

CENTRO DE GRAVEDAD

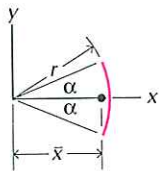
Tabla 5.1 SITUACIÓN DEL CENTRO DE EN ALGUNAS LÍNEAS Y SUPERFICIES.

Arco circular

$$L = 2r\alpha$$

$$\bar{x} = \frac{r \sin \alpha}{\alpha}$$

$$\bar{y} = 0$$

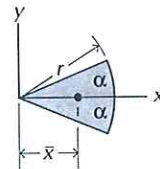


Sector circular

$$A = r^2 \alpha$$

$$\bar{x} = \frac{2r \sin \alpha}{3\alpha}$$

$$\bar{y} = 0$$

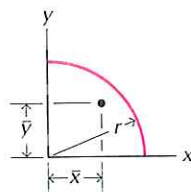


Arco de cuadrante

$$L = \frac{\pi r}{2}$$

$$\bar{x} = \frac{2r}{\pi}$$

$$\bar{y} = \frac{2r}{\pi}$$

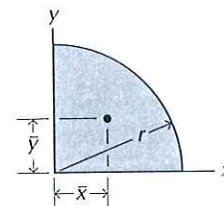


Cuadrante de círculo

$$A = \frac{\pi r^2}{4}$$

$$\bar{x} = \frac{4r}{3\pi}$$

$$\bar{y} = \frac{4r}{3\pi}$$

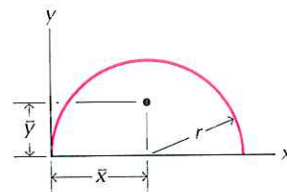


Semicircunferencia

$$L = \pi r$$

$$\bar{x} = r$$

$$\bar{y} = \frac{2r}{\pi}$$

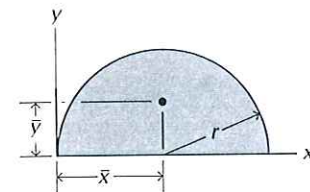


Área semicircular

$$A = \frac{\pi r^2}{2}$$

$$\bar{x} = r$$

$$\bar{y} = \frac{4r}{3\pi}$$

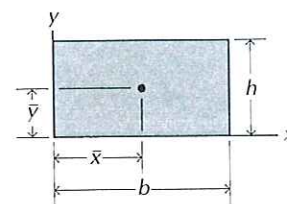


Área rectangular

$$A = bh$$

$$\bar{x} = \frac{b}{2}$$

$$\bar{y} = \frac{h}{2}$$

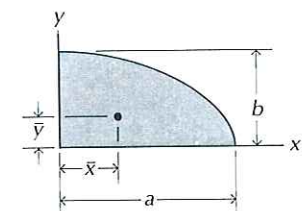


Cuadrante de elipse

$$A = \frac{\pi ab}{4}$$

$$\bar{x} = \frac{4a}{3\pi}$$

$$\bar{y} = \frac{4b}{3\pi}$$

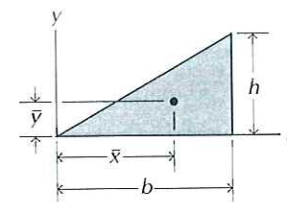


Área triangular

$$A = \frac{bh}{2}$$

$$\bar{x} = \frac{2b}{3}$$

$$\bar{y} = \frac{h}{3}$$

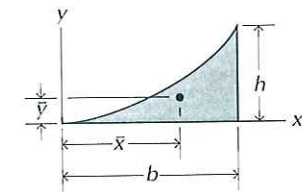


Seno parabólico

$$A = \frac{bh}{3}$$

$$\bar{x} = \frac{3b}{4}$$

$$\bar{y} = \frac{3h}{10}$$

