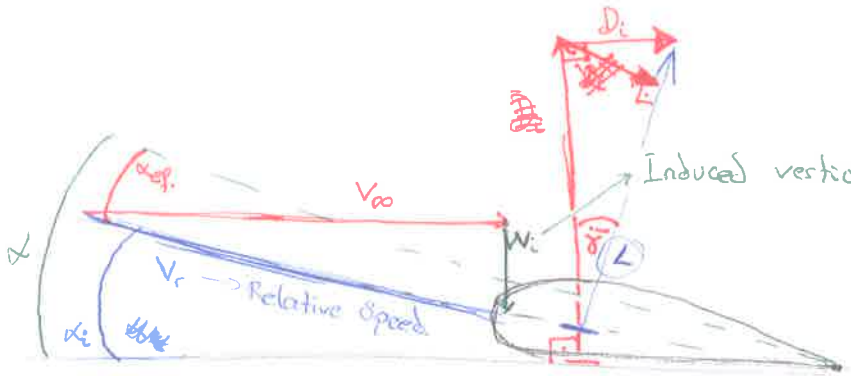
Gradient of P. different than $\emptyset$.

(→ Solve Boundary Layer eqns. eqs.)



$$\frac{\partial \tau}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} = -\rho U \frac{dU}{dx} = \frac{dp}{dx}$$

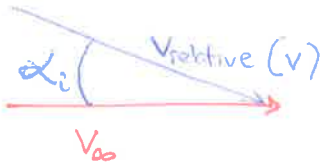
11/2/16



Lift → Perpendicular to relative V.

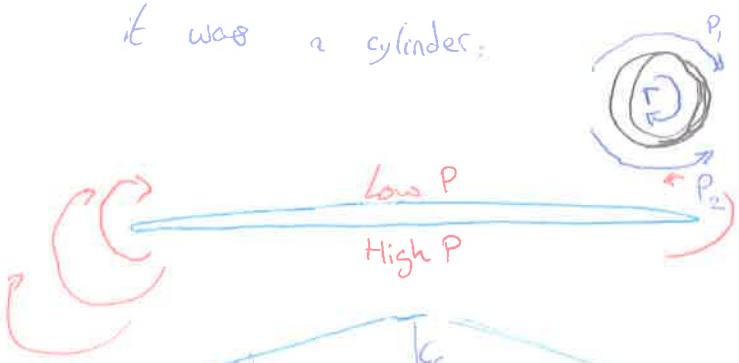
$\alpha = \alpha_i + \alpha_{eff}$

α_{eff} = Effective α



$L = \rho_\infty V_\infty \Gamma$

Assuming (just theoretically) that there is a circulation around an airfoil as it was a cylinder:

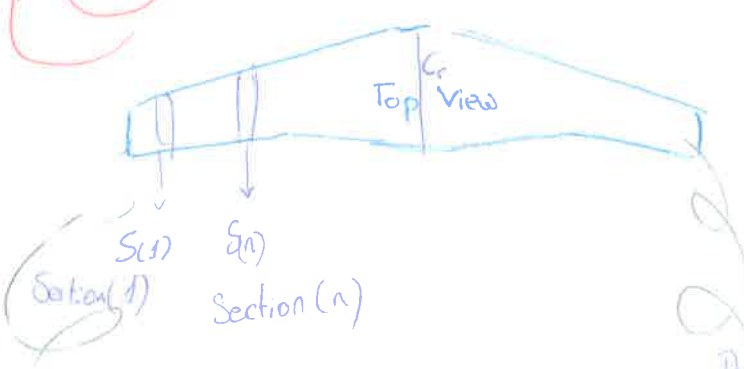


$D_i = L \cdot \sin \alpha_i$

Almost negligible

$\sin \alpha_i = \frac{W_i}{V}$

$D_i = L \cdot \alpha_i$ (Rad)



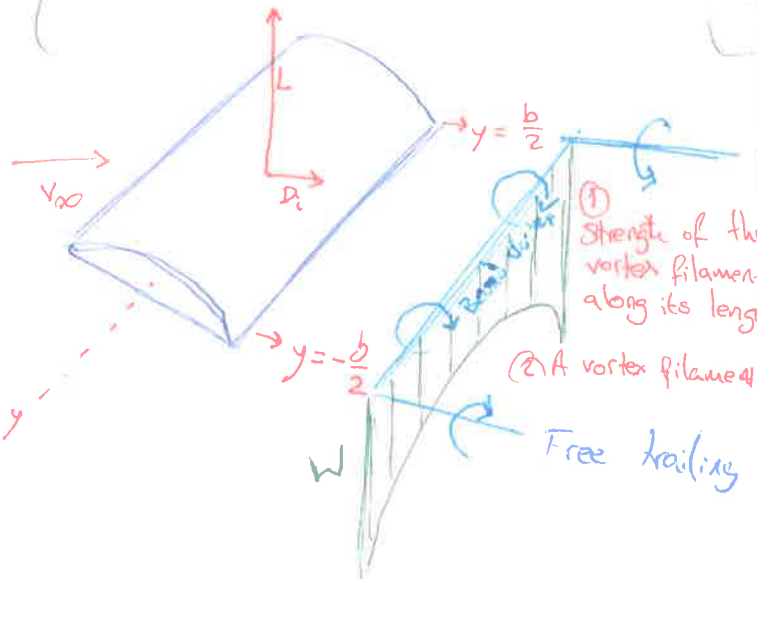
Airfoil Theory.

