ECONOMETRICS
: Problem Sets

ACADEMIC YEAR: 2017-2018
CAMPUS: Segovia - Madrid
Professors: Rodrigo Alegría & Ainara González de San Román

PREFACE

This document contains exercises for you to practice with the content of the course in a practical way. Some of them will be included in the so-called Problem Sets, which will be solved in class. Students are required to work by themselves on these Problem Sets. Exercises to be solved within each Problem Set will be announced in advance to the due date. We will solve five Problem Sets during the course.

Any errors in this document are the responsibility of the author. Corrections and comments regarding any material in this text are welcomed and appreciated.

Authors: Rodrigo Alegría & Ainara González de San Román

e-mail: ralegria@faculty.ie.edu & agod@faculty.ie.edu

CONTENTS

PS1

Descriptive and Correlation Analysis

COURSE CONTENT

-**Chapter 1**: Introduction, Data and Econometric Modelling.
-**Chapter 2**: Review of Statistics, Descriptive Analysis and Correlation Analysis.
-**Chapter 3**: Estimator and its properties.

I asked an econometrician for her phone number…

and she gave me an estimate.

1 *Determine* the type of data of the following variables:

- **a-** Italian monthly salaries in the time period 1980-2005.
- **b-** Gender distribution in each of the OECD countries in 2010.
- **c-** Inflation rate in each of the OECD countries in 2008.
- **d-** R&D expenditure in each of the European Union member states in 2003.
- **e-** Yearly automobile production in France, Italy and Spain in the time period 1980-2010.
- **f-** Yearly race distribution in the United States during the last 20 years.
- **g-** Monthly water consumption during the 20th century in the city of Madrid.

2 Suppose you are asked to conduct a study to determine whether small class sizes lead to increase student performance.

- **a-** *Postulate* an econometric model that allows you to conduct this study.
- **b-** Why might you expect a negative correlation between class size and student performance?
- **c-** Would a negative correlation necessarily show that smaller class sizes cause better performance? *Explain*.

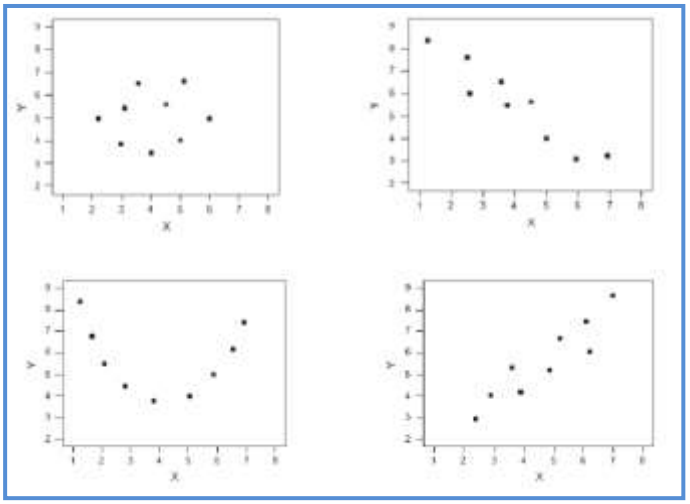
3 A substitute teacher wants to know how students in the class did on their last test. The teacher asks the 10 students sitting in the front row to state their latest test score. He concludes from their report that the class did extremely well.

- **a-** What is the sample? What is the population?
- **b-** Could you *identify* any problems with choosing the sample in this way?

4 Suppose that X is the number of free throws make by a basketball player out of two attempts and assume that the individual probabilities for each outcome of X are the following: $pr(x=0)=0.2$; $pr(x=1)=0.44$ and $pr(x=2)=0.36$

- **a-** *Define* the random variable.
- **b-** *Draw* the probability distribution associated to the above random variable.
- **c-** *Calculate* the expected value of the above random variable.
- **d-** *Calculate* the probability that the player makes at least one free throw.

5 *Interpret* the following graphs in terms of association, correlation and relationship:



6 Suppose x_1 and x_2 are independent random variables with means μ_1, μ_2 and standard deviations σ_1 and σ_2.

a- *Find* the mean and variance for a new random variable $u = x_1 - bx_2$

b- *Find* the mean and variance for a new random variable $v = ax_2 + bx_1$

7 The table below shows data about annual salaries (thousand Euros) and tenure (years) for 8 individual working in a company:

Salary	40	22	19	30	62	32	45	51
Tenure	15	3	1	8	39	13	17	24

a- What is your expectation about the type of relationship that exist between the two variables?

b- *Compute* the linear correlation coefficient between salaries and tenure and interpret your result.

c- Which variable is more dispersed? Why?

8 In the table below, E denotes the employment growth rate and P the productivity growth rate in the manufacturing industry in six countries for the period 1980-1990.

Country	E	P
Austria	2.0	4.0
Belgium	1.7	3.9
Canada	2.0	1.5
Denmark	2.4	3.0
Italy	4.0	2.0
Japan	5.9	9.6

a- *Determine* whether the data is a time series, a cross sectional data or panel data.
b- *Draw* a scatter plot with the data of the table. *Interpret* your graph.
c- *Calculate* the correlation coefficient (E, P) and *interpret* your result.
d- *Calculate* a new correlation coefficient eliminating the Japanese observation and interpret your result.

9 Let X be a random variable knowing that:

$$E(x) = \mu = 1 \; and \; Var(x) = \sigma^2 = 1$$

We have information about four independent observations: $x_1,\ x_2,\ x_3,\ x_4$.

a- Let $\hat{\mu}_1$ y $\hat{\mu}_2$ be two different estimators of μ, find which one is more appropriate according to the mean square error (MSE).

$$\hat{\mu}_1 = \frac{x_1 + x_2 + x_3}{3} \; ; \; \hat{\mu}_2 = \frac{x_1 + x_4}{6}$$

b- *Discuss* the sufficiency of both estimators.
c- *Suggest* a sufficient estimator of μ and with a MSE lower than the above ones.

10 We define a random variable X as tossing three coins. If we define the experiment as the number of tails obtained:

a- *Define* the random variable.
b- *Find* the probability distribution for *X*.
c- *Calculate* $E(X)$.

11 In the table below, P denotes average property prices and S average property sizes in six cities in 2012.

Country	P	S
New York	10.2	6.7
Madrid	7.2	5.5
Rome	9.0	5.8
London	11.6	7.7
Paris	10.8	7.1
Tokyo	17.2	3.1

a- *Determine* whether the data is a time series, a cross sectional data or panel data.
b- *Draw* a scatter plot with the data of the table. *Interpret* your graph.
c- *Find* the correlation coefficient (P, S) and *interpret* your result.
d- *Find* a new correlation coefficient eliminating the Tokyo observation and interpret your result.

12 Let X be a random variable knowing that:

$$E(x) = \mu \; and \; Var(x) = \sigma^2$$

We have information about four independent observations: $x_1,\ x_2,\ x_3,\ x_4$.

Let $\hat{\mu}_1 = \frac{1}{4}(x_1 + x_2 + x_3 + x_4)$ being and estimator for the population mean.

a- What are the expected value and variance of $\hat{\mu}_1$ in terms of μ and σ^2?
b- Now consider a second estimator for the population mean $\hat{\mu}_2$ being defined as:

$$\hat{\mu}_2 = \frac{1}{8}x_1 + \frac{1}{8}x_2 + \frac{1}{4}x_3 + \frac{1}{2}x_4$$

Show that this second estimator is also an unbiased estimator for the population mean. Find its variance.

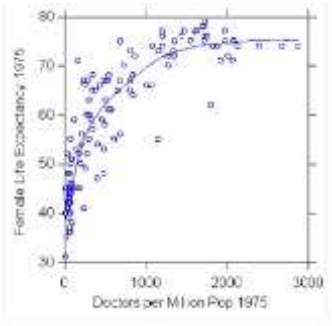
c- *Discuss* the sufficiency of both estimators.
d- Based on all your previous answers, which estimator do you prefer?

13 We define a random variable x as the resulting sum when tossing two dices. .

a- *Find* the probability distribution for x
b- *Compute* the $E(x)$
c- *Calculate* $E(y)$ knowing that $y = 2x - 1$.

14 *Evaluate* in which of the below cases you can say that the presented results are compatible and explain why:

a- $Cov(x, y) = 25.33$ and $\rho = -0.37$
b- $s_x^2 = 1{,}000 \quad n = 50 \quad \sum_{i=1}^{n} x_i = 5{,}000$ and $CV(x) = 0.316$
c-



and $\rho = 0.775$

15 In the table below, U denotes the unemployment rate and I the inflation rate in six American countries in 2011.

Country	U	I
Mexico	5.2	3.4
Argentina	7.2	9.5
Brazil	6.0	6.6
Chile	6.6	3.3
Colombia	10.8	3.4
Venezuela	8.2	26.1

a- *Determine* whether the data is a time series, a cross sectional data or panel data.
b- *Draw* a scatter plot with the data of the table. *Interpret* your graph.
c- *Calculate* the correlation coefficient (U, I) and *interpret* your result.
d- *Calculate* a new correlation coefficient eliminating the Venezuelan observation and interpret your result.

16 Let X be a random variable knowing that:

$$E(x) = \mu = 1 \; and \; Var(x) = \sigma^2 = 1$$

We have information about four observations: $x_1,\ x_2,\ x_3,\ x_4$.

a- Let $\hat{\mu}_1$ y $\hat{\mu}_2$ be two different estimators of μ, *find* which one is more appropriate according to the mean square error (MSE).

$$\hat{\mu}_1 = \frac{x_1 + x_2}{2} \; ; \; \hat{\mu}_2 = \frac{x_1 + x_4}{4}$$

Note: Observations are independent.

b- *Discuss* the sufficiency of both estimators.

c- *Suggest* a sufficient estimator of μ and with a MSE lower than the above ones.

17 A professor teaches a large class and has scheduled an exam for 7:00 pm in a different classroom. She estimates the probabilities in the table for the number of students who will call her at home in the hour before the exam asking where the exam will be held.

Number of calls	0	1	2	3	4	5
P(x)	0.10	0.15	0.19	0.26	0.19	0.11

a- *Draw* the probability distribution associated to the above experiment.

b- *Find* the expected value of the number of calls.

18 A specific company has observed in the last 5 months that their sales depend on the amount invested in advertising. Observe the table below:

Advertising Expenses	**Sales**
$ 100,000	R$ 1,000,000
$ 200,000	R$ 1,000,000
$ 300,000	R$ 2,000,000
$ 400,000	R$ 2,000,000
$ 500,000	R$ 4,000,000

a- *Construct* a scatter plot of the data. Does a clear linear relationship exist between the two variables?

b- *Conduct* a descriptive and correlation analysis of the above data and *interpret* both analysis.

