UNIT 2

Exercises of UNIT 2- Part I: Temporal Characteristics

2.1 A two-level random binary process is defined by

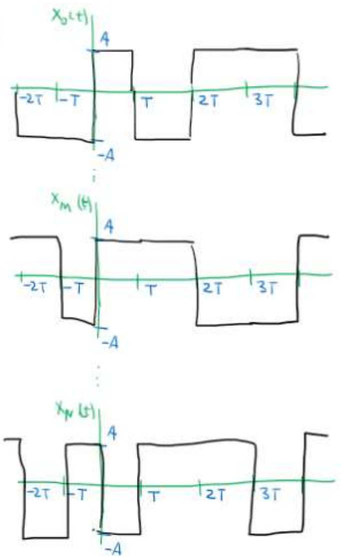
$X(t) = A$ or $-A, (n-1)T < t < nT$ where the levels A and $-A$ occur with equal probability, T is a positive constant, and $n = 0, \pm1, \pm2, \ldots$

(a) Sketch a typical sample function

(b) Classify the process

$X(t) = A$ or $-A$, $(n-1)T < t < nT$

a)



b) NON-INFINITESIMAL DISTANCE BETWEEN AMPLITUDE VALUES => DISCRETE

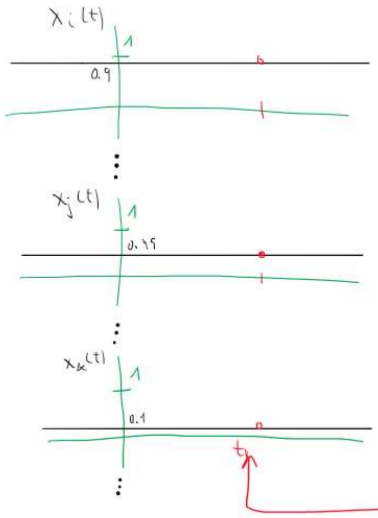
NOT POSSIBLE TO PREDICT FUTURE VALUES => NON-DETERMINISTIC

2.2 A random process is defined by $X(t) = A$, where A is a continuous random variable uniformly distributed on $[0,1]$.

(a) Determine the form of the sample functions

(b) Classify the process

a)



b) INFINITE VALUES WITH NO GAPS $\Rightarrow$ CONTINUOUS

ONCE YOU KNOW $X(t=t_0)$ YOU KNOW THAT $X(t) = X(t=t_0), \forall t$ $\Rightarrow$ DETERMINISTIC

c) EXTRA QUESTION

$E[X(t)]$? $\rightarrow$ $E[X(t)] = E[A] = 0.5$

2.3 Given the random process $X(t) = A\cos(\omega_0 t+\Theta)$ where A and ω_0 are constants and Θ is a uniformly distributed random variable on the interval $[0,2\pi]$. Find

(a) The mean

(b) The autocorrelation

(c) The time average

(d) The time autocorrelation

$x(t) = A\cos(w_0 t + \Theta)$

A, w_0 are constant

$\Theta \sim U(0, 2\pi)$

a) Mean $\rightarrow$ $E[x(t)] = E[\underbrace{A}_{\text{NOT RANDOM}} \cos(w_0 t + \Theta)] = A\, E[\overbrace{\cos(\underbrace{w_0 t}_{\text{NOT RANDOM}} + \Theta)}^{g(\Theta)}] = A \cdot E[g(\Theta)]$

$$= A \cdot \int_{-\infty}^{\infty} g(\sigma) f_\Theta(\sigma)\, d\sigma = A \int_0^{2\pi} \cos(w_0 t + \sigma) \frac{1}{2\pi}\, d\sigma = \frac{A}{2\pi} \sin w_0 t + \sigma \Big|_0^{2\pi} = 0$$

$$f_\Theta(\sigma) = \frac{1}{2\pi}, \quad 0 \le \sigma \le 2\pi$$

Also $\rightarrow$ $E[A\cos(w_0 t + \Theta)] = 0$ IF $\begin{cases} A, w_0 \text{ are constant} \\ \Theta \sim U(\sigma_1, \sigma_2) \\ \sigma_2 - \sigma_1 = 2\pi \cdot k, \; k \in \{1, 2, 3 \dots\} \end{cases}$