$C_D = C_d + C_{D_i}$

Defined by: Airfoil — Finite wing.

Parasitic or profile Drag. — C_{D_i} = Induced Drag

PRANDTL'S classical Lifting-Line Theory.



C_{D_i} → Induced Drag is less, for larger wings.

Proportional to α

S = Wing surface

$C_{D_i} = \frac{D_i}{q_\infty S}$

D_p = Pressure D.

D_f = Skin Friction D.

$= \frac{D_f + D_p}{q_\infty S}$

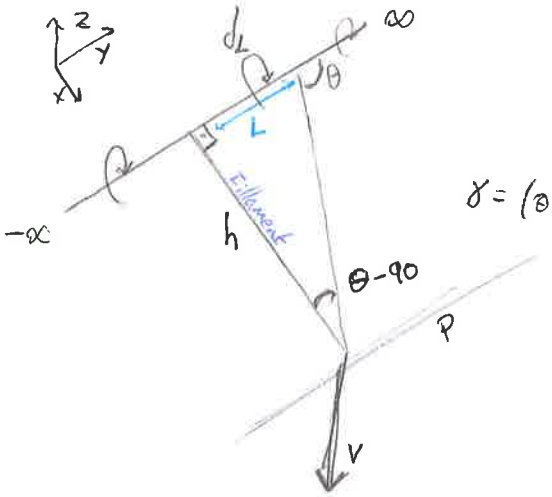
11/2/16

$$dV = \frac{\Gamma\, dL \times r}{4\pi |r|^3}$$

$$\hookrightarrow V = \int_{-\infty}^{\infty} \frac{\Gamma}{4\pi} \cdot \frac{\sin\theta\, dL}{r^2}$$

$$r\underbrace{\cos(\theta - 90)}_{\sin(\theta)} = h$$

$$r = \frac{h}{\sin\theta}$$



$$L = \frac{h}{\tan\theta} \qquad dL = -\frac{h\, d\theta}{\sin^2\theta}$$

$$\gamma = (\theta - 90)$$

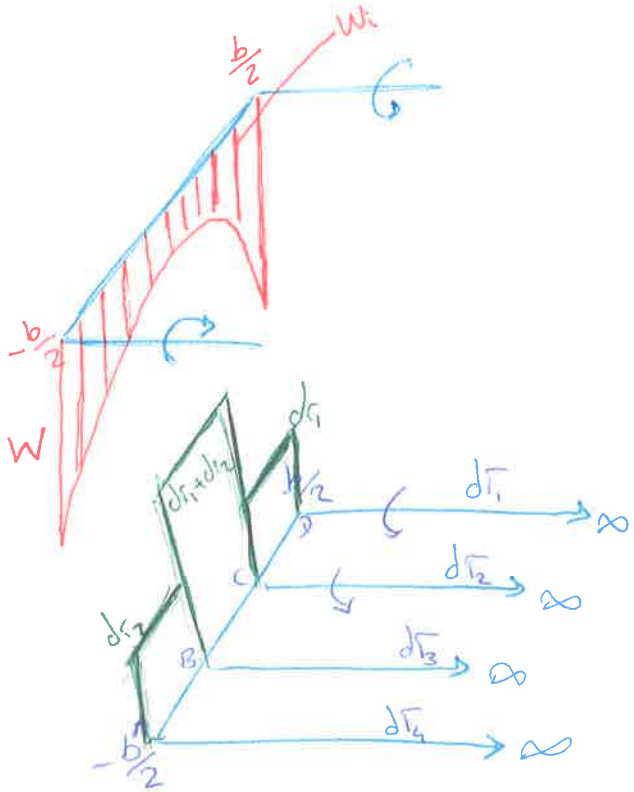
$$V = \frac{-\Gamma}{4\pi} \int_{-\infty}^{\infty} \frac{\sin\theta\, dL}{r^2} = \frac{\Gamma}{4\pi} \int_{-\infty}^{\infty} \frac{\sin\theta}{h^2} \frac{\sin^2\theta}{\sin^2\theta} h\, d\theta$$

$$V = \frac{+\Gamma}{4\pi} \int_{\pi}^{0} \sin\theta\, d\theta \qquad \left(y = \int_a^b dx \quad y = -\int_b^a dx \right)$$

$$V = \frac{\Gamma}{2\pi h}$$ Infinite filament.

Semi infinite filament.

Biot-Savart $V = \frac{\Gamma}{4\pi h}$ $\quad V = W$



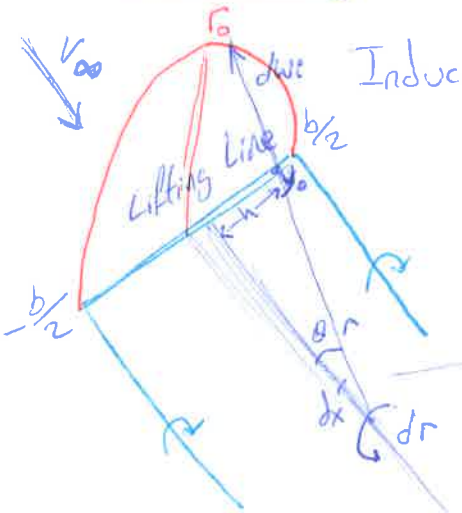
$$-W = \left(\frac{\Gamma}{4\pi\left(\frac{b}{2} + y\right)} + \frac{\Gamma}{4\pi\left(\frac{b}{2} - y\right)} \right)$$