19 In this exercise a researcher uses data on NBA players' salaries and their determinants. She is interested in knowing the effect of performance on NBA players' salaries. The following information is available for 56 NBA players.

Table 1. Variables of the dataset – names and description

SALARY	= Salary earned by players in thousands of dollars.
HT	= Height of the players in inches.
WT	= Weight of each player in pounds.
AGE	= Age of each player
MIN	= Number of minutes that each player played during the season.
STEALS	= Number of times that player stole ball from opponents.
BLOCKS	= Number of blocked shots.
POINTS	= Number of points that the player scored in the full season.

The summary statistics for all the variables in Table 1, as well as the correlation matrix, are presented in Tables 2 and 3 respectively.

Table 2. Summary statistics

Variable	Mean	Std. Dev.	Minimum	Maximum
SALARY	1668.04	667.910	1000	3750
HT	80.6250	3.66091	73	88
WT	226.804	26.7034	175	290
AGE	28.4107	3.03181	23	36
MIN	2538.96	670.669	189	3255
STEALS	97.9821	85.9193	6	564
BLOCKS	70.5179	74.0361	5	315
POINTS	1369.68	578.186	116	2633

Table 3. Correlation matrix

	SALARY	HT	WT	AGE	MIN	STEALS	BLOCKS	POINTS
SALARY	1	0.003	0.048	-0.075	0.094	0.082	0.088	0.233
HT		1	0.832	0.2926	-0.345	-0.349	0.556	-0.365
WT			1	0.086	-0.184	-0.225	0.473	-0.289
AGE				1	-0.112	-0.346	0.169	-0.081
MIN					1	0.319	0.048	0.793
STEALS						1	-0.128	0.386
BLOCKS							1	-0.078
POINTS								1

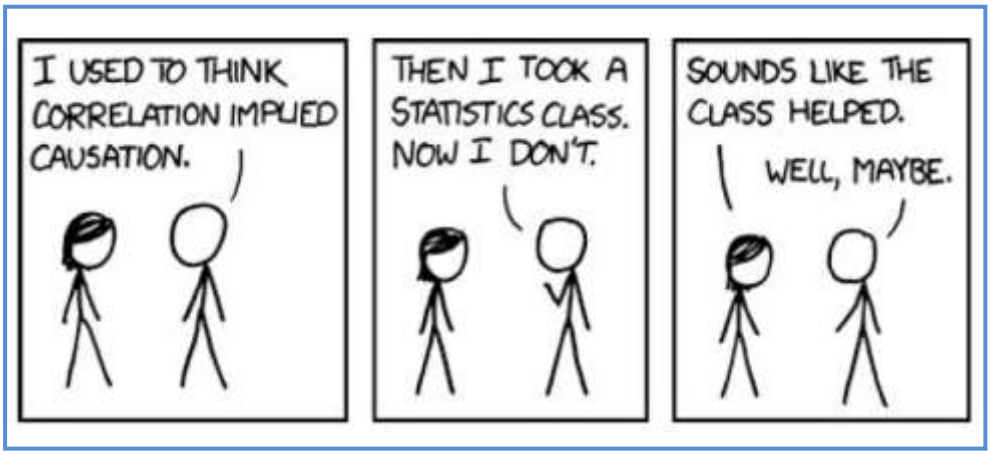
Answer the following questions:

- **a-** Is this a cross-section or a time series? Why? Which is the unit of analysis in this data set? And the sample size?
- **b-** How old is the youngest basketball player of this sample?
- **c-** Could you tell which variable is more dispersed by looking at the values of the standard deviations in Table 2?
- **d-** Could you say, by looking at Table 3, that there is a penalty in terms of lower wages associated with age? *Explain*.
- **e-** Which variables have the highest correlation (positive or negative) with wage? *Explain*.

20 Let *Y1*, *Y2*, *Y3*, and *Y4* be independent, identically distributed random variables from a population with mean μ and variance σ2. Let *Y be*:

$$Y = \frac{1}{4}Y_1 + \frac{1}{6}Y_2 + \frac{1}{2}Y_3 + \frac{1}{4}Y_4$$

denoting the average of these four random variables. *Show* that *Y* is a biased estimator of μ.



PS2
Linear Regression Analysis

COURSE CONTENT

-**Chapter 4**: Linear Regression Analysis
-Simple Linear Regression Model.
-Multiple Linear Regression Model.

An econometrician was asked about the meaning of life. He replied: It depends on the parameter values.

1 Assume that in order to establish the linear relationship between Y (percentage variation in the real wages) and X (unemployment rate) we consider the following equation:

$$\hat{Y}_i = 8.33 - 0.84X_i$$

Interpret the meaning of the estimated coefficients.

2 The *per capita* consumption of electric energy, in thousands of kWh (C), and the *per capita* income (X), in thousands of Euros for the countries belonging to the European Union in 2001 are explained for the following linear model:

$$\hat{C}_i = -0.154 + 0.571X_i$$

Compute the *per capita* income elasticity for a *per capita* income of 6,000 Euros.

3 Review exercise 11 in Problem Set 1. Find (using the OLS equations) the simple regression line that explains the behavior of P through the information contained in S. Use firstly the six city observations and then, estimate the same regression line but eliminating the Tokyo observation. *Interpret and explain your results*. Which is the difference between the **linear correlation analysis** discussed in exercise 11 of Problem Set 1 and the **linear regression analysis** performed in this exercise?

4 *Analytically show* that $\sum_{i=1}^{n} \hat{u}_i = 0$ is a descriptive property, which is satisfied when estimating a SLRM using OLS.

5 We have a dataset containing data about births to women in the United States. Two variables of interest are the dependent variable, infant birth weight in onces (*bw*), and an explanatory variable, average number of cigarettes the mother smoked per day during pregnancy (*cigs*). The following simple regression was estimated using data on 1,388 births:

$$\widehat{bw}_i = 119.77 - 0.514cigs_i$$

a- *Think* about possible factors contained in u_i.
b- *Interpret* the above regression results.
c- What is the predicted birth weight when *cigs* =10? What about when *cigs* = 20 (one pack per day)? *Comment* on the difference.

6 A company A operates with the following production function:

$$Y_t^A = 110 + 0.65K_t^A \qquad (R_A^2 = 0.37)$$

Such that Y_t^A measures total production in thousand Euros in year t and K_t^A measures the use of capital in thousand Euros in year t.

a- *Interpret* the coefficients of the estimated production function.

b- A competitor of company A, company B, operates according to a different production function defined as:

$$Y_t^B = 80 + 0.50K_t^B \qquad (R_B^2 = 0.48)$$

Interpret the coefficients of the estimated production function for company B in comparison to the coefficients for company A.

c- In 2010 ($t = 2010$), the use of capital in company A had a value of 320,000 Euros and 280,000 Euros in company B. Both companies are planning to expand their businesses to the Brazilian market in 2015. Therefore, their capital levels will increase 20% respect to 2010. *Find* the total production prediction in 2015 ($t = 2015$) for each company using the estimated cost functions. *Explain* which company will obtain a more accurate prediction in your opinion.

d- Do you think the relationship between production and the use of capital has constant returns (whether linearity assumption is satisfied)? If no, specify a more realistic regression model.

7 Are rent rates influenced by the city population? Using 2005 data for 70 cities, the following equation relates rent rates (*rent*) to total city population (*pop*):

$$\log(\widehat{rent}) = 9.40 + 0.0312\log(pop) \qquad R^2 = 0.192 \qquad n = 70$$

a- *Interpret* the coefficient on log(*pop*). Is the sign of this estimate what you expect it to be?

b- *Interpret* the determination coefficient. Why do you think is such a low value?

c- What other factors about a property may affect its rental price?

8 We have the following information regarding the average growth rates of employment (*e*) and real GDP (*g*) for 25 OECD countries for the period 1988-2007:

$$\bar{e} = 0.83 \quad \bar{g} = 2.82 \quad SST = 14.57 \quad SSR = 6.12$$

$$\sum_{i=1}^{25}(e_i - \bar{e})(g_i - \bar{g}) = 29.76 \quad \sum_{i=1}^{25}(g_i - \bar{g})^2 = 60.77$$

a- *Find* the regression coefficients for a regression model that investigates the behaviour of *e* through the behaviour of *g*.
b- *Interpret* your regression coefficients.
c- *Find* and *interpret* the value of the determination coefficient.
d- *Calculate* the predicted *e* when *g*=3.15.

9 Review exercise 8 in Problem Set 1. Find (using the OLS equations) the simple regression line that explains the behavior of E through the information contained in P. Use firstly the six country observations and then, estimate the same regression line but eliminating the Japanese observation. *Interpret and explain your results*. Which is the difference between the **linear correlation analysis** discussed in exercise 8 of Problem Set 1 and the **linear regression analysis** performed in this exercise?

10 *Analytically show* that if you estimate a SLRM using OLS method of estimation the $Cov(\hat{y}_i, \hat{u}_i) = 0$

11 The CAMP (Capital Asset Pricing Model) is an equilibrium model explaining the expected returns for assets. The regression for the excess of return (over the free-risk asset) has the following econometric specification:

$$\left(R_t - r_t^f\right) = \beta_0 + \beta_1\left(R_t^M - r_t^f\right) + u_t$$

Where, for the $t - th$ month, R_t represents the return of the asset, r_t^f is the monthly return of the risk-free asset (for example, the Treasury bills with a maturity of 30 days), R_t^M is the

return of the market available assets, and u_t is the random perturbance term that captures the random fluctuations that are independent on the market portfolio.

a- *Interpret* β_1.
b- What can be say about an asset with $\beta_1 = 1$? And one with $\beta_1 > 1$? And with $\beta_1 < 1$?
c- *Explain* the G-M condition that is being described above.

12 Review exercise 18 in Problem Set 1.

a- *Estimate* the Simple Linear Regression Model associated to the data. *Interpret* your estimation results.
b- If the company invests 355,000 in advertising, what is the forecasted amount of sales?

13 Observe the table below:

X	Y
62	8.1
70	9.0
76	9.2
82	10.5
88	10.8
74	9
75	8.1

a- *Estimate* the relationship between X and Y using OLS; that is obtain the intercept and slope estimates in the regression equation.
b- *Compute* the fitted values and residuals for each observation, and verify if residuals (approximately) sum to zero.
c- What is the predicted value of Y when X=58?
d- How much of the variation in Y for these 7 observations is explained by X?