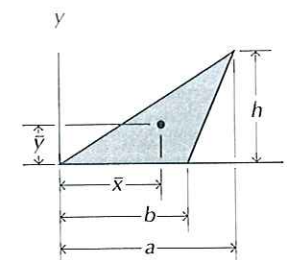


Área triangular

$$A = \frac{bh}{2}$$

$$\bar{x} = \frac{a+b}{3}$$

$$\bar{y} = \frac{h}{3}$$

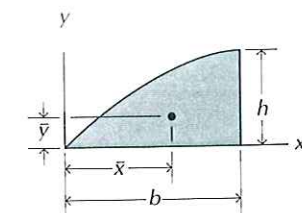


Cuadrante de parábola

$$A = \frac{2bh}{3}$$

$$\bar{x} = \frac{5b}{8}$$

$$\bar{y} = \frac{2h}{5}$$



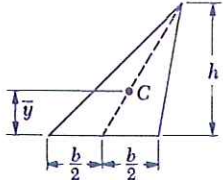
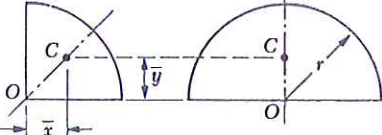
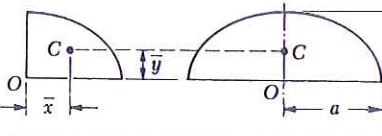
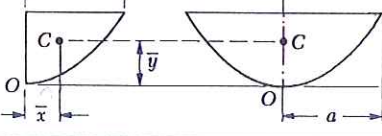
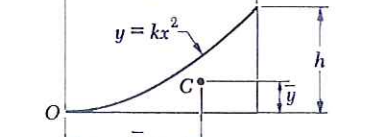
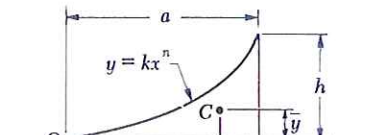
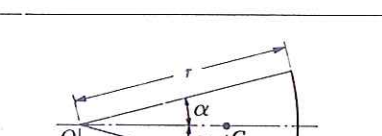
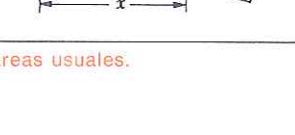
Forma		$\bar{x}$	$\bar{y}$	Área
Área triangular			$\frac{h}{3}$	$\frac{bh}{2}$
Un cuarto de área circular		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Área semicircular		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Cuarto de área elíptica		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Área semielíptica		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Área semiparabólica		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Área parabólica		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Extradós parabólico		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Extradós de forma general		$\frac{n+1}{n+2} a$	$\frac{n+1}{4n+2} h$	$\frac{ah}{n+1}$
Sector circular		$\frac{2r \text{ sen } \alpha}{3\alpha}$	0	αr^2

Fig. 5.8A Centros de gravedad de áreas usuales.

CENTRO DE GRAVEDAD

Tabla 5.2 SITUACIÓN DEL CENTRO DE DE ALGUNOS VOLÚMENES.