$y = -\frac{b}{2} \to \infty$ (Physically impossible)

$y = \frac{b}{2} \to \infty$

16/2/16

Finite wing



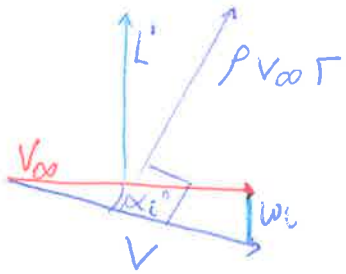
Induced velocity $(\omega(y)) = \dfrac{-\Gamma b}{4\pi\left[\left(\frac{b}{2}\right)^2 - y^2\right]}$

→ Biot-Savart's Law

$$d\omega = \frac{1}{4\pi}\frac{d\Gamma}{(y_0 - y)}$$

$$\left[d\omega = -\frac{1}{4\pi}\frac{(d\Gamma/dy)\,dy}{(y_0 - y)} \right]$$

$$\omega(y_0) = -\frac{1}{4\pi}\int_{-b/2}^{b/2}\frac{(d\Gamma/dy)\cdot dy}{(y_0 - y)}$$



$$\alpha_i = \tan^{-1}\left(-\frac{w}{V_\infty}\right) = -\frac{w}{V_\infty}$$

$$\alpha_i(y_0) = -\frac{w(y_0)}{V_\infty}$$

$$\alpha_i(y_0) = -\left(-\frac{1}{4\pi}\cdot\frac{1}{V_\infty}\right)\int_{-b/2}^{b/2}\frac{(d\Gamma/dy)dy}{(y_0 - y)}$$

Lift Coeff. for the a/f section located at (y_0)

$$\left[C_L = \alpha_0\left(\alpha_{eff}(y_0) - \alpha_{L=0}\right) \right]$$

$$\left[C_L = 2\pi\left(\alpha_{eff}(y_0) - \alpha_{L=0}\right) \right]$$

For non-twisted wing $\alpha_{L=0}$ is K.

$$L = \rho_\infty V_\infty \Gamma(y_0)$$

$$= \frac{1}{2} \rho_\infty V_\infty^2 C(y_0) \cdot c_L \quad \leftarrow \text{chord}$$

$$C_L = \frac{2\Gamma(y_0)}{V_\infty c(y_0)} \qquad C_L = 2\pi \left(\alpha_{eff}(y_0) - \alpha_{L=0}\right)$$

$$\frac{2\Gamma(y_0)}{V_\infty c(y_0)} = 2\pi \left(\alpha_{eff}(y_0) - \alpha_{L=0}\right)$$

$$\alpha_{eff}(y_0) = \frac{\Gamma(y_0)}{V_\infty \pi c(y_0)} + \alpha_{L=0}$$

$$\alpha(y_0) = \frac{\Gamma(y_0)}{\pi V_\infty c(y_0)} + \alpha_{L=0}(y_0) + \frac{1}{4\pi V_\infty} \int_{-b/2}^{b/2} \frac{(d\Gamma/dy)\, dy}{(y_0 - y)}$$

Solution $\Gamma = \Gamma(y_0)$ gives the 3 main aerodynamic characteristics of the finite wing.

1) Lift distribution is obtained from Kutta-Jukowski theorem.

$$L'(y_0) = \rho_\infty V_\infty \Gamma(y_0)$$

2) The total lift is obtained by integration of the above equation over the span

- $L = \int_{-b/2}^{b/2} L'(y)\,dy = \rho_\infty V_\infty \int_{-b/2}^{b/2} \Gamma(y_0)\,dy$

- $C_L = \dfrac{L}{q_\infty S}$ $\qquad S = (b \cdot c)$

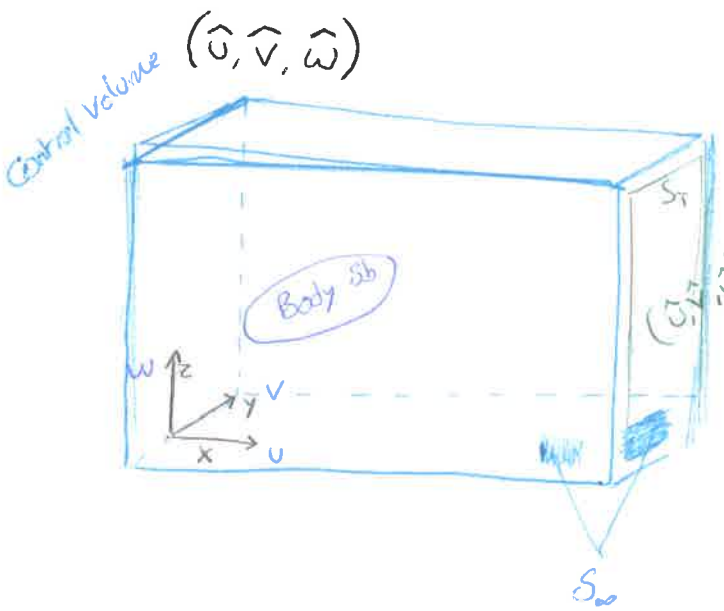
3) Induced Drag is obtained by

$$D'_i = L'_i \cdot \alpha_i$$

- $D_i = \int_{-b/2}^{b/2} \Gamma(y)\,\alpha_i(y)\,dy \cdot (\rho_\infty V_\infty)\,dy$

- $C_{D_i} = \dfrac{D_i}{q_\infty S} = \dfrac{2}{V_\infty S} \int_{-b/2}^{b/2} \Gamma(y) \cdot \alpha_i(y)\,dy$

Ex. Cosider a flow over a body of V_∞. Calculate the induce drag as a function of the velocity perturbation.