14 The following data give X, the price charged per piece of playwood, and Y, the quantity sold (in thousands).

Price per Piece	**Thousands of Pieces Sold**
$6	80
$7	60
$8	70
$9	40
$10	0

a- *Draw* a scatter plot and interpret it.
b- *Compute* SST, SSR and SSE and explain the difference between SSE and SSR.
c- *Compute* the coefficient of determination and the value of the sample correlation coefficient. *Explain* the difference between them.

15 A company A operates with the following production function:

$$Y_t^A = 120 + 0.75L_t^A \qquad (R_A^2 = 0.38)$$

Such that Y_t^A measures total production in thousand Euros in year t and L_t^A measures the use of labour in number of workers in year t.

a- *Interpret* the coefficients of the estimated production function.
b- A competitor of company A, company B, operates according to a different production function defined as:

$$Y_t^B = 70 + 0.45L_t^B \qquad (R_A^2 = 0.58)$$

Interpret the coefficients of the estimated production function for company B in comparison to the coefficients for company A.

c- In 2010 ($t = 2010$), the use of labour in company A had a value of 3,500 workers and 2,800 workers in company B. Both companies are planning to expand their businesses to the Chinese market in 2015. Therefore, their labor levels will increase 20% respect to 2010. *Find* the total production prediction in 2015 ($t = 2015$) for each company using the estimated production functions. *Explain* which company will obtain a more accurate prediction in your opinion.

d- Do you think the relationship between production and the use of labor has constant returns (whether linearity assumption is satisfied)? If no, *specify* a more realistic regression model.

16 *Analytically show* that the OLS estimator for the intercept in a Simple Linear Regression Model is an unbiased estimator.

17 We denote I_i as total investment in a country (million dollars) and IR_i represents the interest rate. We consider the following linear regression model that yields the relationship between *I* and *IR*:

$$I_i = \beta_0 + \beta_1 IR_i + u_i$$

such that u_i denotes an unobservable error (random perturbances).

d- *Think* about possible factors contained in u_i.

e- Knowing that $\bar{I} = 0.25$, $\overline{IR} = 5$, *Cov(I, IR)=-0.7* and *Var(IR)=0.45*, *find* the estimated value for the intercept and slope coefficients and *interpret* your results.

f- Could you *specify and explain* a theoretical regression model of the above relationship in order to take into account decreasing returns in the effect of IR_i on I_i?

18 A company A operates with the following cost function:

$$TC_t^A = 220 + 0.45P_t^A \qquad (R_A^2 = 0.57)$$

Such that TC_t^A measures total production costs in thousand Euros in year t and P_t^A measures the level of production in thousand Euros in year t.

a- *Interpret* the coefficients of the estimated cost function.

b- A competitor of company A, company B, operates according to a different cost function defined as:

$$TC_t^B = 280 + 0.38P_t^B \qquad (R_B^2 = 0.42)$$

Interpret the coefficients of the estimated cost function for company B in comparison to the coefficients for company A.

c- In 2010 ($t = 2010$), the level of production in company A had a value of 430,000 Euros and 380,000 Euros in company B. Both companies are planning to expand their businesses to the Indian market in 2015. Therefore, their production levels will increase 20% respect to 2010. *Find* the total costs prediction in 2015 ($t = 2015$) for each company using the estimated cost functions. *Explain* which company will obtain a more accurate prediction in your opinion.

19 A political party is investigating whether spending in marketing (me_t), measured in thousand Euros, is an appropriate strategy in order to gain more members in the parliament (M_t) for the next elections:

$$M_t = \beta_0 + \beta_1 me_t + u_t$$

In order to estimate the above regression model, data of the last five elections is collected obtaining the following estimated regression:

$$\widehat{M_t} = 2.684 + 0.025 me_t \qquad T = 5 \qquad R^2 = 0.392$$

e- *Interpret* the constant term.
f- *Interpret* $\hat{\beta}_1$ (slope-estimated coefficient).
g- *Interpret* the value of the determination coefficient.
h- *Find* the predicted members in the parliament if the political party is thinking in spending about 750,000 Euros in marketing for the next elections.

20 Review exercise 15 in Problem Set 1. Find (using the OLS equations) the simple regression line that explains the behavior of I through the information contained in U. Use firstly the six country observations and then, estimate the same regression line but eliminating the Venezuelan observation. *Interpret and explain your results*. Which is the difference between the **linear correlation analysis** discussed in exercise 15 of Problem Set 1 and the **linear regression analysis** performed in this exercise?

21 A production function for a company is estimated using yearly observations for 20 years and we obtain the following estimated regression model:

$$\widehat{log(production)}_t = 4.822 + 0.257\log(capital)_t \qquad R^2 = 0.311 \qquad T = 20$$

Where both production and the use of capital are measured in thousand Euros.

a- *Interpret* the the coefficient on log(*capital*). Is the sign of this estimate what you expect it to be?
b- *Interpret* the determination coefficient.
c- What other factors may affect production levels?
d- Do you think the relationship between production and the use of capital has constant returns? If no, *specify* a more realistic regression model.

22 Using data from 1988 for houses sold in Andover, MA, from Kiel and McClain (1995), the following equation relates housing price (*price*) to the distance from a recently built garbage incinerator (*dist*):

$$\widehat{\log(price)}_i = 9.40 + 0.312\log(dist)_i \qquad R^2 = 0.162 \qquad n = 135$$

a- *Interpret* the the coefficient on log(*dist*). Is the sign of this estimate what you expect it to be?
b- *Interpret* the determination coefficient. Why do you think is such a low value?
c- What other factors about a house affect its price? Might these be correlated with distance from the incinerator?

23 Let sales be annual firm sales, measured in million dollars and salary annual salary measured in thousand dollars. We estimate the following regression model:

$$\widehat{\log(salary)}_t = 4.822 + 0.257\log(sales)_t \qquad R^2 = 0.211 \qquad T = 209$$

a- *Interpret* the coefficient on log(*sales*). Is the sign of this estimate what you expect it to be?
b- *Interpret* the determination coefficient. Why do you think is such a low value?
c- What other factors about an individual affect her salary? Might these be correlated with firm sales?

24 We have the following students' econometrics grade function:

$$G_i = \beta_0 + \beta_1 TH_i + u_i$$

Such that G_i represents the student's grade (points) obtained in Econometrics course and TH_i measures the total number of hours invested in studying Econometrics during the course. Using a sample of 50 students at IE University, the following estimated regression is obtained:

$$\widehat{G_i} = 0.25 + 0.08TH_i \qquad R^2 = 0.672$$

- **a-** *Think* about possible factors contained in u_i.
- **b-** *Interpret* the estimated regression coefficients.
- **c-** *Interpret* the value of the determination coefficient.
- **d-** *Find* the predicted grade in Econometrics if a student invests 75 hours studying the course.

25 We denote I_i as sale incomes for the shops located in a mall (thousand Euros) and NS_i represents the number of shop assistants working in each shop. We consider the following linear regression model that yields the relationship between *I* and *NS*:

$$I_i = \beta_0 + \beta_1 NS_i + u_i$$

such that u_i denotes an unobservable error (random perturbances).

- **a-** *Think* about possible factors contained in u_i.
- **b-** Knowing that $\hat{\beta}_1 = 4.245$, $\bar{I} = 46.75$ and $\overline{NS} = 5.75$, *find* the estimated value for the intercept coefficient and *interpret* your result.
- **c-** Would you use an OLS estimation of the above model to study the relationship between I and NS? Why?

26 In the linear consumption function:

$$C_i = \beta_0 + \beta_1 inc_i + u_i$$

the (estimated) marginal propensity to consume (MPC) out of income is simply the slope of the above regression model. Using observations for 100 families on annual income and consumption (both measured in dollars), the following estimated equation is obtained:

$$\widehat{G}_i = -124.84 + 0.853inc_i \qquad R^2 = 0.692$$

a- *Interpret* the estimated regression coefficients.
b- *Interpret* the value of the determination coefficient.
c- *Find* the predicted consumption when a family income is $30,000.

27 The data used for this exercise contains information on births for women in the United States. Two variables of interest are the dependent variable, infant birth weight in ounces (*bwght*) and an explanatory variable, average number of cigarettes the mother smoked per day during pregnancy (*cigs*). The following simple regression was estimated using data on $n = 1388$ births:

$$\widehat{bwght}_i = 119.77 - 0.514cigs_i$$

a- What is the predicted birth weight when *cigs* = 0? What about when *cigs* = 20 (one pack per day)? Comment on the difference.
b- Does this simple regression necessarily capture a causal relationship between the child's birth weight and the mother's smoking habits? *Explain*.
c- To predict a birth weight of 125 ounces, what would *cigs* have to be? *Comment*.
d- The proportion of women in the sample who do not smoke while pregnant is about 0.85. Does this help reconcile your finding from part (c)?

28 The econometrics team of the ministry of labor wants to investigate the relationship between unemployment duration and job search effort. For that purpose, they collect information from the Spanish Employment office (INEM) for 680 unemployed on the following variables:

unem = individual's unemployment duration measured as the number of weeks the individual remains unemployed
effort = individual's job search effort – it ranges from 0 (not effort at all) to 10 (the highest level of effort)

a- *Specify* the econometric model. Which relationship do you expect that holds in the population between unemployment duration and effort? Why? *Explain* relating your arguments to the elements of the postulated model.

b- Could you think of variables included in the error component and correlated with job search effort? *Give* two examples and *discuss* the consequences in terms of SLR assumptions.

c- The estimation result is presented next:

$$\widehat{unem}_i = 24.5 - 1.86\, effort_i$$

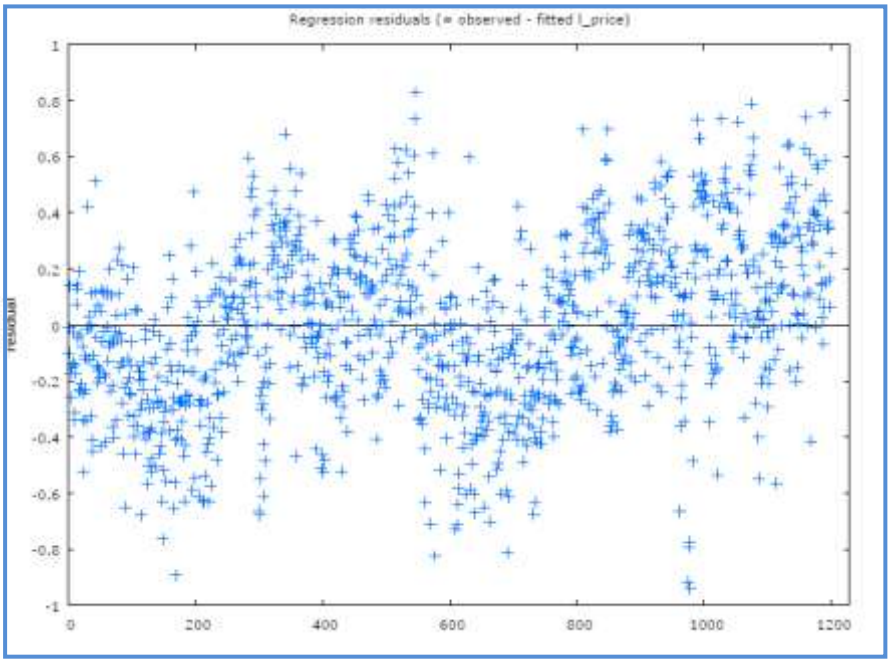
$$n = 680 \qquad R^2 = 0.28$$

Interpret the estimated coefficients and the *R*-squared.

d- Given the estimation in (c), *compute* the predicted unemployment duration for an individual making the maximum job search effort. Does this model help explain long term unemployment (we consider long-term as one year or more)?

