REGLAS DERIVADAS DE GENTZEN PARA LPO

T31 (Cambio de variable cuantificada): T31.1: $\forall x \varphi(x) \vdash \forall y \varphi(y)$
T31.2: $\forall y \varphi(y) \vdash \forall x \varphi(x)$, T31.3: $\exists x \varphi(x) \vdash \exists y \varphi(y)$, T31.4: $\exists y \varphi(y) \vdash \exists x \varphi(x)$

T32 (Descenso cuantificacional): $\forall x \varphi(x) \vdash \exists x \varphi(x)$

T33 (Conmutatividad): T33.1: $\exists x \exists y \varphi(x,y) \vdash \exists y \exists x \varphi(x,y)$,
T33.2: $\exists y \exists x \varphi(x,y) \vdash \exists x \exists y \varphi(x,y)$, T33.3: $\forall x \forall y \varphi(x,y) \vdash \forall y \forall x \varphi(x,y)$
T33.4: $\forall y \forall x \varphi(x,y) \vdash \forall x \forall y \varphi(x,y)$

T34 (Reglas de la conjunción):
T34.1: $\exists x(\varphi(x) \wedge \psi(x)) \vdash \exists x \varphi(x) \wedge \exists x \psi(x)$
T34.2: $\forall x \varphi(x) \wedge \forall x \psi(x) \vdash \forall x(\varphi(x) \wedge \psi(x))$ T34.3: $\forall x(\varphi(x) \wedge \psi(x)) \vdash \forall x \varphi(x) \wedge \forall x \psi(x)$
T34.4: $\exists x(\varphi \wedge \psi(x)) \vdash \varphi \wedge \exists x \psi(x)$, T34.5: $\varphi \wedge \exists x \psi(x) \vdash \exists x(\varphi \wedge \psi(x))$
T34.6: $\forall x(\varphi \wedge \psi(x)) \vdash \varphi \wedge \forall x \psi(x)$, T34.7: $\varphi \wedge \forall x \psi(x) \vdash \forall x(\varphi \wedge \psi(x))$

T35 (Reglas de la disyunción):
T35.1: $\exists x(\varphi(x) \vee \psi(x)) \vdash \exists x \varphi(x) \vee \exists x \psi(x)$, T35.2: $\exists x \varphi(x) \vee \exists x \psi(x) \vdash \exists x(\varphi(x) \vee \psi(x))$
T35.3: $\forall x \varphi(x) \vee \forall x \psi(x) \vdash \forall x(\varphi(x) \vee \psi(x))$
T35.4: $\varphi \vee \exists x \psi(x) \vdash \exists x(\varphi \vee \psi(x))$, T35.5: $\exists x(\varphi \vee \psi(x)) \vdash \varphi \vee \exists x \psi(x)$
T35.6: $\varphi \vee \forall x \psi(x) \vdash \forall x(\varphi \vee \psi(x))$, T35.7: $\forall x(\varphi \vee \psi(x)) \vdash \varphi \vee \forall x \psi(x)$

T36 (Reglas de la implicación):
T36.1: $\exists x \varphi(x) \to \exists x \psi(x) \vdash \exists x(\varphi(x) \to \psi(x))$, T36.2: $\forall x(\varphi(x) \to \psi(x)) \vdash \forall x \varphi(x) \to \forall x \psi(x)$
T36.3: $\exists x \psi(x) \to \varphi \vdash \exists x(\psi(x) \to \varphi)$, T36.4: $\forall x(\psi(x) \to \varphi) \vdash \forall x \psi(x) \to \varphi$
T36.5: $\forall x(\varphi \to \psi(x)) \vdash \varphi \to \forall x \psi(x)$

T37 (Reglas de la eliminación de la negación de un cuantificador)
T37.1 ($E\neg\forall$): $\neg \forall x \varphi(x) \vdash \neg \varphi(a)$ a es cte.
T37.2 ($E\neg\exists$): $\neg \exists x \varphi(x) \vdash \neg \varphi(y)$ y es variable libre