Assumptions

- Inviscid → Bernoulli equation
- Incompressible
- Steady state
- Potential flow

Perturbation non-zero

Applying the momentum integral in the x-direction: ↓

$$\frac{d(mV)}{dt} = \frac{d}{dt}\iiint_V \rho \vec{u}\cdot dV + \iint_{S_\infty + S_b} \rho\vec{u}(\vec{u}\cdot\vec{n})\,ds + \iint_{S_\infty+S_b} P\cdot\vec{n}\,ds$$

Momentum of Volum. Surface Momentum

$$\iint_{S_\infty+S_b} \rho\vec{u}\vec{u}\cdot\vec{n}\,ds = -\iint_{S_\infty+S_b} P\vec{n}\cdot ds \quad (*) \longrightarrow \iint_{S_\infty} \rho\vec{u}\vec{u}\,ds = \iint_{S_\infty+S_b} P\vec{n}\cdot ds$$

On body surface $\vec{u}\cdot\vec{n} = 0$

$$\iint_{S_b} P\vec{n}\,ds = \text{Force of body acting on fluid.}$$

x-direction → Drag

z-direction → Lift

y-direction → Yaw (Side force)

Example continuation

* Replacing in * $-\iint p\vec{n}\,ds$

$$-\iint_{S_b} p\vec{n}\,ds = -D\hat{i} - Y\hat{j} - L\hat{k}$$

$$-\iint_{S_\infty} \rho\vec{v}\vec{v}\cdot\vec{n}\,ds - \iint p\vec{n}\,ds = D\hat{i} + Y\hat{j} + L\hat{k}$$

Looking only for the drag.

$$D = -\iint \rho u\cdot\vec{v}\cdot\vec{n}\,ds - \iint p\vec{n}\hat{i}\,ds$$

Bernoulli equation (as its invisicid flow)

$$P + \frac{1}{2}V^2\rho = P_\infty + \frac{1}{2}V_\infty\rho \qquad \underbrace{(u^2+v^2+w^2)}_{} = V^2$$

$$P = P_\infty + \frac{1}{2}V_\infty\rho - \frac{1}{2}\rho V^2$$

$$D = -\iint_{S_\infty} \rho u\vec{v}\cdot\vec{n}\,ds - \left(\iint_{S_\infty} P\vec{n}\hat{i}\,ds\right) \rightarrow P_\infty + \frac{1}{2}\rho V_\infty^2 \iint_{S_\infty} \vec{n}\hat{i}\,ds$$

$$D = -\iint_{S_\infty} \rho u\cdot\vec{v}\cdot\vec{n}\,ds + \iint_{S_\infty} \frac{\rho}{2}(u^2+v^2+w^2)\,n\hat{i}\,ds$$

The perturbation flow $(\hat{u},\hat{v},\hat{w})$

$$\begin{cases} u = V_\infty + \hat{u} \\ v = \hat{v} \\ w = \hat{w} \end{cases}$$

$$D = \iint_{S_\infty} \frac{1}{2}\rho\left[(V_\infty + \hat{u})^2 + \hat{v}^2 + \hat{w}^2\right] n\,r\,ds - \rho \iint_{S_\infty} (V_\infty + \hat{u})\cdot \vec{v}\,\vec{n}\,ds$$

$$D = \iint_{S_\infty} \frac{1}{2}\rho\left(\underbrace{V_\infty^2}_{=0} + 2V_\infty \hat{u} + \hat{u}^2 + \hat{v}^2 + \hat{w}^2\right) n\,r\,ds -$$

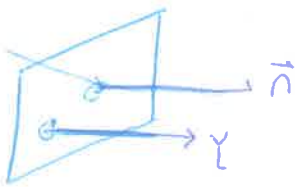
Conservation of mass

Incompressible flow

$$- \rho\left(\iint_{S_\infty} (V_\infty \vec{u}\cdot\vec{n}\,ds)_{=0} + \iint_{S_\infty} \hat{u}\,\vec{u}\,\vec{n}\,ds\right)$$

$$D = \iint_{S_\infty} \cancel{\frac{1}{2}\rho(2V_\infty \hat{u}}\,\vec{n}\,ds + \frac{1}{2}\rho \iint_{S_\infty} (\hat{u}^2 + \hat{v}^2 + \hat{w}^2)\,\hat{n}\,r\,ds - \rho \iint_{S_\infty} \hat{v}\,\vec{v}\,\vec{n}\,ds$$

$\vec{n}\,ds = \cos(0) = 1 \leftrightarrow \cos\theta$



$$D = \rho \iint_{S_T} \cancel{\frac{1}{2} V_\infty \hat{u}\,ds} +$$

$$+ \frac{1}{2}\rho \iint_{S_T} (\hat{u}^2 + \hat{v}^2 + \hat{w}^2)\,ds - \rho \iint_{S_T} \cancel{V_\infty \hat{u}\,ds}$$

$$D = \frac{1}{2}\rho \iint_{S_T} (\hat{v}^2 + \hat{w}^2 - \cancel{\hat{u}^2})\,ds \quad \rightarrow \text{No velocity in z direction}$$

$$\downarrow$$

$$D_i = \frac{1}{2}\rho \iint_{S_T} (\hat{v}^2 + \hat{w}^2)\,ds$$

Gauss - Theorem

$$\iint_S \vec{V} F \cdot ds = \oint_C F \cdot \vec{n}\, dL$$

Potential Flow

$$D_i = \frac{1}{2} \rho \int_{wake} \Gamma \cdot \vec{V}_n \, dL$$

$V_n \rightarrow$ Velocity en la dirección del vector normal al plano. S_1

SPECIAL CASE (ELLIPTICAL LIFT DISTRIBUTION)