29 *Explain*, with your own words and using the graph below, Ordinary Least Squares (OLS) estimation method.

Graph 1: Estimation residuals by observation number for a SLRM



30 Suppose the following model describes the relationship between annual salary (*salary*) and the number of previous years of labour market experience (*exper*)

$$\log(salary)_i = 10.6 + 0.027 exper_i$$

a- What is *salary* when *exper* = 0? When *exper* = 5 *Interpret* the intercept. [*Hint*: you will need to exponentiate].

b- *Draw* the shape (approximately) of the Population Regression Function for the *salary* conditional on *exper*. *Comment* on the advantages of the semi-logarithmic function for this particular example.

c- *Approximate* the percentage increase in *salary* when *exper* increases by five years. [*Hint*: you can use the formula: $\%\Delta y \approx (100 \cdot \beta_1)\Delta x$].

d- Use the results of part (a) to *compute* the exact percentage difference in *salary* when *exper* = 5 and *exper* = 0. *Comment* on how this compares with the approximation in part (c).

31 We have annual data, from 1963 until 1972, about the amount of money in a country (M_t) and the national income (Y_t), in million Euros, that can be summarised in the following:

$$\sum_{t=1}^{T} M_t = 37.2 \quad \sum_{t=1}^{T} M_t^2 = 147.18 \quad \sum_{t=1}^{T} M_t Y_t = 295.95$$

$$\sum_{t=1}^{T} Y_t = 75.5 \quad \sum_{t=1}^{T} Y_t^2 = 597.95$$

a- Could you *specify* a linear regression model representing the theory that states that the national income is determined by the amount of money in a country?

b- *Think* about possible factors contained in u_i of your econometric specification representing the above theory.

c- *Find* the OLS estimated values for the parameters of your econometric model and *interpret* your results.

32 A recent marketing department study of TV ad minutes vs profits (in thousand Euros) at a large company are shown below:

TV Ad Minutes	Money earned from Ad time
11	80
8	60
15	55
10	62

The average of TV Ad Minutes is 11; the average of money earned from Ad time is 64.25. The standard deviation of TV Ad minutes is 2.94 and the standard deviation of money earned from Ad time is 10.9. The covariance between variables is -7.33.

a- *Draw* a scatter plot and *interpret* it.

b- *Determine* the regression equation. What is the *interpretation* of $\hat{\beta}_0$ and$\hat{\beta}_1$?

c- How much profit does the company make if they advertise for 20 minutes in a month?

d- R-Squared is 0.05, what does this mean?

33 One company in the aeronautics industry wants to calculate the number of working hours that are required to finish the design of a new airplane. They think that the relevant explanatory variables are the top speed of the airplane, its weight and the number of pieces that are shared with other airplane models that the company builds. In order to do this, a sample of 35 airplanes is taken and the following model is estimated:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + u_i$$

Such that:

y_i= design effort in million working hours.

x_{1i}= airplane´s top speed in kilometres per hour.

x_{2i}= airplane´s weight in tons.

x_{3i}= percent number of pieces that are shared with other airplane models.

The estimated regression coefficients are:

$$\hat{\beta}_1 = 0.661\ ;\ \hat{\beta}_2 = 0.065\ ;\ \hat{\beta}_3 = -0.018$$

Interpret the above estimated values.

34 A district manager of an important chain that sells electronic products is currently analyzing why sales figures for its local outlets within the district are different among them (some outlets are performing better than others in terms of annual sales figures). She selects 20 random outlets located in different localities within the district and considers the following econometric specification:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + u_i$$

Such that for each outlet in the sample: y_i measures total annual sales (thousand dollars), x_{1i} is the number of competitor outlets in the locality where the outlet is located, x_{2i} measures the local population (millions) and x_{3i} indicates annual marketing expenditures in each sample outlet (thousand dollars).

a- Which would be, in your opinion, the expected signs for β_1, β_2 and β_3? Why?

b- *Interpret* the estimated intercept coefficient if the above model is estimated such that:
$\widehat{y_i} = 14 - 1x_{1i} + 0.3x_{2i} + 0.2x_{3i} \qquad R^2 = 0.5809$

c- *Find* the estimated change in annual sales for an outlet having 5 additional competitor outlets within its local market, maintaining the population and marketing expenditures as constant terms.

d- *Interpret* the value $\hat{\beta}_3 = 0.2$.

e- *Discuss* the explanatory power of the above estimated regression model.

f- The sixth sample outlet has 7 local competitors, is placed in a locality with 2,750,300 inhabitants and its marketing expenditures are 150,000 dollars. *Find* the estimated annual sales for this outlet.

g- The true annual sales for the sixth sample outlet are 890,000 dollars each. *Find* the estimated residual for this outlet.

35 We estimate a model that relates the salary for business managers with the sales of the firm and the market value of the firm such that:

$$log(wage)_i = 4.62 + 0.162\log(sales)_i + 0.106\log(mv)_i$$

$$n = 220 \qquad R^2 = 0.3481$$

a- *Interpret* the estimated model.

b- We have a second estimation in which we include a third explanatory variable (firm´s profits) such that:

$$\log(wage)_i = 4.734 + 0.165\log(sales)_i + 0.084\log(mv)_i + 0.003profits_i$$

$$n = 220 \qquad R^2 = 0.3541$$

Why $profits_i$ variable is not included in the model in logs? Which is the model with a better goodness-of-fit? Do these firm specific variables explain the behaviour of the wage variable? Why?

36 Consider the regression model in which the dependent variable (television viewing hours per week) is to be explained in terms of three explanatory variables:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + u_i$$

Such that:

x_{1i}= income in thousand Dollars.

x_{2i}= hours per week spent at work.

x_{3i}= number of people living in the household.

The estimated regression coefficients are:

$$\hat{\beta}_1 = -1.28\,;\ \hat{\beta}_2 = -0.13\,;\ \hat{\beta}_3 = 2.45$$

Interpret the above estimated values.

37 Assume that we have the following theoretical specification:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{1i}^2 + u_i$$

Explain, in your opinion, whether the following statements are true or false:

a- One unit change in x_{1i} always produces the same effect on the value of the independent variable.

b- One unit change in x_{1i} does not produce the same effect on *y*, but depends on the value of x_{1i}.

38 A company in the financial sector wants to rent an office space in Madrid. The following regression model estimates the rent prices for office space in Madrid:

$$price_i = \beta_0 + \beta_1 sqfeet_i + \beta_2 dis_i + u_i$$

Such that *sqfeet* is the office space in square feet, *dis* is the distance between the place the office is located and the city centre, measured in kilometres, and *price* is the monthly rental price in thousand Euros.

a- Which would be the expected signs for β_1 and β_2? Why?

b- The above model is estimated such that:

$$\widehat{price}_i = -19.315 + 1.1284 sqfeet_i - 0.8819 dis_i$$

$$n = 120 \qquad R^2 = 0.6319$$

Interpret the constant term.

c- *Find* the estimated increment in the rental price for an office space with 100 additional square feet, maintaining the distance to the city centre as a constant term.

d- *Find* the change in the rental price for an office located 5 additional kilometres away from the city centre, maintaining the size of the office as a constant term.

e- *Discuss* the explanatory power of the regression model.

f- The fifth sample office has a size of 120 square feet and is located at a distance of 5.4 kilometres from the city centre. *Find* the estimated rental price using the above OLS regression line.

g- The true rental price for the fifth sample office is 89,000 Euros each month. *Find* the estimated residual for this office. Could this suggest that the company is over paying or under paying its office space?

39 A consultancy firm is analyzing monthly transportation costs in the manufacturing sector for several companies. The following regression model estimates the transportation costs for a firm:

$$tc_i = \beta_0 + \beta_1 oilp_i + \beta_2 dis_i + u_i$$

Such that *oilp* is the price of oil in Dollars per barrel, *dis* is the distance between the location of the manufacturing company and the location of its main supplier, measured in kilometers and *tc* denotes transportation costs in thousand Dollars.

a- Which would be the expected signs for β_1 and β_2? Why?
b- The above model is estimated such that:

$$\widehat{tc_i} = 23.385 + 10.588 oilp_i + 0.777 dis_i$$

$$n = 400 \qquad R^2 = 0.6319$$

Interpret the constant term.
c- *Find* the estimated change in transportation costs if oil price decreases in 5 Dollars per barrel, maintaining the distance to the main supplier as a constant term.
d- *Find* the change in transportation costs for a company located 10 additional kilometers away from its main supplier, maintaining oil price as a constant term.
e- *Discuss* the explanatory power of the regression model.
f- The tenth manufacturing firm pays oil at 97.16 Dollars per barrel and is located at a distance of 23.4 kilometers from its main supplier. *Find* the estimated transportation costs using the above OLS regression line.
g- The true transportation costs for the tenth sample manufacturing firm is 28,000 Dollars each month. *Find* the estimated residual for this company.

40 The initial wage for just graduated lawyers is determined by the following estimated regression model:

$$\widehat{\log(wage)}_i = 6.77 + 0.104 \log(book)_i + 0.44 \log(cost)_i$$

$$n = 200 \qquad R^2 = 0.278$$

Such that $wage_i$ measures initial monthly wage in thousand Euros, $book_i$ indicates the number of law books in the university library where the graduated studied and $cost_i$ measures the annual cost (thousand Euros) of the university where the graduated got her law title.

a- *Interpret* the above estimated model.
b- We have a second estimation in which we include a third explanatory variable: rank of the law faculty (being $rank = 1$ the best one) such that:

$$\widehat{\log(wage)}_i = 6.34 + 0.095 \log(book)_i + 0.38 \log(cost)_i - 0.0033 rank_i$$

$$n = 200 \qquad R^2 = 0.294$$

Why $rank_i$ variable is not included in the model in logs? Which is the model with a better goodness-of-fit? Do these university specific variables explain the behaviour of the wage variable? Why?