b) Autocorrelation $\rightarrow$ $R_{XX}(t_1, t_2) = R_{XX}(t, t+\tau) = E[X(t) X(t+\tau)]$

$$= A^2 E\left[\cos(\underbrace{w_0 t + \Theta}_{\alpha}) \cos(\underbrace{w_0(t+\tau) + \Theta}_{\beta})\right]$$

$$\cos\alpha \cdot \cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

$$= \frac{A^2}{2}\left[\underbrace{E[\cos(w_0(2t+\tau) + 2\Theta)]}_{0 \text{ since } 2\Theta \sim U(0, 4\pi)} + \underbrace{E[\overbrace{\underbrace{\cos(-w_0\tau)}_{\cos w_0 \tau}}^{\text{NOT RANDOM}}]}_{\cos w_0 \tau}\right]$$

$$= \frac{A^2}{2} \cos w_0 \tau$$

c) Time average $\rightarrow$ $A[x(t)] = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)\, dt = \lim_{T \to \infty} \frac{1}{2T} \underbrace{\int_{-T}^{T} \cos(w_0 t + \sigma)\, dt}_{\substack{\alpha \\ \text{BOUNDED VALUE}}}$

$$= \lim_{T \to \infty} \frac{\alpha}{2T} = 0$$

d) Time autocorrelation $\Rightarrow$ $\mathcal{A}[x(t)x(t+\tau)] = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \underbrace{x(t)x(t+\tau)}_{\frac{A^2}{2}[\cos(\omega_0(2t+\tau)+2\theta)+\cos\omega_0\tau]} dt$

$\mathbb{R}_{XX}(\tau)$

$= \lim_{T\to\infty} \frac{1}{2T}\frac{A^2}{2}\underbrace{\int_{-T}^{T}\cos(\omega_0(2t+\tau)+2\theta)dt}_{\substack{\alpha \\ \text{BOUNDED VALUE}}} + \lim_{T\to\infty}\frac{1}{2T}\frac{A^2}{2}\cos\omega_0\tau\underbrace{\int_{-T}^{T}dt}_{2T}$

$= \frac{A^2}{2}\cos\omega_0\tau$

e) SOME CONCLUSIONS

- $X(t)$ is w.s.s. since $\begin{cases} E[X(t)] = 0 = \text{constant} & 1^{ST} \text{ ORDER} \\ R_{XX}(t,t+\tau) = \frac{A^2}{2}\cos\omega_0\tau = R_{XX}(\tau) & 2^{ND} \text{ order} \end{cases}$

- $E[X(t)] = \mathcal{A}[x(t)] \Rightarrow$ ERGODIC IN THE MEAN

- $R_{XX}(\tau) = \mathbb{R}_{XX}(\tau) \Rightarrow$ ERGODIC IN THE AUTOCORRELATION

2.4 Given the random process

$X(t) = Asin(\omega_0 t + \Theta)$ where A and ω_0 are constants and Θ is a uniformly distributed random variable on the interval $[-\pi,\pi]$. Define a new random variable $Y(t) = X^2(t)$.

(a) Find the autocorrelation function of $Y(t)$

(b) Find the cross-correlation function of $X(t)$ and $Y(t)$

(c) Are $X(t)$ and $Y(t)$ wide-sense stationary?

(d) Are $X(t)$ and $Y(t)$ jointly wide-sense stationary?

a) $R_{YY}(t, t+\tau) = E[Y(t)Y(t+\tau)] = E\left[\frac{A^2}{2}[1+\cos(2\omega_0 t + 2\Theta)]\,\frac{A^2}{2}[1+\cos(2\omega_0(t+\tau)+2\Theta)]\right]$

$$= \frac{A^4}{4}\Big[1 + \underbrace{\cancel{E[\cos(\omega_0 t + 2\Theta)]}}_{2\Theta \sim U(-2\pi, 2\pi)} \xrightarrow{} 0 + \underbrace{\cancel{E[\cos(2\omega_0(t+\tau)+2\Theta)]}}_{2\Theta \sim U(-2\pi,2\pi)} \xrightarrow{} 0 + E[\cos(\underbrace{2\omega_0 t + 2\Theta}_{\alpha})\cos(\underbrace{2\omega_0(t+\tau)+2\Theta}_{\beta})]\Big]$$