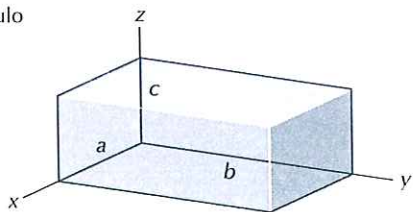
Paralelepípedo rectángulo

$$V = abc$$

$$\bar{x} = \frac{a}{2}$$

$$\bar{y} = \frac{b}{2}$$

$$\bar{z} = \frac{c}{2}$$



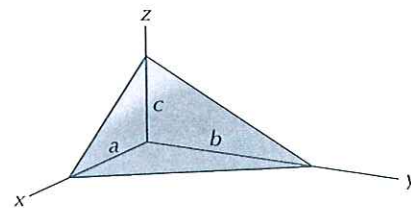
Tetraedro rectángulo

$$V = \frac{abc}{6}$$

$$\bar{x} = \frac{a}{4}$$

$$\bar{y} = \frac{b}{4}$$

$$\bar{z} = \frac{c}{4}$$



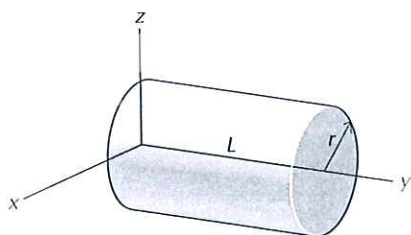
Cilindro de revolución

$$V = \pi r^2 L$$

$$\bar{x} = 0$$

$$\bar{y} = \frac{L}{2}$$

$$\bar{z} = 0$$



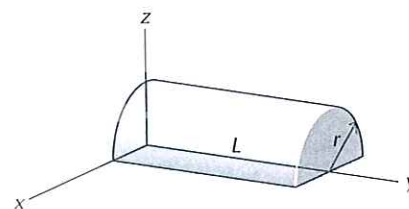
Semicilindro

$$V = \frac{\pi r^2 L}{2}$$

$$\bar{x} = 0$$

$$\bar{y} = \frac{L}{2}$$

$$\bar{z} = \frac{4r}{3\pi}$$



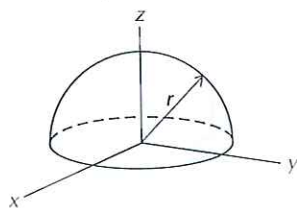
Semiesfera

$$V = \frac{2\pi r^3}{3}$$

$$\bar{x} = 0$$

$$\bar{y} = 0$$

$$\bar{z} = \frac{3r}{8}$$



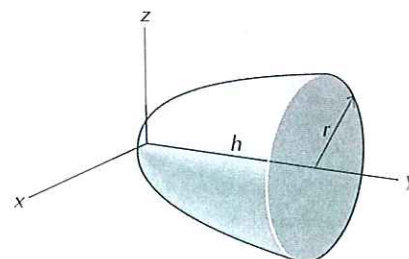
Paraboloide

$$V = \frac{\pi r^2 h}{2}$$

$$\bar{x} = 0$$

$$\bar{y} = \frac{2h}{3}$$

$$\bar{z} = 0$$



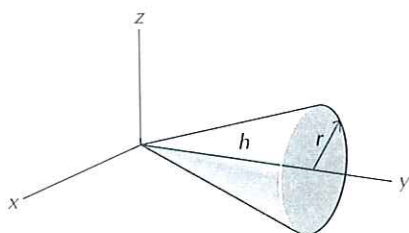
Cono de revolución

$$V = \frac{\pi r^2 h}{3}$$

$$\bar{x} = 0$$

$$\bar{y} = \frac{3h}{4}$$

$$\bar{z} = 0$$



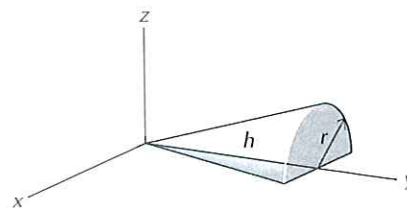
Semicono

$$V = \frac{\pi r^2 h}{6}$$

$$\bar{x} = 0$$

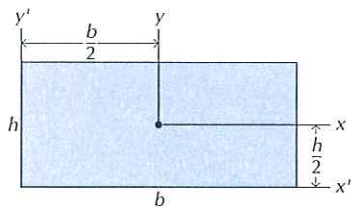
$$\bar{y} = \frac{3h}{4}$$

$$\bar{z} = \frac{r}{\pi}$$



MOMENTOS DE INERCIA

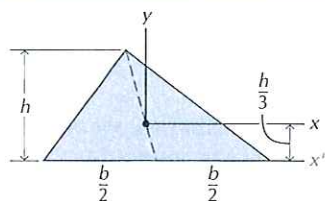
Tabla 10.1 MOMENTOS SEGUNDOS DE SUPERFICIES PLANAS