41 A consultancy firm is analyzing property prices in the city of Madrid using a sample of 88 properties using the following regression model:

$$p_i = \beta_0 + \beta_1 sqrft_i + \beta_2 bdrms_i + u_i$$

Such that *p* is property price in thousand dollars, *sqrft* is the size of the property in squared feet, and *bdrms* is the number of bedrooms.

a- Which would be the expected signs for β_1 and β_2? Why?

b- The above model is estimated such that:

$$\widehat{p_i} = -19.315 + 0.128 sqrft_i + 15.198 bdrms_i$$

$$n = 88 \qquad R^2 = 0.6319$$

Interpret the constant term.

c- *Find* the estimated change in property prices if there is an increment of 3 bedrooms, maintaining the size of the property as a constant term.

d- *Find* the change in property prices for each additional 10 square feet in size, maintaining the number of bedrooms as a constant term.

e- *Discuss* the explanatory power of the regression model (goodness-of-fit).

f- The tenth property has a size of 2,438 square feet and has 4 bedrooms. *Find* the estimated property price using the above OLS regression line.

g- The true property price for the tenth sample property is 300,000 Dollars. *Find* the estimated residual for this company. Does this suggest that the buyer underpaid this property?

42 Consider Y (logarithm of real money demand), X1 (logarithm of real GDP) and X2 (logarithm of the interest rate of Treasury bills). Consider the following regression results:

$$\widehat{Y_i} = 2.3296 + 0{,}5573 X1_i - 0.2032 X2_i$$

Interpret the above estimated equation.

43 The CEO salary is determined by the following estimated regression model:

$$\log(\widehat{salary})_i = 6.77 + 0.904 \log(sales)_i + 1.44 ceoten_i$$

$$n = 400 \qquad R^2 = 0.388$$

Such that $salary_i$ measures monthly wage in thousand Euros, $sales_i$ indicates monthly firm sales in thousand Euros and $ceoten_i$ measures CEO tenure with the firm in years.

a- *Interpret* the above estimated model.
b- Why $ceoten_i$ variable is not included in the model in logs?

We re-estimate the above model including a new explanatory factor, CEO education in years and we obtain the following estimation results:

$$\widehat{\log(salary)}_i = 5.77 + 0.774\log(sales)_i + 1.15ceoten_i + 0.54ceoedu_i$$

$$n = 400 \qquad R^2 = 0.498$$

c- Which is the model with a better goodness-of-fit? Why?

44 The Data on U.S. working men was used to estimate the following equation:

$$\widehat{educ}_i = 10.30 - 0.094sibs_i + 0.131meduc_i + 0.210feduc_i$$

$$n = 722 \qquad R^2 = 0.214$$

where *educ* is years of schooling, *sibs* is number of siblings, *meduc* is mother's years of schooling, and *feduc* is father's years of schooling.

a- Does *sibs* have the expected effect? *Explain*. Holding *meduc* and *feduc* fixed, by how much does *sibs* have to increase to reduce predicted years of education by one year? (A non-integer answer is acceptable here)
b- *Discuss* the interpretation of the coefficient on *meduc*.
c- Suppose that Man A has no siblings, and his mother and father each have 12 years of education. Man B has no siblings, and his mother and father each have 16 years of education. What is the *predicted difference* in *educ* between A and B?
d- Would you say *sibs*, *meduc* and *feduc* explain much of the variation in *educ*? What other factors might affect men's years of schooling? Are these likely to be correlated with *sibs*? *Explain*.

45 For a child i living in a particular school district, let $voucher_i$ be a dummy variable equal to one if a child is selected to participate in a school voucher program, and let $score_i$ be that child's score on a subsequent standardized exam. Suppose that the participation

variable, $voucher_i$, is completely randomized in the sense that it is independent of both observed and unobserved factors that can affect the test score.

a- If you run a simple regression $score_i$ on $voucher_i$, using a random sample of size n, which sign do you expect to find on the coefficient associated to the dummy variable? *Explain* the intuition.

b- Does the OLS estimator provide an unbiased estimator of the effect of the voucher program?

c- Suppose you can collect additional background information, such as family income, family structure and parent's education levels. Do you need to control for these factors to obtain an unbiased estimator of the effects of the voucher program? *Explain*.

46 Observe the equation below:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u$$

Write and explain at least three characteristics that this model need to have to not violate Gauss-Markov theorem.

47 Consider the multiple regression model containing three independent variables, under Assumptions *MLR*.1 through *MLR*.4:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u$$

You are interested in estimating the sum of the parameters on x_1 and x_2; call this $\theta_1 = \beta_1 + \beta_2$

a- *Show* that $\hat{\theta}_1 = \hat{\beta}_1 + \hat{\beta}_2$ is an unbiased estimator of θ_1.

b- *Find* $Var(\hat{\theta}_1)$ in terms of $Var(\hat{\beta}_1)$, $Var(\hat{\beta}_2)$ and $Corr(\hat{\beta}_1, \hat{\beta}_2)$

48 *Review* exercise 19 in Problem Set 1. In order to exploit the data, the researcher decides to estimate two different multiple linear regression models. They are presented next:

Model 1: OLS, Dependent variable: SALARY

	Coefficient	*Std. Error*	*t-ratio*	*p-value*
const	−573.466	2753.14	−0.2083	0.8358
POINTS	0.330204	0.166822	1.9794	0.0532
AGE	−22.1496	32.429	−0.6830	0.4977
HT	29.6911	49.4528	0.6004	0.5509
WT	0.10876	6.33779	0.0172	0.9864

SSR = 22,576,134 **SST=** 24,512,631

Model 2: OLS, Dependent variable: SALARY

	Coefficient	*Std. Error*	*t-ratio*	*p-value*
const	1804.65	3556.23	0.5075	0.6142
POINTS	0.587788	0.276637	2.1248	0.0388
AGE	−22.1622	34.1573	−0.6488	0.5195
HT	−1.77246	57.2917	−0.0309	0.9754
WT	2.40206	6.69018	0.3590	0.7211
MIN	−0.305162	0.243823	−1.2516	0.2168
STEALS	−0.142169	1.24054	−0.1146	0.9092
BLOCKS	1.06044	1.56953	0.6756	0.5025

SSR = 21,816,981 **SST =** 24,553,568

a- *Interpret* the estimated regression coefficients from **Model 1**. Are the signs of the coefficients the expected ones? *Compute* the R-squared and *interpret* its meaning.

b- Is there a penalty in terms of lower wages associated with age? *Explain*. Is this result consistent with the correlation analysis carried out in exercise 19 PS1.

c- *Compare* the estimated slope parameters associated to the explanatory variable of interest (`POINTS`) between Models 1 and 2. Does the estimated slope change from Model 1 to 2? Could you give an *explanation* for this happening?

d- Which model is the best in terms of goodness-of-fit?

48 We have information about the following variables: (1) p_i (rental office prices in thousand Euros per month, (2) s_i (size of the office space in square meters), (4) d_i (distance from the city centre in kilometres) and (4) n_i (number of floors of the building in which the office space is placed). We use regression analysis to obtain some insights about the

behaviour of rental office prices using a sample of 150 offices located within the city of Barcelona in 2012. You can see in the below table estimation results:

Table 1: OLS Estimation Results

Variable	Model 1 coefficients	Model 2 coefficients	Model 3 coefficients	Model 4 coefficients
constant	-34.65	1.58	1.22	0.38
size	0.076	0.025	0.022	0.042
distance		-0.098	-0.088	
numfloors			-0.127	
log(distance)				-3.14
n	150	150	150	150
R squared	0.243	0.457	0.422	0.527

Note: Models 1, 2 and 3 use as dependent variable prices and Model 4 uses as dependent variable log(prices).

- **a-** What happens to the coefficient of size when comparing Model 1 and Model 2? Why?
- **b-** Which model do you prefer when comparing Model 1 and Model 2?
- **c-** Do you think Model 3 is a better specification than Model 2? Why?
- **d-** *Interpret* the coefficients associated to Model 4?
- **e-** Is Model 4 the best model? Why?

PS3
Hypothesis Testing

COURSE CONTENT

-**Chapter 5**: Hypothesis Testing
-Hypothesis Testing in the SLRM.
-Hypothesis Testing in the MLRM.

Three econometricians went out hunting, and came across a large deer. The first econometrician fired, but missed, by a meter to the left. The second econometrician fired, but also missed, by a meter to the right. The third econometrician didn't fire, but shouted in triumph, "We got it! We got it!"

1 We are interested in examining the relationship between Cabinet duration and Polarization where CD_i denotes the number of months a cabinet government survives until its fall (this variable ranges from 0.5, half a month, to 59 months) and P_i measures the support in the country for extremist political parties (this variable ranges from 0, 0%, to 43, 43% support. It is hypothesized that polarization will be negatively related to cabinet duration: the more support there is for extremist parties, the more difficult it will become for the governing party to bargain and hence, maintain a government. The sample size is 314 and OLS estimation result is the following (standard errors in parenthesis):

$$\widehat{CD}_i = 26.652 - 0.537P_i \qquad R^2 = 0.41$$

$$(1.189) \qquad (0.06)$$

- **a-** *Interpret* the estimated regression model and the value of the determination coefficient
- **b-** *Test* the null hypothesis that the polarisation coefficient is zero at a 1% significance level.
- **c-** Knowing that a country has a 25% support for extremist parties, *find* the predicted cabinet duration.
- **d-** In your opinion, *explain* one application of the above model from the perspective of a non-extremist political party.

2 Annual profits evolution in an Italian company in the aeronautics sector follows an exponential growth model which was estimated for the time period 1981-2010 (both years included in the sample) such that:

$$\widehat{logy_t} = 32.555 + 0.0534t$$

$$(33.2) \qquad (0.00211)$$

- **a-** *Interpret* the estimated slope coefficient.
- **b-** *Test* the null hypothesis that the true value for the slope coefficient is zero at a 5% significance level. What about at 1% significance level? Which of the two t-test is more informative?

3 Population time evolution in the United States follows an exponential growth model which was estimated for the time period 1970-1999 (both years included in the sample) such that:

$$\widehat{logy_t} = 201.9727 + 0.0284t$$

$$(743.2) \qquad (0.00211)$$

a- *Interpret* the estimated slope coefficient.
b- *Test* the null hypothesis that the true value for the slope coefficient is zero at a 5% significance level. What about at 1% significance level?
c- In your opinion, could you use the above model to predict the United States´ population in the current decade? Why?