Circulation

Circulation at origin

$$\Gamma = \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2} \longrightarrow \text{Ellipse}$$

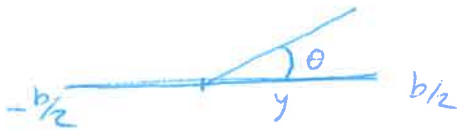
$$L'(y) = \rho_0 V_\infty \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

Downwash v.

$$w = \frac{1}{4\pi} \int_{-b/2}^{b/2} \frac{(d\Gamma/dy)}{(y_0 - y)}$$

$$\frac{d\Gamma}{dy} = \Gamma_0 \frac{1}{2} \left[1 - \left(\frac{2y}{b}\right)^2\right]^{-1/2} \cdot \frac{4 \cdot 2y}{b^2}$$

$$w = -\frac{\Gamma_0}{4\pi b^2} \int_{-b/2}^{b/2} \frac{4\, y\, dy}{\sqrt{1 - \left(\frac{2y}{b}\right)^2}\,(y_0 - y)}$$



$$w(y_0) = \frac{\Gamma_0}{\pi b^2} \int_{\pi}^{0} \frac{-\frac{b}{2}\cos\theta \cdot \frac{b}{2}\sin\theta\, d\theta}{\sqrt{1 - \left(\frac{2}{b}\cdot\frac{b}{2}\right)^2 \cos^2\theta}\,(\cos\theta_0 - \cos\theta)\frac{b}{2}}$$

$\sin\theta$

$$y = \frac{b}{2} \cdot \cos\theta$$

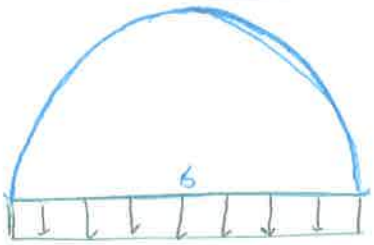
$$dy = -\frac{b}{2} \sin\theta\, d\theta$$

$$w(y_0) = \frac{+\Gamma_0}{2\pi b} \int_{\pi}^{0} \frac{\cos\theta \, d\theta}{\cos\theta_0 - \cos\theta}$$

$$\int_0^{\pi} \frac{\cos n\theta \, d\theta}{\cos\theta - \cos\theta_0} = \pi \frac{\sin(n\theta_0)}{\sin\theta}$$

$$w(y_0) = \frac{-\Gamma_0}{2\pi b} \int_0^{\pi} \frac{\cos\theta \, d\theta}{\cos\theta - \cos\theta_0}$$

$$\boxed{w(y_0) = \frac{-\Gamma_0}{2b}}$$ Downwash is K along b.



$$\alpha_i = -\frac{w(y_0)}{V_\infty} = \frac{S \cdot C_L}{\pi \cdot b^2} = \frac{C_L}{\pi AR}$$

$$AR = \frac{b^2}{S} = \frac{b^2}{b\bar{c}} = \frac{b}{\bar{c}}$$ ↓ Avg Chord

$$\boxed{\alpha_i = \frac{\Gamma_0}{2bV_\infty}}$$ → The induced drag is K along the span

$$L(y) = \frac{\rho_\infty V_\infty \Gamma_0 \, b\pi}{4}$$

$$\boxed{C_{Di} = \frac{\alpha_i \Gamma_0 \, b\pi}{V_\infty S \, 2}} = \frac{C_L^2}{\pi AR} = \frac{C_L^2 \cdot c}{\pi \cdot b}$$

19/2/16

Elliptical wing

$$C_{Di} = \frac{C_L^2}{\pi AR e}$$

Span efficiency factor

$$e = \frac{1}{(1+\delta)}$$

↳ Longer wing = Bigger AR → Reduces C_{Di}

With tappered wings, stall occurs first in the w/ tip. while in the rest of the wing, there is still lift.

Ex. 1

Tip: ① Calculate coefficient of Fourier Series in function of α.

$$\alpha(\theta) = \sum_{n=1}^{N} A_n \sin(\theta n)\left(1 + \frac{n\pi}{2AR\sin\theta}\right)$$

$$\alpha = A_1 \sin\theta\left(1 + \frac{\pi}{2 \cdot AR \cdot \sin\theta}\right) + \cancel{A_2 \sin(2\theta)\left(1 + \frac{2\pi}{2 \cdot AR \cdot \sin(\theta)}\right)} + \quad (0 \text{ (even n°)})$$

$$+ A_3 \sin(\theta 3)\left(1 + \frac{3\pi}{2AR\sin(\theta)}\right) + A_5 \ldots + A_7 \ldots$$

(The more terms, the more accurate)

Is a rectangular wing, so can be calculated just the half of it.