$$= \frac{A^4}{4}\Big[1 + \frac{1}{2}\underbrace{\cancel{E[\cos(2\omega_0(2t+\tau)+4\Theta)]}}_{4\Theta \sim U(-4\pi,4\pi)} \xrightarrow{} 0 + \frac{1}{2}\underbrace{E[\cos(-2\omega_0\tau)]}_{\cos 2\omega_0\tau}\Big]$$

$$= \frac{A^4}{4}\left[1 + \frac{\cos 2\omega_0\tau}{2}\right]$$

b) $R_{XY}(t, t+\tau) = E[X(t)Y(t+\tau)] = E\left[A\cos(\omega_0 t + \Theta)\,\frac{A^2}{2}[1+\cos(2\omega_0(t+\tau)+2\Theta)]\right]$

$$= \frac{A^3}{2}\Big[\cancel{E[\cos(\omega_0 t + \Theta)]} \xrightarrow{} 0 + E[\cos(\underbrace{\omega_0 t + \Theta}_{\alpha})\cos(\underbrace{2\omega_0(t+\tau)+2\Theta}_{\beta})]\Big]$$

$$= \frac{A^3}{2}\,\frac{1}{2}\Big[\cancel{E[\cos(\omega_0(3t+\tau)+3\Theta)]} \xrightarrow{} 0 + \cancel{E[\cos(-\omega_0(t+2\tau)-\Theta)]} \xrightarrow{} 0\Big]$$

$= 0 \implies X, Y$ ORTHOGONAL

c) X(t) w.s.s.?
Y(t) w.s.s.?

$X(t) \rightarrow E[X(t)] = \cancel{E[A\cos(\omega_0 t + \Theta)]} \xrightarrow{} 0 = 0 \implies$ 1ST ORDER

$R_{XX}(t, t+\tau) = E[A\cos(\omega_0 t + \Theta)\,A\cos(\omega_0(t+\tau)+\Theta)]$

$= \frac{A^2}{2}\left[\cancel{E[\cos(\omega_0 t + \Theta)]} \xrightarrow{} 0 + \underbrace{E[\cos(-\omega_0\tau)]}_{\cos\omega_0\tau}\right]$

$R_{XX}(\tau)$

$= \frac{A^2}{2}\cos\omega_0\tau \implies$ 2ND ORDER

w.s.s.

$Y(t) \rightarrow E[Y(t)] = E[X^2(t)] = R_{XX}(0) = \frac{A^2}{2} \implies$ 1ST ORDER

$R_{YY}(t, t+\tau) = R_{YY}(\tau) \implies$ 2ND ORDER

w.s.s.

d) X(t) and Y(t) are w.s.s. and $R_{XY}(t, t+\tau) = 0$.

Since $R_{XY}(t, t+\tau) = 0$, the function can be a function of τ as well as a function of $(t_1 = t, t_2 = t+\tau)$. However, if it is constant changing the values of the couple

(t_1, t_2) does not affect the outcome of R_{XY}. Thus, we can perfectly state that X(t) and Y(t) are j.w.s.s.

2.5 Given the random process

$Y(t) = X(t)cos(\omega_0 t + \Theta)$ where $X(t)$ is a wide-sense stationary random process that amplitude-modulates a carrier of constant angular frequency ω_0 with a random phase Θ independent of $X(t)$ and uniformly distributed on $[-\pi, \pi]$

(a) Find $E[Y(t)]$

(b) Find the autocorrelation of $Y(t)$

(c) Is $Y(t)$ wide-sense stationary?

$$Y(t) = X(t)\cos(w_0 t + \Theta)$$

- $X(t)$ w.s.s.
- w_0 constant
- $\Theta \sim U(-\pi, \pi)$
- $X(t)$, Θ are INDEPENDENT

a) $E[Y(t)] = E[X(t)\cos(w_0 t + \Theta)]$ IND.