4 Consider a SLRM relating the annual number of crimes on college campuses (*crime*) to student enrollment (*enroll*) with the following estimation results:

$$\widehat{\log(crime)}_i = -6.63 + 1.27\log(enroll)_i \qquad n = 97 \quad R^2 = 0.585$$
$$(1.03) \qquad (0.11)$$

a- *Interpret* the estimated slope coefficient.
b- *Calculate* two-tailed test to find whether the variable *enroll* should be included in the regression model (at 1% significance level).
c- *Test* that the elasticity of crime with respect to enrolment is 1 (at 5% significance level).
d- What could you say about the explanatory power of the above model? *Test* the whole model fit at 5% significance level.

5 We have a sample $T = 27$ with data for the following variables:

Y: housing expenditure in USA (dollars)

X: household income (dollars)

The following regression model is estimated through OLS:

$$\widehat{logy_t} = 1.20 + 0.55 logx_t \qquad SST = 330 \quad SSR = 51$$
$$(0.11) \quad (0.02)$$

a- *Interpret* the estimated slope coefficient.
b- *Calculate* one-tailed test to find whether the variable $logx_t$ should be included in the regression model (at 1% significance level).
c- *Test* that housing expenditure elasticity respect to household income is 1 (at 5% significance level).
d- What could you say about the explanatory power of the above model? *Test* the whole model fit at 5% significance level.

6 Given the following regression model:

$$Inflation_i = \beta_0 + \beta_1 InterestRate_i + u_i$$

Where both variables are measured in percentage points, a sample of 100 countries is used in order to estimate the above model and the following information is given:

$$Var(Inflation_i) = 100;\ Var(InterestRate_i) = 50;$$

$$Cov(Inflation_i, InterestRate_i) = -25;\ SSR = 49$$

a- *Find* the OLS estimation of the effect of interest rates on inflation and the estimated standard error.
b- *Interpret* your estimation results.
c- *Calculate* a one-tailed t-test in order to validate the significance of the estimated slope coefficient at 1% significance level.
d- What could you say about the explanatory power of the above model? *Test* the whole model fit at 5% significance level.

7 We have a sample of 45 workers employed in a company. We ask to each worker to evaluate her/his satisfaction level at work (x) from 0 to 10. We also know, for each worker, the number of labor absenteeism days (y) last year. A linear regression line is estimated such that:

$$\hat{y}_i = 12.6 - 1.2x_i \qquad R^2 = 0.321$$

$$(0.112)\ \ (0.088)$$

a- *Interpret* the estimated regression model and the value of the determination coefficient
b- *Test* the null hypothesis that work satisfaction does not produce any significant effect on labour absenteeism at a 1% significance level.
c- The level of work satisfaction of a different worker is 6. *Find* the predicted labour absenteeism days per year for this worker.
d- In your opinion, *explain* one application of the above model from the perspective of the Human Resources department of the company.

8 The following theoretical model is the so-called characteristic line for investment analysis:

$$r_{it} = \beta_0 + \beta_1 r_{mt} + u_t$$

Such that the dependent variable measures return rate for an asset and the explanatory variable denotes return rate for the market portfolio. In this type of model, we can interpret the slope coefficient as a risk indicator. The above model was estimated using 240 monthly return rates for the period 1956-1976 (both years included) related to IBM assets and USA market portfolio:

$$\widehat{r_{it}} = 0.7264 + 1.059 r_{mt} \qquad R^2 = 0.551$$

$$(0.3001) \quad (0.0728)$$

a- *Interpret* the above estimation results.
b- *Test* the null hypothesis that the true value for the slope coefficient is zero at a 1% significance level.
c- It is said that an asset with a slope coefficient greater than 1 is a volatile asset. Could you say IBM assets are significantly volatile assets?
d- What could you say about the explanatory power of the above model? *Test* the whole model fit at 5% significance level.

9 The French Ministry of Education is analyzing the evolution of university tuition fees in the last 20 years. Using a sample of 55 public universities, the following estimated model is obtained:

$$\widehat{\log(y_i)} = 38.03 + 0.07t \qquad R^2 = 0.41$$

$$(14.2) \quad (0.05)$$

a- *Interpret* the estimated slope coefficient.
b- *Test* the null hypothesis that the true value for the slope coefficient is zero at a 5% significance level. What about at 1% significance level?
c- Given that: $\widehat{\log(y_i)} = 12.23 + 2.01t$ for all European countries, in your opinion, could you think in an *application* of the above estimated models from the perspective of the French Ministry of Education?

10 The OLS estimation for a model that relates annual household expenditures in thousand Euros (G_i) with annual household disposable income in thousand Euros (I_i) and number of individuals within the household (N_i) is given by the following regression line ($n = 38$ households):

$$\widehat{G}_i = 2.24 + 0.16I_i + 1.45N_i \qquad R^2 = 0.45$$

$$(2.666)\ \ (0.0345)\ \ (0.5253)$$

a- *Test* the individual significance of each explanatory variable at 5% significance level.
b- *Test* the overall significance of the model at 5% significance level.
c- *Test* if the coefficient associated to N_i equals to one at 5% significance level.
d- *Interpret* the value of the determination coefficient. What would you change in the above specification in order to increase the explanatory power of the model?

11 An econometric study for the period 1960-2004 relates production costs in USA (*y*) and time (*x*) such that *t=1* (1960), *t=2* (1962), and ... *t=23* (2004). The following exponential model is obtained:

$$\widehat{\log(y_t)} = 95.3 + 0.0253t$$

$$(4.15)\ \ (0.008)$$

a- *Interpret* the estimated slope coefficient.
b- *Test* the null hypothesis that the true value for the slope coefficient is zero at a 5% significance level.
c- *Test* the above but at a 1% significance level. Why this second hypothesis test is more informative than the first one?

12 Are rent rates influenced by the student population in a college town? Let *rent* be the average monthly rent paid on rental units in a college town. Let *pop* denote the total city population, *avginc* the average city income, and *pctstu* the student population as a percent of the total population. We get the following estimation results:

$$\widehat{\log(rent)}_t = -0.043 + 0.066\log(pop)_t + 0.507\log(avginc)_t + 0.0056pctstu_t$$

$$(0.844)\qquad (0.039)\qquad\qquad (0.081)\qquad\qquad (0.0056)$$

$$R^2 = 0.458 \qquad T = 64$$

a- What signs do you expect for the beta parameters? Why?
b- What is wrong with the statement: A 10% increase in population is associated with about a 6.6% increase in rent?
c- Is *pop* an individual significant explanatory variable at 1% significance level?
d- *Interpret* the coefficient associated to *pctstu* variable. Does it have a significant effect on rent?
e- *Test* the overall significance of the above regression model at 5% significance level.

13 Consider the following estimated model:

$$\widehat{Y}_i = 2.613 + 0.30X_{1i} - 0.090X_{1i}^2 \qquad R^2 = 0.1484 \quad n = 32$$

$$(0.429) \quad (0.14) \qquad (0.037)$$

Test whether we should keep the quadratic term in the model at 1% significance level.

14 We are interested in an equation to explain a CEO wage as a function of the firm´s annual sales, the firm´s bond yield (*roe*, in percentage) and the firm´s equity value *(ros*, in percentage):

$$\log(salary_i) = \beta_0 + \beta_1 \log(sales_i) + \beta_2 roe_i + \beta_3 ros_i + u_i$$

a- *Specify*, in terms of the model parameters, the null hypothesis that, once that *sales* and *roe* are accounted for, *ros* does not influence the CEO wage. As alternative hypothesis, consider that, other things equal, a higher equity value tends to increase the CEO wage.

Consider now the following OLS results:

$$\widehat{\log(salary)}_i = 4.32 + 0.280\log(sales)_i + 0.0174 roe_i + 0.00024 ros_i$$

$$(0.32) \qquad (0.035) \qquad (0.0041) \qquad (0.00054)$$

$$R^2 = 0.283 \qquad n = 209$$

b- In what predicted percentage would the wage increase if *ros* increased by 50 points?
c- *Test*, at the 5% significance level, the null that *ros* has no effect on salary, against the alternative, that it has a positive effect.
d- Would you include *ros* in the final model to explain the CEO wage as a function of the firm performance? *Justify*.

15 Using a dataset for 46 states of the United States in 1992, the following estimated regression line was obtained:

$$\widehat{logC_i} = 4.30 - 1.34logP_i + 0.17logY_i \qquad R^2 = 0.37$$

$$(0.91) \quad (0.32) \qquad (0.20)$$

Such that:

C_i denotes cigarettes consumption (number of packets per year).

P_i measures price per packet (dollars).

Y_i denotes average annual income in state *i* (thousand dollars)

- **a-** *Find* the elasticity of cigarettes consumption respect to price. Is it statistically significant at 1% significance level? If it is statistically significant, is it statistically different to -1 at 1% significance level?
- **b-** *Find* the elasticity of cigarettes consumption respect to income. Is it statistically significant at 1% significance level? If it is not statistically significant, in your opinion, explain why.
- **c-** *Find* the value for the determination coefficient. *Interpret* its value and *test* the overall significance of the model at 1% significance level.

16 The goal of this exercise is to test the rationality of assessments of housing prices. We use a model that relates the assessment of the house with its price for a sample of 88 houses.

a- In the SLRM:

$$price_i = \beta_0 + \beta_1 assess_i + u_i$$

the assessment is rational if $\beta_0 = 0$ and $\beta_1 = 0$. The estimated equation is:

$$\widehat{price} = -14.47 + 0.976assess_i \qquad R^2 = 0.82 \quad SSR = 165{,}644.$$

$$(16.27) \qquad (0.049)$$

First, *test* whether assess is a significant variable at 5% significance level. Then, *test* $H_0: \beta_1 = 1$. What do you conclude?

- **c-** Now *test* whether the addition of new variables in the model below is a significant improvement respect the first model:

$$price_i = \beta_0 + \beta_1 assess_i + \beta_2 lotsise_i + \beta_3 sqrf_i + \beta_4 bdrms_i + u_i$$

Knowing that the determination coefficient of this model using the 88 houses is 0.829.