Unknowns:

A_1 ; A_3 ; A_5 ; A_7 ; θ

Sustituir en la ecuación α_θ cada valor

Solo cambia el n para α_{θ_1}

$\alpha_{\theta_1} \Rightarrow \theta_1 = \frac{\pi}{8}$ → $\alpha_{\theta_1} = 0'644\,A_1 + 2'82\,A_3 + 4.0841A_5 + 2'21A_7$

$\alpha_{\theta_2} \Rightarrow \theta_2 = \frac{\pi}{4}$ → $\alpha_{\theta_2} = 0'9689\,A_1 + 1'4925A_3 + 2'0161\,A_5 - \ldots$

$\alpha_{\theta_3} \rightarrow \theta_3 = \frac{3\pi}{8}$ →

$\alpha_{\theta_4} \rightarrow \theta_4 = \frac{\pi}{2}$

4 Unknowns for 4 equations

$A_1 = 0'9174\alpha$ $\quad A_5 = 0'0218\alpha$

$A_3 = 0'1104\alpha$ $\quad A_7 = 0'0038\alpha$

$C_L = \frac{A_1 \pi \cdot AR}{h}$ → We are calculating half of the wing.

→ $C_{L_1} = 17'29\alpha / 2 = 8'64\alpha$

$C_{L_1} = \frac{A_1 \pi \cdot AR}{2}$

For elliptical wing:

$C_{D_i} = \frac{C_L^2 (1+\delta)}{\pi AR} = \frac{(8'64\alpha)^2}{\pi \cdot \frac{AR}{2}} = 7'98\alpha^2 (1+\delta)$

$\delta = \sum_2^N n \left(\frac{A_n}{A_1}\right)^2$ → $\frac{3A_3^2 + 5A_5^2 + 7A_7^2}{A_1^2}$ $\quad \delta = 0'0464$

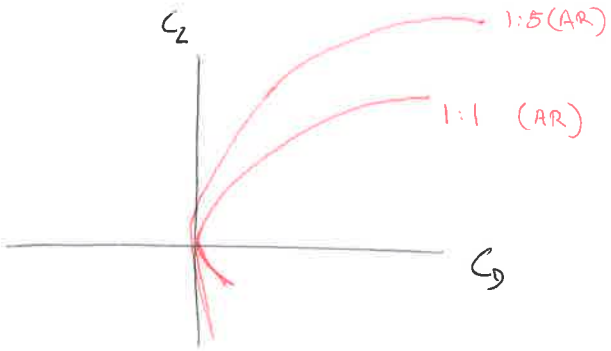
$C_{D_i} = 8'29\alpha^2$

EFFECT OF THE AR IN THE WING

$$AR = \frac{b}{c}$$

For general wing:

$$C_D = C_d + \frac{C_L^2}{\pi AR\, e}$$

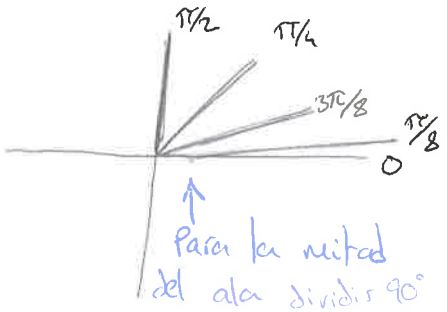


Correction for the fourier coeff

$$\frac{dC_L}{d\alpha} = a_e = \frac{a_0}{1 + a_0/\pi AR} \quad \text{(Elliptical w)}$$

$$a_{GW} = \frac{a_0}{1 + \left(\frac{a_0}{\pi AR}\right)(1+\tau)} \qquad 0.05 < \tau < 0.25 \leftarrow \text{General Wing.}$$

$$\alpha_i = \frac{\omega_0}{V_\infty}$$



Para la mitad del ala dividis 90° en 4 ángulos (el total sería 180°)

Induced Velocity

$$\omega(\theta_0) = V_\infty \sum_1^N \frac{n \cdot A_n \cdot \sin(n\theta_0)}{\sin(\theta_0)}$$

$\alpha = 5° = 0'087$ rad

$V_\infty = 41'67$ m/s

$\theta = \pi/8$

$$\omega\left(\frac{\pi}{8}\right) = 41'67 \left[0'9174\alpha \cdot \frac{\sin(\theta_0)}{\sin(\theta_0)} + \ldots + A_n \right] \rightarrow \alpha_i$$

$\omega(\pi/8) = 41'67 \times \overbrace{0'11?}^{\alpha_i}$ (→ Might be wrong.)

$\omega\left(\frac{\pi}{8}\right) = 4'6$ m/s

$\omega(\pi/4) = ?$

$\omega(3\pi/8) = ?$

$\omega\left(\frac{\pi}{2}\right) = ?$

$$\alpha_{eff} = \alpha - \alpha_{i(\theta)}$$

For elliptical wing ~~to~~ α_i is K

HOMEWORK 7

(2) NACA 23012 $C_L = a(\alpha - \alpha_{L=0})$ $C_L = 0'894$

a = slope $0'108$/degree → Slope in C_L vs α

$\alpha_{L=0} = -1'8°$

L?

C_{D_i}?

$\alpha = 7°$

$AR = 8$

T. prat. $= 0'8$

$\delta = \tau = 0'054$

(3)

$$a = \frac{a_0}{1 + \frac{a_0}{\pi AR}(1+\tau)} = 4'39 \text{ rad}^{-1}$$ C_L

$C_L = 0'3415$

$$D_i = \frac{1}{2} \cdot \rho_0 V_\infty^2 \cdot \delta \cdot C_{D_i}$$

$$C_{D_i} = \frac{C_L^2}{\pi AR}$$ $V = 2\pi\ m^2$

(4) $e = 0'64$

D_i?