$$= E[X(t)]\,\underbrace{\cancel{E[\cos(w_0 t + \Theta)]}}_{0} \qquad \Theta \sim U(-\pi,\pi)$$

$$= 0$$

b) $R_{YY}(t, t+\tau) = E[Y(t)Y(t+\tau)] = E[X(t)\cos(w_0 t + \Theta)\, X(t+\tau)\cos(w_0(t+\tau) + \Theta)]$

$$= E[X(t)X(t+\tau)\cos(w_0 t + \Theta)\cos(w_0(t+\tau) + \Theta)]$$

IND.

$$= \underbrace{E[X(t)X(t+\tau)]}_{R_{XX}(\tau)}\, E[\cos(w_0 t + \Theta)\cos(w_0(t+\tau) + \Theta)]$$

$$= R_{XX}(\tau) \cdot \frac{1}{2}\Big[\underbrace{\cancel{E[\cos(w_0(2t+\tau) + 2\Theta)]}}_{0} + \underbrace{E[\cos(-w_0\tau)]}_{\cos w_0\tau}\Big] \qquad 2\Theta \sim U(-2\pi, 2\pi)$$

$$= R_{XX}(\tau)\cos w_0\tau = R_{YY}(\tau)$$

c) Y(t) is w.s.s. since E[Y(t)] is constant and its autocorrelation is

a fuction of τ.

2.6 The random processes $X(t)$ and $Y(t)$ are statistically independent, have zero mean and have autocorrelation functions

$$R_{XX}(\tau) = e^{-|\tau|}$$
$$R_{YY}(\tau) = cos(2\pi\tau)$$

(a) Find the autocorrelation function of $W_1(t) = X(t) + Y(t)$

(b) Find the autocorrelation function of $W_2(t) = X(t) - Y(t)$

(c) Find the cross-correlation function of $W_1(t)$ and $W_2(t)$

$R_{XX}(\tau) = e^{-|\tau|}$ $\quad \bar{X} = 0$

$R_{YY}(\tau) = \cos 2\pi\tau$ $\quad \bar{Y} = 0$

X, Y are INDEPENDENT

INDEPENDENT + $\bar{X} = \bar{Y} = 0$ $\Rightarrow$ ORTHOGONAL $\Rightarrow$ $R_{XY}(t, t+\tau) = 0$, $R_{YX}(t, t+\tau) = 0$

a) $W_1(t) = X(t) + Y(t)$

$$R_{W_1W_1}(t, t+\tau) = E[W_1(t)W_1(t+\tau)] = E[(X(t) + Y(t)) \cdot (X(t+\tau) + Y(t+\tau))]$$

$$= \underbrace{E[X(t)X(t+\tau)]}_{R_{XX}(\tau)} + \cancel{E[Y(t)X(t+\tau)]}^{0} + \cancel{E[X(t)Y(t+\tau)]}^{0} + \underbrace{E[Y(t)Y(t+\tau)]}_{R_{YY}(\tau)}$$

$$= R_{XX}(\tau) + R_{YY}(\tau) = e^{-|\tau|} + \cos 2\pi\tau = R_{W_1W_1}(\tau)$$

b) $W_2(t) = X(t) - Y(t)$

$$R_{W_2W_2}(t, t+\tau) = E[W_2(t) W_2(t+\tau)] = E[(X(t) - Y(t))(X(t+\tau) - Y(t+\tau))]$$

$$= E[X(t) X(t+\tau)] - \cancel{E[Y(t) X(t+\tau)]}^{\,0} - \cancel{E[X(t) Y(t+\tau)]}^{\,0} + E[Y(t) Y(t+\tau)]$$

$$= R_{XX}(\tau) + R_{YY}(\tau) = e^{-|\tau|} + \cos 2\pi\tau = R_{W_2W_2}(\tau) = R_{W_1W_1}(\tau)$$

c) $$R_{W_1W_2}(t, t+\tau) = E[W_1(t) W_2(t+\tau)] = E[(X(t) + Y(t))(X(t+\tau) - Y(t+\tau))]$$

$$= R_{XX}(\tau) - \cancel{R_{XY}(\tau)}^{\,0} + \cancel{R_{YX}(\tau)}^{\,0} - R_{YY}(\tau)$$

$$= R_{XX}(\tau) - R_{YY}(\tau) = e^{-|\tau|} - \cos 2\pi\tau = R_{W_1W_2}(\tau)$$

