

Construir el polinomio interpolador para los siguientes datos de una función  $f$ :

$$f(0) = 5, f(1) = -2, f'(1) = 5, f''(1) = 10, f'''(1) = 3, f(4) = 1 \text{ y } f'(4) = -2$$

$x_i$	$f[x_i]$	$f[x_i, x_{i+1}]$	$f[x_i, x_{i+1}, x_{i+2}]$	$f[x_i, x_{i+1}, x_{i+2}, x_{i+3}]$	$f[x_i, x_{i+1}, x_{i+2}, x_{i+3}, x_{i+4}]$	$f[x_i, x_{i+1}, x_{i+2}, x_{i+3}, x_{i+4}, x_{i+5}]$	$f[x_i, x_{i+1}, x_{i+2}, x_{i+3}, x_{i+4}, x_{i+5}, x_{i+6}]$
0	5						
		$\frac{-2-5}{1-0} = -7$					
1	-2		$\frac{5-(-7)}{1-0} = 12$				
		$f[1,1] = f'(1) = 5$		$\frac{5-12}{1-0} = -7$			
1	-2		$f[1,1,1] = \frac{f''(1)}{2!} = 5$		$\frac{\frac{1}{2} - (-7)}{1-0} = \frac{15}{2}$		
		$f[1,1] = f'(1) = 5$		$f[1,1,1,1] = \frac{f'''(1)}{3!} = \frac{3}{3!} = \frac{1}{2}$		$\frac{\left(-\frac{47}{54}\right) - \frac{15}{2}}{4-0} = \frac{-113}{54}$	
1	-2		$f[1,1,1] = \frac{f''(1)}{2!} = 5$		$\frac{\left(-\frac{19}{9}\right) - \frac{1}{2}}{4-1} = -\frac{47}{54}$		$\frac{\frac{87}{162} - \left(-\frac{113}{54}\right)}{4-0} = \frac{71}{108}$
		$f[1,1] = f'(1) = 5$		$\frac{\left(-\frac{4}{3}\right) - 5}{4-1} = \frac{-19}{9}$		$\frac{\frac{20}{27} - \left(-\frac{47}{54}\right)}{4-1} = \frac{87}{162}$	
1	-2		$\frac{1-5}{4-1} = \frac{-4}{3}$		$\frac{\frac{1}{9} - \left(-\frac{19}{9}\right)}{4-1} = \frac{20}{27}$		
		$\frac{1-(-2)}{4-1} = 1$		$\frac{-1 - \left(-\frac{4}{3}\right)}{4-1} = \frac{1}{9}$			
4	1		$\frac{-2-1}{4-1} = -1$				
		-2					
4	1						

$$\begin{aligned}
P_6(x) &= f[0] + f[0,1](x-0) + f[0,1,1](x-0)(x-1) + f[0,1,1,1](x-0)(x-1)^2 + f[0,1,1,1,1](x-0)(x-1)^3 + f[0,1,1,1,4](x-0)(x-1)^4 + f[0,1,1,1,4,4](x-0)(x-1)^4(x-4) \\
&= 5 - 7x + 12x(x-1) - 7x(x-1)^2 + \frac{15}{2}x(x-1)^3 - \frac{113}{54}x(x-1)^4 + \frac{71}{108}x(x-1)^4(x-4) =
\end{aligned}$$

$$\frac{71x^6 - 794x^5 + 3276x^4 - 6530x^3 + 7349x^2 - 4128x + 540}{108} = \frac{71}{108}x^6 - \frac{397}{54}x^5 + \frac{91}{3}x^4 - \frac{3265}{54}x^3 + \frac{7349}{108}x^2 - \frac{344}{9}x + \frac{185}{27}$$