(6)

$AR = 5$

$c = 1'2\,m$

$b = 6\,m$

$\alpha = 4°$

(Parasitic D) $C_{d_0} = 0'002$

L?

C_{D_i}?

$V_\infty = 200\,m/s$

$\rho_\infty = 0'98\,Kg/m^3$

D_i?

V_i?

α_i?

D?

$AR = 9$

$$\alpha(\theta) = \sum_{n=1}^{N} A_n \sin(\theta n)\left(1 + \frac{n\pi}{2AR\sin\theta}\right)$$

3'4688

$$A_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{dz}{dx}\cos(n)\,d\theta$$

$$\theta = \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{2}$$

$\alpha_{\theta_1} = 0'6968 A_1 + 3'1992 A_3 + 4'716 A_5$

$\alpha_{\theta_2} =$

$\alpha_{\theta_3} = 1'02 A_1 + 1'6498 A_3 + (-1'033) A_5$

Sistema de ecuaciones

$A_1 = 0'05475$	$A_3 = 0'0085$
$A_5 = 0'0025$	$A_7 = -0'002187$

(2) $C_L = A_n \cdot \pi \cdot AR$ $\qquad C_L = 0'43$

$$\delta = \sum_{2}^{N} n\left(\frac{A_n}{1}\right)^2 \qquad \delta = 3\left(\frac{0'0085}{A_1}\right)^2 + \ldots = 0'0723 + 0'010425 + 0'011169$$

$= 0'0938$

$$C_{D_i} = \frac{C_L^2}{\pi \frac{AR}{2}}(1+\delta) \qquad C_D = C_{D_i} + C_{D_0}$$

$D_T =$

$$D_i = \frac{1}{2}\cdot\rho_\infty V_\infty^2 \cdot S \cdot C_{D_i} \quad (N)$$

Potencia (P) $\longrightarrow P = D_i \cdot V$

(6) For AR = 9 — Cambia el C_D en función del nuevo AR.

$$C_{D_2} = C_{D_1} - \frac{C_L^2}{\pi e}\left(\frac{1}{AR_1} - \frac{1}{AR_2}\right)$$

5"

Induced velocity.

$$w_{(\theta_0)} \; V_\infty \sum_{1}^{N} n \, A_n \, \frac{\sin(n\theta_0)}{\sin(\theta_0)}$$

Subsonic - Airfoil - incompressible flow

3 Mar 16

Ex 2.

At a given point on a surface of an a/f, the pressure coefficient is 0'3. At very low speed. If the $M = 0'6$ calculate C_p at this point.

Air at $M=0'6$ is compressible.
(Above $M = 0'3$ is compressible)

$$C_{p_c} = \frac{C_{p_i}}{\sqrt{1 - M_\infty^2}}$$

$C_{p_i} = 0'3$

$M = 0'6$

$C_{p_c} = -0'5$

Suppose that the a/f is thin and symmetric. Determine C_L for $M = 0'6$, $0'7$, $0'8$ and $0'9$

$$C_L = \frac{C_{L_i}}{\sqrt{1 - M_\infty^2}}$$

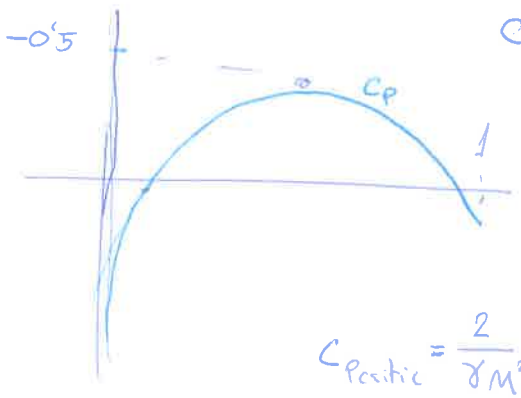
$$C_L = \frac{2\pi\alpha}{\sqrt{1 - M_\infty^2}}$$

M	C_L
.6	~~0'547~~ $7'85\alpha$
.7	$8'8\alpha$
.8	$10'46\alpha$
.9	$14'40\alpha$

If it wasn't symmetric → calculate Fourier series coeff. (A_n.)

Ex 3

Determine M_c (critical) for NACA 0012 at $\alpha = 0°$



$C_{p\,min} = -0'43$

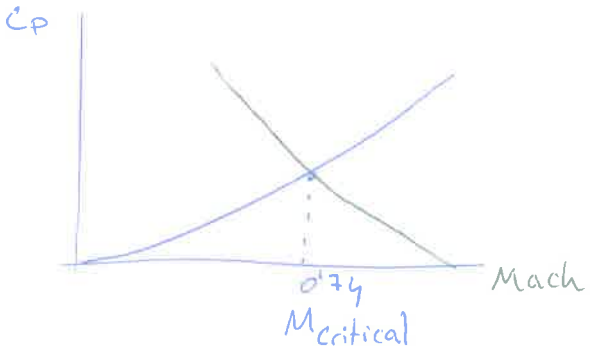
$Re = 365 \times 10^6$

$$C_{p\,critic} = \frac{2}{\gamma M^2_{critic}} \left[\left(\frac{\gamma - 1}{\gamma + 1} M_c^2 + \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} - 1 \right]$$

$\gamma = 1'4$

M	0'4	5	6	7	8	9	1
$C_{p\,crit.}$	-3'66	-2'13	-1'29	-0'779	-0'435	-0'188	0

$$C_{p\,critic} = \frac{(C_{p_c})}{\sqrt{1 - M_\infty^2}} = \frac{-0'43}{\sqrt{1 - M_\infty^2}}$$



Aerodynamics for a/c Lift and Drag

Produced by wing-Body Combination

HTP ; Canards

$$C_D = C_{D_0} + \frac{C_L}{\pi AR e}$$

Parasitic D. Coeff. (Not only P and skin drag also friction an P Drag from the Tail, fuselage etc...)