17 For a sample of 506 communities in the Boston area, we estimate a model relating median housing prices (*price*) in the community with two housing characteristics: *dist* is a weighted distance of the community from five employment centres, in miles and *rooms* is the average number of rooms in house in the community:

$$\widehat{\log(rent)}_i = 15.87 + 0.355 rooms_i - 0.22 \log(dist_i)$$
$$(0.342) \qquad (0.020) \qquad (0.055)$$
$$R^2 = 0.399 \quad n = 506 \quad SSR = 11.1$$

We try to improve the above specification by introducing two new independent factors related to community characteristics: *nox* is the amount of nitrous oxide in the air, in parts per million and *stratio* is the average student-teacher ratio of schools in the community:

$$\widehat{\log(rent)}_i = 11.8 + 0.25 rooms_i - 0.13 \log(dist_i) - 0.95 \log(nox_i) - 0.052 stratio_i$$
$$(0.32) \quad (0.019) \qquad (0.043) \qquad (0.117) \qquad (0.006)$$
$$R^2 = 0.506 \quad n = 506 \quad SSR = 4.88$$

a- Which is the most accurate model in terms of goodness-of-fit?
b- *Test* the joint significance of the two additional explanatory variables that are included in the second model at 1% significance level.

18 We estimate a model aiming to study the annual salary, measured in thousand dollars (1980-2007 time period) as a function of labor experience and education level, both of them measured in years. The estimation results are the following:

$$\widehat{w_t} = 100.25 + 0.87 le_t + 1.85 edu_t \qquad R^2 = 0.95$$
$$(2.75) \quad (0.025) \quad (0.075)$$

a- Are the signs of the coefficients consistent with theory? *Explain*.
b- *Interpret* the explanatory power of the model.
c- *Test* the statistical significance of the model both individually and globally.

19 We have the following equation representing the behavior of salaries in the British economy for the time period 1950-1969:

$$\widehat{w_t} = 1.073 + 0.288v_t + 0.116x_t + 0.054m_t + 0.056m_{t-1} \qquad R^2 = 0.934$$

$$(0.797) \quad (0.812) \quad (0.011) \quad (0.022) \quad (0.018)$$

Where:

w_t: Salary per employee (thousand pounds).

v_t: Unemployment rate (percentage).

x_t: GDP per head (thousand pounds).

m_t: Import prices (,00 pounds per imported unit).

m_{t-1}: Import prices lagged one period.

- **a-** *Interpret* the estimated equation.
- **b-** *Find* the explanatory variables that can be eliminated from the equation. Why?
- **c-** *Test* the global significance of the model at 1% significance level.

20 A company in the railway transportation industry is analyzing the factors affecting company´s income levels. Using a sample of 17 years, the following estimated regression model is obtained:

$$\hat{Y} = -2759.26 + 1.525N - 1.856C - 0.672L + 2.753NC$$

$$(2645.4) \quad (0.402) \quad (0.439) \quad (0.369) \quad (0.868)$$

$$R^2 = 0.879 \qquad SSR = 6.09$$

Where Y denotes annual income levels (thousand pounds), N measures the number of trains belonging to the company in each year, C is annual electricity consumption (thousand pounds), L denotes annual labor costs (thousand pounds) and NC is total number of clients in each year.

- **a-** *Test* the individual significance of each explanatory variable at 5% significance level.
- **b-** *Test* the overall significance of the model at 5% significance level.
- **c-** *Test* if the coefficient associated to N equals to one at 5% significance level.
- **d-** *Interpret* the values of the determination coefficient and SSR.

21 We have the following regression model:

$$logy_t = \beta_0 + \beta_1 logx_{1t} + \beta_2 logx_{2t} + \beta_3 logx_{3t} + u_t$$

a- *Analytically show* that imposing the linear restriction $\beta_1 = -\beta_3$, the model can be rewritten as:

$$logy_t = \beta_0 + \beta_2 logx_{2t} + \beta_3 logz_t + u_t$$

Knowing that: $z_t = \frac{x_{3t}}{x_{1t}}$

We estimate both models using 50 observations such that:

$$\widehat{logy_t} = 3.45 - 0.45 logx_{1t} + 0.28 logx_{2t} + 0.55 logx_{3t}$$

(0.18) (0.81) (0.04) (0.02)

$$SSR = 0.186 \qquad R^2 = 0.47$$

And

$$\widehat{logy_t} = 0.29 + 0.87 logx_{2t} + 0.39 logz_t$$

(0.17) (0.01) (0.05)

$$SSR = 0.204 \qquad R^2 = 0.8$$

b- Statistically *validate* the linear restriction at 1% significance level.

22 Consider the linear regression model:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_4 x_{4i} + \beta_5 x_{5i} + u_i$$

Explain how you would test the following hypothesis:

a- $\beta_1 = 0$
b- $\beta_4 = \beta_5$
c- $\beta_3 = \beta_4 = \beta_5 = 0$

23 We have the following theoretical regression model:

$$y_t = \alpha + \beta x_t + \gamma z_t + \delta s_t + u_t$$

We obtain the following estimated model through OLS using 10 observations:

$$\widehat{y_t} = 4.1 + 2x_t + 0.4z_t + 0.35s_t \qquad R^2 = 0.79 \quad SSR = 2$$

A new variable *m* is built such that:

$$m_t = z_t + s_t$$

And a new theoretical model is defined:

$$y_t = \bar{\alpha} + \bar{\beta} x_t + \bar{\varphi} m_t + u_t$$

We estimate the above model such that:

$$\widehat{y_t} = 4.0 + 1.8x_t + 0.47m_t \qquad R^2 = 0.77 \quad SSR = 4$$

Test the null hypothesis that the coefficients for z_t and s_t are the same at 5% significance level. What about at 1% significance level?

24 A marketing consultancy firm is investigating the behavior of sales in the pharmacy industry. Using a sample of 75 companies within the industry, the following two regression models are obtained:

$$\widehat{y_i} = \underset{(7.089)}{22.163} + \underset{(0.0971)}{0.363x_{1i}} \qquad R^2 = 0.424 \quad SSR = 78$$

$$\widehat{y_i} = \underset{(9.986)}{7.059} + \underset{(0.3699)}{1.0847x_{1i}} - \underset{(0.0019)}{0.004x_{1i}^2} - \underset{(0.111)}{0.245x_{2i}} \qquad R^2 = 0.567 \quad SSR = 47$$

Such that y_i denotes sales (thousand Euros), the first explanatory variable measures marketing expenditures (thousand Euros) and the second explanatory variable denotes production costs (thousand Euros).

a- *Interpret* both estimated models.
b- Which is the most accurate model in terms of goodness-of-fit?
c- *Test* the significance of the two additional explanatory variables that are included in the second model at 1% significance level.

25 We have the following estimated regression models for $t = 1, 2, \dots 20$:

$$\widehat{y_t} = 14.1 + 0.6x_{1t} + 0.7x_{2t} \qquad R^2 = 0.67 \qquad SSR = 10$$

(2,1) (1,2) (0,1)

$$\widehat{y_t} = 10.4 + 0.4x_{1t} + 0.65x_{2t} + 0.4x_{3t} + 0.9x_{4t} \qquad R^2 = 0.84 \qquad SSR = 2$$

(2.0) (0.1) (0.14) (0.1) (0.02)

a- Could we *compare* both models in terms of their determination coefficients? Why?
b- Which is the most accurate model in terms of goodness-of-fit?
c- *Test* the joint significance of the two additional explanatory variables that are included in the second model at 1% significance level.

26 Observe the equations below:

$$\log(salary) = \beta_0 + \beta_1 years + \beta_2 gamesyr + \beta_3 bavg + \beta_4 hrunsyr + \beta_4 rbisyr + u$$

$$\log(salary) = \beta_0 + \beta_1 years + \beta_2 gamesyr + u$$

Where: Salary (major league baseball players salary), years (years in the league), gamesyr (average games played by year), bavg (career batting average), hrunsyr (home runs per year) and rbisyr (runs batted in per year).

Having a sample of 352 players, we estimate both models and obtain a SSR for the first model of 183.186 and 198.311 for the second one. Knowing that the R-squared of the first model is 0.6278 and for the second one 0.5971:

a- Which model do you prefer in terms of explanatory power? *Explain*.
b- Are the three more variables in the first model adding a significant predictive power to the model if compared with the second model? *Explain*.

27 A laboratory collected data about the cost of material used for testing necessary products over a one year period. They want to know if the cost of materials A, B and C have a significant value on the overall cost of testing. Observe the following tables and answer to the questions below:

REGRESSION STATISTICS	
R Squared	0.861831639
Adjusted R Squared	0.723663279
Standard Error	18.44727874
Observations	7
F-statistic	6.237546965

REGRESSION RESULTS

	Coefficients	*Stand Error*	*T Stat*	*P value*
Intercept	2921.794805	1189.334796	2.456663013	0.091137493
A	-5.647542515	5.750644311	-0.982071262	0.398482345
B	4.037563072	5.180492629	0.77937821	0.492589441
C	-20.5971781	5.573745294	-3.695392776	0.034387629

a- *Specify* the MLR equation.
b- *Determine* and *interpret* the determination coefficient.
c- Using a significance level of 10%, *analyse* the global significance of the model.
d- Which of the three coefficients can be considered as the most efficient? Why?
e- Which regressor(s) should we keep in our equation? Why?

28 We have information about mortality rates (MORT=total mortality rate per 100,000 population) in a specific year for 51 States of the United States combined with information about potential determinants: INCC (per capita income by State in Dollars), POV (proportion of families living below the poverty line), EDU (proportion of population completing 4 years of high school), TOBC (per capita consumption of cigarettes by State) and AGED (proportion of population over the age of 65). Estimation results are presented in the following table:

OLS Estimation Results

Variable	Model 1 coefficients	Model 2 coefficients	Model 3 coefficients
Constant	194.747	531.608	-9.231
	(53.915)	(94.409)	(176.795)
Aged	5,546.56	5,024.38	5,311.4
	(445.727)	(358.218)	(334.415)
Incc		0.014	0.015
		(0.0038)	(0.0037)
Edu		-682.591	-285.715
		(114.812)	(152.926)
Pov			854.178
			(302.345)
Tobc			0.989
			(0.342)
n	51	51	51
Adjusted R squared	0.759	0.856	0.884
SSR	228,770.3	128,260.1	99,303.73

a- *Interpret* the slope coefficient in Model 1 and validate it at 1% significance level.

b- *Validate* the individual and global significance in Model 2 at 1% significance level?

c- *Comment* on the effect of INCC on MORT in the second model. Why do you think is a positive and significant effect?

d- In Model 3 we add two new explanatory variables: POV and TOBC. *Test* whether this inclusion helps to improve the quality of the model at 1% significance level. Is model 3 the best in terms of goodness-of-fit?

e- Are the effects of these two new variables the expected ones? Are they individually significant at 1% significance level?

f- What about the individual significance of EDU in model 3 if compared with model 2? Why?