2.7 Given two w.s.s. random processes $X(t)$ and $Y(t)$. Find expressions for the autocorrelation function of $W(t) = X(t) + Y(t)$ if:

(a) $X(t)$ and $Y(t)$ are correlated

(b) $X(t)$ and $Y(t)$ are uncorrelated

(c) $X(t)$ and $Y(t)$ are uncorrelated with zero mean

Try to use means over cross-correlations if possible.

w.s.s. $\Rightarrow$ $E[X(t)] = \bar{X}$, $R_{XX}(t, t+\tau) = R_{XX}(\tau)$

$E[Y(t)] = \bar{Y}$, $R_{YY}(t, t+\tau) = R_{YY}(\tau)$

$W(t) = X(t) + Y(t)$

a) $X(t)$, $Y(t)$ are correlated

$$R_{WW}(t,t+\tau) = E[W(t)W(t+\tau)] = E[(X(t)+Y(t))\cdot(X(t+\tau)+Y(t+\tau))]$$
$$= R_{XX}(\tau) + R_{XY}(\tau) + R_{YX}(\tau) + R_{YY}(\tau)$$

b) uncorrelated $\Rightarrow$ $R_{XY}(\tau) = E[X(t)Y(t+\tau)] = E[X(t)]\,E[Y(t+\tau)] = \bar{X}\cdot\bar{Y}$
$R_{YX}(\tau) = \bar{Y}\bar{X}$

$$R_{WW}(\tau) = R_{XX}(\tau) + R_{YY}(\tau) + 2\bar{X}\cdot\bar{Y}$$

c) uncorrelated with zero mean $\Rightarrow$ ORTHOGONAL $\Rightarrow$ $R_{XY}(\tau) = R_{YX}(\tau) = 0$

$$R_{WW}(\tau) = R_{XX}(\tau) + R_{YY}(\tau)$$

2.8 Consider random processes

$$Y_1(t) = X(t)cos(\omega_0 t)$$
$$Y_2(t) = Y(t)cos(\omega_0 t + 2\Theta)$$

where $X(t)$ and $Y(t)$ are jointly wide-sense stationary processes.

(a) If $\Theta \sim U(\theta_0, \theta_1)$ and independent of $X(t)$ and $Y(t)$, are there any conditions on Θ that will make $Y_1(t)$ and $Y_2(t)$ orthogonal?

$Y_1(t) = X(t)\cos w_0 t$

$Y_2(t) = Y(t)\cos(w_0 t + 2\Theta)$

$Y_1(t), Y_2(t)$ are j.w.s.s. $\Rightarrow$ $E[X(t)] = \bar{X}$ $\quad R_{XX}(\tau)$

$E[Y(t)] = \bar{Y}$ $\quad R_{YY}(\tau)$

$R_{XY}(\tau)$

$\Theta \sim U(\sigma_0, \sigma_1)$

Θ, $X(t)$ are independent

Θ, $Y(t)$ are independent

Looking for σ_0, σ_1 / $R_{Y_1Y_2}(t, t+\tau) = 0$

$$R_{Y_1Y_2}(t, t+\tau) = E[Y_1(t)Y_2(t+\tau)] = E[X(t)\cos w_0 t \cdot Y(t+\tau)\cos(w_0(t+\tau)+2\Theta)]$$

$$= E[X(t)\,Y(t+\tau)\,\underbrace{\cos w_0 t}_{\text{NOT RANDOM}} \cdot \cos(w_0(t+\tau)+2\Theta)]$$

IND.

$$= E[X(t)Y(t+\tau)]\,E[\cos(w_0(t+\tau)+2\Theta)] \cdot \cos w_0 t$$

$$= \cos w_0 t \cdot R_{XY}(t, t+\tau) \cdot E[\cos(w_0(t+\tau)+2\Theta)]$$

$R_{Y_1Y_2}(t, t+\tau) = 0$ if $E[\cos(w_0(t+\tau)+2\Theta)] = 0$

$\Theta' = 2\Theta \sim U(2\sigma_0, 2\sigma_1)$

$E[\cos(w_0(t+\tau)+2\Theta)] = 0$ if $2\sigma_1 - 2\sigma_0 = k \cdot 2\pi$ $\quad k \in \{1, 2, \dots\}$