Total D.

$$C_D = C_{D_e} + \frac{C_L}{\pi AR e}$$

$$C_{D_e} = C_{D_0} + \Gamma C_L^2$$

Empirical constant

Parasite C_D at $L=0$ drag coeff.

$L = f(\alpha)$ The C_{D_e} is also function of C_L

Drag coeff. for the entire a/c (Considering HTP, engine, etc...)

$$C_D = C_{D_0} + \Gamma C_L^2 + \frac{C_L}{\pi AR e}$$

$$\Rightarrow C_D = C_{D_0} + C_L^2 \left(\Gamma + \frac{1}{\pi e AR} \right)$$

e includes the effect of variations of parasite Drag with Lift.

~~$e = 1'78 (1 - 0'04$~~

Drag Polar

$$C_D = C_{D_0} + \frac{C_L^2}{\pi e AR}$$

↳ Represents C_L vs C_D

$C_L \rightarrow$ of the full a/c

$AR \rightarrow$ of the wing.

Oswald Factor

$$e = 1'78 \left(1 - 0'045\ AR^{0'68} \right) - 0'64$$

4 Mar 16

Seversky P-35.

w. Area = 20'1 m^2

w. span (b) = (10'8 m)

$AR = 5'7176$

At 18° $\longrightarrow C_{D,T} = 0'0275$

At $\alpha = ?$

↳ $C_L = 0'15$

$C_{D_0} = ?$

$e = 0'878$

$$C_{D,T} = C_{D_0} + \frac{C_L^2}{\pi e AR}$$

$$AR = \frac{b^2}{S}$$

$$C_{D,T} = 0'0275 + \frac{0'15^2}{0'878 \cdot AR}$$

$$C_{D_0} = 0'026$$

$$\left(\frac{C_L}{C_D}\right)_{max} = 11'8$$

(5) Obtain (and explain how to) the wing parameters of an a/c flying at sea level cndt. $V = 500\ ^{km}/_{h}$ and a wing load $^{W}/_{S} = 7\ldots\ ^{N}/_{m^2}$

Wing Data

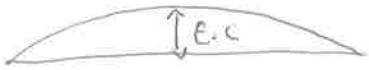
- TR :=
- NACA
- $\alpha_{L=0}$
-

(1)

$$z = c\varepsilon\left[1 + \left(\frac{x}{c}\right)^2\right]$$

Calculate:

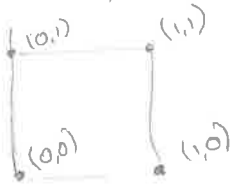
- Ideal α (Explain)
- $\alpha_{L=0}$
- C_L ; C_m
- x_{cp}



(2) Are similar flows for equal μ and c (s.o sound)? Both proportional to $\sqrt{T}$.

Airfoil 1	Airfoil 2
$c = x$	$c = 2x$
$\rho = 1'23$	$\rho = 1'74$
$V = 100$	$V = 200$
$T = 200K$?	$T = 800k$

(3) Circulation w/ velocity vectors $u = xy^2$ $\quad v = xy$



(4) Obtain the geometric α

Data: • V • Wing Area • A_0 • Weight

• $\alpha_{L=0}$ • Wing Span (b)

$\theta° = \frac{\text{Rad} \cdot 180°}{\pi}$? $\text{Rad} = \frac{\theta° \cdot \pi}{180°}$

$P_a + \frac{1}{2} \rho V^2$

Simmetric.

$\alpha = 1'5°$

$\alpha = 1'5° = 0'0261 \text{ Rad}$

$C_l = 0'164$

Γ = Circulation $\Gamma = V_\infty c \alpha \pi$

Elemental vortex strength $\gamma(\xi) d\xi$

$q_\infty = \frac{1}{2} \rho V^2 \, (Pa)$

ξ Distance from LE.

$C_{M,LE} = -\frac{C_L}{4} = -0'041$

$dL = \rho_\infty V_\infty d\Gamma$ $(L' = \text{Lift/unit span}) = \pi \alpha c \rho_\infty V_\infty^2$

Increment of L created M about the LE $\rightarrow dM = -\xi (dL)$

Moment about LE

$$M'_{LE} = -\int_0^c \xi (dL) = -\rho_\infty V_\infty \int_0^c \xi \gamma(\xi) d\xi$$

$$M'_{LE} = -q_\infty c^2 \frac{\pi \alpha}{2}$$

Lift C_l → in rad.

$C_l = 2\pi\alpha$

$C_l = \frac{L'}{q_\infty S}$

Coefficient $\Rightarrow$ $(C_m)_{LE} = \frac{M'_{LE}}{q_\infty S_c}$ $S = c(1)$

$C_{M,LE} = \frac{M'_{LE}}{q_\infty c^2} = \frac{-\pi \cdot \alpha}{2} \rightarrow \pi\alpha = \frac{C_l}{2}$

$C_{M,LE} = \frac{-C_L}{4}$

For simmetric a/f. center of pressure and ac. are both the quarter-chord point. $C_{M,c/4}$

$C_{M,c/4} = C_{M,LE} + \frac{C_L}{4} = 0$