$$I_x = \frac{bh^3}{12}$$

$$I_{x'} = \frac{bh^3}{3}$$

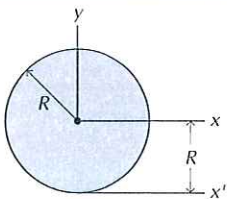
$$A = bh$$



$$I_x = \frac{bh^3}{36}$$

$$I_{x'} = \frac{bh^3}{12}$$

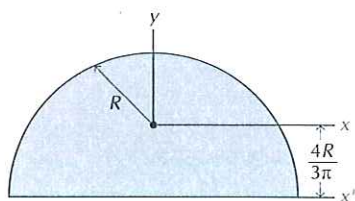
$$A = \frac{1}{2}bh$$



$$I_x = \frac{\pi R^4}{4}$$

$$I_{x'} = \frac{5\pi R^4}{4}$$

$$A = \pi R^2$$

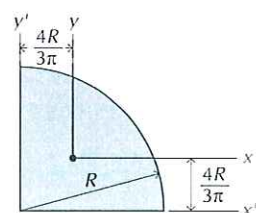


$$I_x = \frac{\pi R^4}{8} - \frac{8R^4}{9\pi}$$

$$I_{x'} = \frac{\pi R^4}{8}$$

$$I_y = \frac{\pi R^4}{8}$$

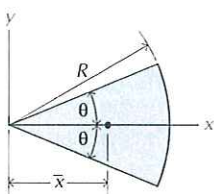
$$A = \frac{1}{2}\pi R^2$$



$$I_x = \frac{\pi R^4}{16} - \frac{4R^4}{9\pi}$$

$$I_{x'} = \frac{\pi R^4}{16}$$

$$A = \frac{1}{4}\pi R^2$$



$$I_x = \frac{R^4}{4} \left(\theta - \frac{1}{2} \sin 2\theta \right)$$

$$\bar{x} = \frac{2}{3} \frac{R \sin \theta}{\theta}$$

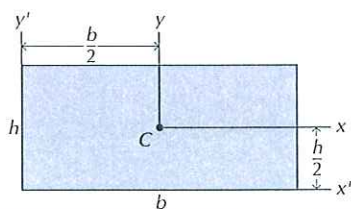
$$I_y = \frac{R^4}{4} \left(\theta + \frac{1}{2} \sin 2\theta \right)$$

$$A = \theta R^2$$

MOMENTOS DE INERCIA

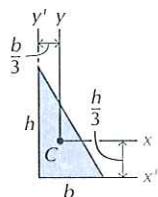


Tabla 10.3 MOMENTOS SEGUNDOS MIXTOS DE SUPERFICIES PLANAS



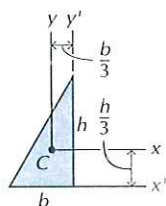
$$I_{xy} = 0$$

$$I_{x'y'} = \frac{b^2 h^2}{4}$$



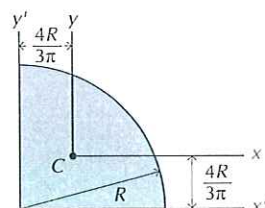
$$I_{xy} = -\frac{b^2 h^2}{72}$$

$$I_{x'y'} = \frac{b^2 h^2}{24}$$



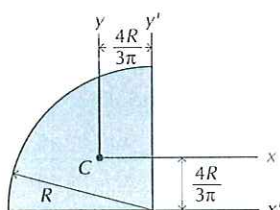
$$I_{xy} = \frac{b^2 h^2}{72}$$

$$I_{x'y'} = -\frac{b^2 h^2}{24}$$



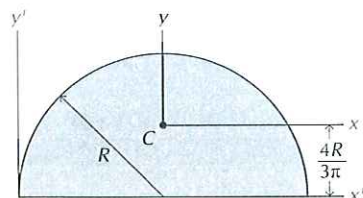
$$I_{xy} = \frac{(9\pi - 32) R^4}{72\pi}$$

$$I_{x'y'} = \frac{R^4}{8}$$



$$I_{xy} = -\frac{(9\pi - 32) R^4}{72\pi}$$

$$I_{x'y'} = -\frac{R^4}{8}$$



$$I_{xy} = 0$$

$$I_{x'y'} = \frac{2R^4}{3}$$

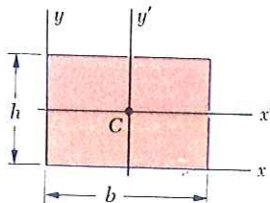
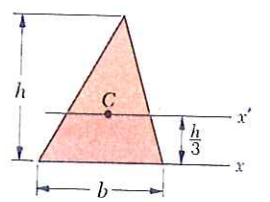
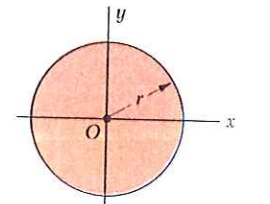
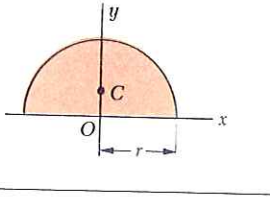
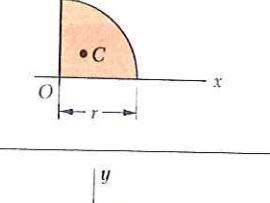
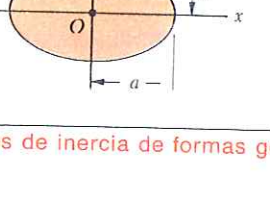
Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

Fig. 9.12 Momentos de inercia de formas geométricas usuales.