$\sigma_1 - \sigma_0 = k \cdot 2\pi$

2.9 Consider the random processes

$$X(t) = A cos(\omega_0 t + \Theta)$$

$$Y(t) = B sin(\omega_0 t + \Theta)$$

where A, B, and ω_0 are constants while Θ is a random variable uniform on $[0,2\pi]$. $X(t)$ and $Y(t)$ are zero-mean, wide-sense stationary with autocorrelation functions:

$$R_{XX}(\tau) = (A^2/2)cos(\omega_0\tau)$$

$$R_{YY}(\tau) = (B^2/2)cos(\omega_0\tau)$$ Are

$X(t)$ and $Y(t)$ jointly wide-sense stationary?

$X(t) = A\cos(w_0 t + \Theta) \rightarrow$ w.s.s.

$Y(t) = B\sin(w_0 t + \Theta) \rightarrow$ w.s.s.

$\Theta \sim U(0, 2\pi)$

$A, B \rightarrow$ constant

$X(t), Y(t)$ j.w.s.s ? $\Rightarrow$ $X(t)$ w.s.s. ✓

$Y(t)$ w.s.s. ✓

$R_{xy}(t, t+\tau) = R_{xy}(\tau)$

$R_{xy}(t, t+\tau) = E[X(t)Y(t+\tau)]$

$= E[A\cos(w_0 t + \Theta) B\cos(w_0(t+\tau) + \Theta)]$

= WE CAN FOLLOW THE TRADITIONAL ELABORATION OR ...

$= AB \; E[\cos(w_0 t + \Theta)\cos(w_0(t+\tau) + \Theta)]$

$= AB \dfrac{R_{xx}(\tau)}{A^2} = \dfrac{B}{A} R_{xx}(\tau) = R_{xy}(\tau)$

Exercises to UNIT 2- Part II: Spectral Characteristics

2.10

Given that $X(t) = \sum_{i=1}^{N} \alpha_i X_i(t)$ where α_i are real constants, show that

$$\mathcal{S}_{XX}(\omega) = \sum_{i=1}^{N} \alpha_i^2 \mathcal{S}_{X_i X_i}(\omega)$$

if

(a) the processes $X_i(t)$ are orthogonal

(b) the processes are independent with zero mean

2.11

If $X(t)$ is a stationary process, find the power spectrum of $Y(t) = A_0 + B_0 X(t)$ in terms of the power spectrum of $X(t)$ if A_0 and B_0 are real constants.

2.12 The autocorrelation function of a random process $X(t)$ is

$R_{XX}(\tau) = 3 + 2e^{-4\tau^2}$

Find

(a) The power spectrum of $X(t)$

(b) The average power of $X(t)$

(c) The fraction of power that lies in the frequency band $-1/\sqrt{2} \leq \omega \leq 1/\sqrt{2}$

2.13 Given a random process with autocorrelation $R_{XX}(\tau) = P\cos^4(\omega_0\tau)$, find

(a) $S_{XX}(\omega)$

(b) P_{XX} from $S_{XX}(\omega)$ (c) P_{XX} from $R_{XX}(\tau)$

2.14 Given a random process with autocorrelation $R_{XX}(\tau) = Ae^{-\alpha|\tau|}\cos(\omega_0\tau)$ where $A > 0$, $\alpha > 0$, and ω_0 are real constants, find the power spectrum.

2.15 A random process is given by

$W(t) = AX(t) + BY(t)$ where A and B are real constants and $X(t)$ and $Y(t)$ are jointly widesense stationary processes. Find

(a) The power spectrum of $W(t)$ as a function of $S_{XX}(\omega)$, $S_{YY}(\omega)$, $S_{XY}(\omega)$ and $S_{YX}(\omega)$

(b) The power spectrum of $W(t)$ as a function of $S_{XX}(\omega)$, $S_{YY}(\omega)$, $\overline{X}$ and $\overline{Y}$, if $X(t)$ and $Y(t)$ are uncorrelated

(c) $S_{XW}(\omega)$ and S_{WX} as functions of $S_{XX}(\omega)$, $S_{YY}(\omega)$, $S_{XY}(\omega)$ and $S_{YX}(\omega)$

2.16 A wide-sense stationary $X(t)$ is applied to an ideal differentiator, so that $Y(t) = dX(t)/dt$. The cross-correlation of the input-output processes is known to be

$$R_{XY}(\tau) = dR_{XX}(\tau)/dt$$

(a) Determine $S_{XY}(\omega)$ in terms of $S_{XX}(\omega)$

(b) Determine $S_{YX}(\omega)$ in terms of $S_{XX}(\omega)$

2.17 The cross-correlation of jointly wide-sense stationary processes $X(t)$ and $Y(t)$ is assumed to be

$R_{XY}(\tau) = Be^{-W\tau}u(\tau)$ where $B > 0$ and $W > 0$

are constants. Find

(a) $R_{YX}(\tau)$

(b) $S_{XY}(\omega)$ (use appendix C from Peebles' book)

(c) $S_{YX}(\omega)$ (use cross-power density properties)

2.18 Consider two random process $X_1(t)$ and $X_2(t)$. The mean of $X_1(t)$ is equal to A ($A > 0$) and $X_2(t)$ is a white noise with power density 5 W/(rad/s). Given an LTI system with impulse response $h(t) = e^{-\alpha t}u(t)$.

with $\alpha > 0$. Find

(a) The mean value of the response of the LTI system if the input is $X_1(t)$

(b) The average power (second-order moment) of the response of the system if the input is $X_2(t)$.

2.19 A random process $X(t)$ with known mean $\bar{X}$ is the input of an LTI system with impulse response

$$h(t) = te^{-Wt}u(t).$$

Find

(a) The mean value of the response of the LTI system

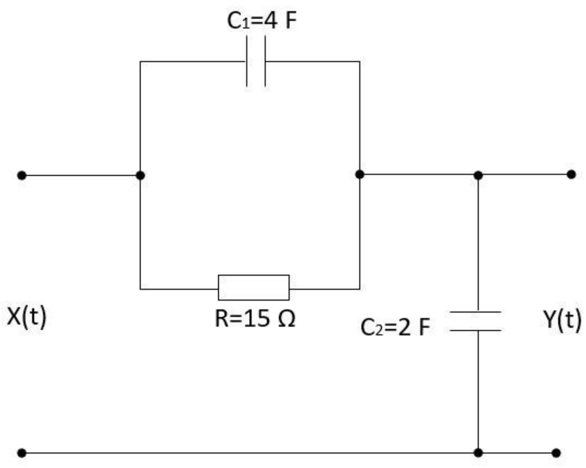
(b) The average power (second-order moment) of the response of the system if $X(t)$ is a white noise with power density 5 W/(rad/s).

2.20 A white noise with power density $N_0/2$ is applied to a network with impulse response of a system with impulse response

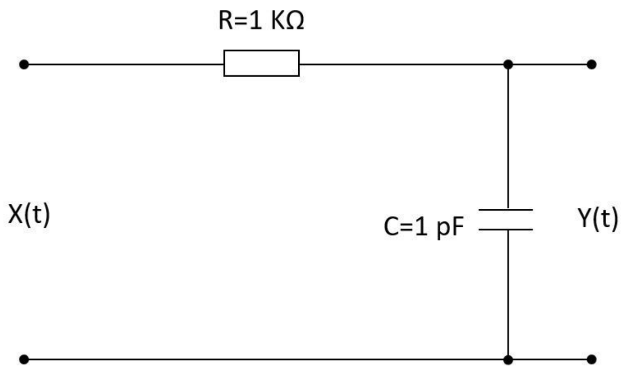
$$h(t) = Wte^{-Wt}u(t)$$

where W is a real positive constant. Find the cross-correlation of the reponse of input and the output of the system.

2.21 A stationary random process $X(t)$, having an autocorrelation function $R_{XX} = 2e^{-4|\tau|}$ is applied to the network of the figure below. Find the power spectrum of the output of the system.



2.22 A white noise $X(t)$ with $R_{XX} = 4\cdot 10^{-3}\cdot\delta(\tau)$ is filtered with the network of the figure below. Find the average power of the input and output of the system.


R=1 KΩ
X(t)
C=1 pF
Y(t)