29 We have information about families below poverty level (POVRATE=percentage of families with income below the poverty level) in a specific year for 58 counties in California combined with information about potential determinants: UNEMP (percentage of unemployment rate), FAMSIZE (persons per household), EDU (percent that completed four years of college or higher), URBAN (percentage of urban population). Estimation results are presented in the following table:

OLS Estimation Results

Variable	Model 1 coefficients	Model 2 coefficients	Model 3 coefficients
Constant	2.637	1.906	4.309
	(0.987)	(4.292)	(4.535)
Unemp	0.731	0.721	0.424
	(0.092)	(0.106)	(0.166)
Famsize		0.305	2.388
		(1.742)	(1.871)
Edu			-0.177
			(0.081)
Urban			-0.051
			(0.022)
n	58	58	58
Adjusted R squared	0.518	0.510	0.548
SSR	421.692	421.457	374.675

a- *Interpret* the slope coefficient in Model 1 and validate it at 1% significance level.

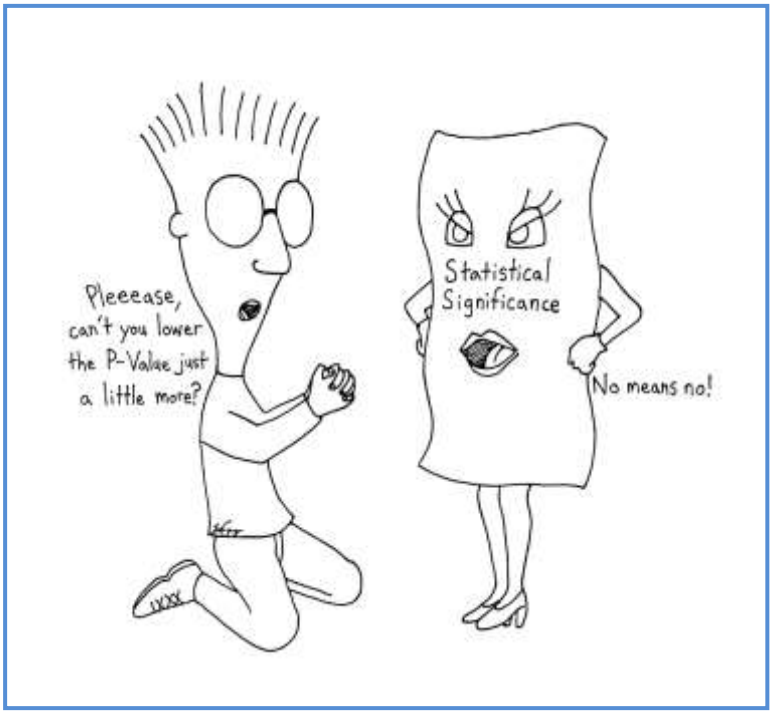
b- *Validate* the individual and global significance in Model 2 at 1% significance level?

c- *Comment* on the effect of FAMSIZE on POVRATE in the second model. Why do you think is a positive and insignificant effect? Does this effect affect the explanatory power of model 2 if compared with model 1? Why?

d- In Model 3 we add two new explanatory variables: EDU and URBAN. *Test* whether this inclusion helps to improve the quality of the model at 5% significance level. Is model 3 the best in terms of goodness-of-fit?

e- Are the effects of these two new variables the expected ones? Are they individually significant at 5% significance level?

f- What about the individual significance at 1% significance level of UNEMP in model 3 if compared with model 2? *Explain*.


Pleeease, can't you lower the P-Value just a little more?
Statistical Significance
No means no!

PS4

Categorical Analysis

(Dummy Variables)

COURSE CONTENT

-**Chapter 6**: Categorical Variables
-Dummy Variables.
-Structural Break.

How many econometricians does it take to change a light bulb?
Eight. One to screw it and seven to hold everything else constant.

1 We have information about the average annual salary (dollars) for teachers in public secondary schools in 45 states in the USA. Using this information the following model is estimated:

$$\widehat{y_i} = 28{,}694.918 - 2{,}954.127D_{1i} - 3{,}112.194D_{2i} - 2.34x_i \qquad R^2 = 0.4977$$

$$(3{,}262.521) \qquad (1{,}862.576) \qquad (1{,}112.873) \qquad (0.359)$$

Such that x_i is expenditures in public secondary schools per pupil (dollars), D_{1i} is a dummy variable being 1 if the state is a North-eastern or North central state and D_{2i} is a dummy variable being 1 if the state is a Southern state.

Interpret this estimated regression model and *calculate* the appropriate tests to validate the model at 1% significance level.

2 Suppose you have survey data on wages, education, professional experience and gender. Additionally, you have answers to the following question: how many times have you smoked marihuana in the last month?

- **a-** *Write down* an equation that allows us to estimate the effect of marihuana consumption on wages, considering the effect of other factors. The objective is to be able to make statements of this short: "Increasing the consumption of marihuana in %, would change on average wages on %"
- **b-** *Specify a model* that allows us to test whether the consumption of drugs has different effects on males´ wages and females´ wages. How would you *test* for this difference to be non-existent?
- **c-** Assume that marihuana consumption is measured by dividing people into 4 categories: no consumer, occasional consumer (1 to 5 times a month), moderate consumption (6 to 10 times a month) and regular consumer (more than 10 times a month). *Write down* a model that allows us to estimate the effects of consuming marihuana on wages.

3 We are analyzing quarterly ice-cream consumption during ten years and estimate the following regression model:

$$\widehat{\log(c_t)} = 4.27 - 0.33\log(p_t) - 0.48S_{1,t} - 0.12S_{2,t} - 0.25S_{3,t}$$

$$(2.33) \qquad (0.11) \qquad (0.08) \qquad (0.02) \qquad (0.06)$$

Where the dependent variable is ice-cream consumption, the quantitative explanatory variable is ice-cream prices and three seasonal dummies variables with the third quarter of the year as the reference category.

a- *Explain* why this model is introducing those seasonal components.
b- *Interpret* the above estimation results.
c- Is ice-cream consumption in the first quarter significantly different to ice-cream consumption in the third quarter of the year?

4 To test the effectiveness of a job-training program on the subsequent wages of workers, we specify the model

$$log(wage) = \beta_0 + \delta_0 train + \beta_1 educ + \beta_2 exper + u$$

where $train$ is a binary variable equal to unity if a worker participated in the program. Think of the error term u as containing unobserved worker ability. If less able workers have a greater chance of being selected for the program, and you use an OLS analysis, what can you say about the likely bias in the OLS estimator of δ_0?

5 Using the data of eight firms, a regression model was estimated to analyze the relationship between investment in thousand Euros (y_i) and production growth rate in % (x_i):

$$\widehat{y_i} = 3.841 - 0.0812x_i \qquad R^2 = 0.466 \quad SSR = 39.21 \quad n = 8$$
$$(2.12) \quad (0.038)$$

Additionally, two different regressions are estimated. The first one only takes into account European firms within the original sample:

$$\widehat{y_i} = -0.372 + 0.108x_i \qquad R^2 = 0.976 \quad SSR = 0.949 \quad n = 4$$
$$(0.782) \quad (0.012)$$

And the second one only takes into American firms within the original sample:

$$\widehat{y_i} = 1.259 + 0.171x_i \qquad R^2 = 0.933 \quad SSR = 1.407 \quad n = 4$$
$$(1.43) \quad (0.032)$$

Find whether making the distinction between European and American firms helps to understand better the behavior of investment and *interpret* your results.

6 We have the following estimated regression model that explains the behavior of profits:

$$\widehat{profit}_i = 215 - 25pc_i + 14sector_i - 22home_i - 50south_i + 45urban_i$$

Such that *profit* is monthly profits in thousand dollars, *pc* is monthly production costs in thousand dollars, *sector* is a sector dummy variable with a value of 1 if the sampled company belongs to the tertiary sector, *home* is a nationality dummy variable equals to 1 if the sampled company is a national company, *south* is a dummy variable with a value of 1 if the sampled company is located in the south of the country and *urban* is a dummy variable with a value of 1 if the sampled company is located in an urban area.

a- *Find* the predicted average profit for a foreign manufacturing company that is located in a rural area at the north of the country independently of *pc*.

b- Taking two companies of our sample with the same production costs, *find* the estimated average difference in their monthly profit if we know that one of them is a national manufacturing company located in a southern city of the country and the other one is a foreign services company located in a northern city of the country.

7 The Chinese Ministry of Education is performing an analysis about the recurrent expenditures in secondary schools in the city of Shanghai. Using a sample of 74 secondary schools, the following estimation results are obtained:

$$\hat{y}_i = 23{,}953.3 + 339.0432x_i \qquad R^2 = 0.394 \qquad SSR = 8.916 \qquad n = 74$$

$$(27{,}167.96) \qquad (49.551)$$

Where the dependent variable is recurrent expenditures and the explanatory variable is number of students in each secondary school.

However, it is believed that the type of school affects completely the behavior of recurrent expenditures and two different regression models are estimated distinguishing between regular secondary schools (40 observations) and occupational secondary schools (34 observations) such that:

$$\hat{y}_i = 47{,}974.07 + 436.7769x_i \qquad R^2 = 0.634 \qquad SSR = 3.4895 \qquad n = 34$$

$$(33.879{,}03) \qquad (58{,}621)$$

$$\hat{y}_i = 51{,}475.25 + 152.2982x_i \qquad R^2 = 0.263 \qquad SSR = 1.215 \qquad n = 40$$

$$(21{,}599.14) \qquad (41.398)$$

Is there a significant difference in the behaviour of recurrent expenditures between the two types of schools? *Interpret* your result at 1% significance level.

8 Male babies tend to weigh more than female babies do. If we define a dummy variable $M = 1$ for male babies and $M = 0$ for female babies, the regression that explains baby´s weigh in grams (Y) as a function of the number of cigarettes per day smoked by the mother (x) and the dummy variable M is the following (sample size $n = 964$):

$$\widehat{y_i} = 3{,}354 + 119M_i - 7x_i \qquad R^2 = 0.033$$

$$(20) \qquad (26) \qquad (2.1)$$

Interpret this estimated regression model and *calculate* the appropriate tests to validate the model.

9 Using the data of the previous exercise, a new regression model is estimated such that (strategy 1):

$$\widehat{y_i} = 3{,}418 - 7.2x_i \qquad R^2 = 0.012 \quad SSR = 158.6 \quad n = 964$$

$$(143) \qquad (2.1)$$

Strategy 2 consists on performing two different regressions. The first one only takes into account babies that are first-born (their mothers do not have previous births):

$$\widehat{y_i} = 3{,}363 - 4.0x_i \qquad R^2 = 0.004 \quad SSR = 91.2 \quad n = 584$$

$$(18) \qquad (2.8)$$

And the second one only takes into account babies that are not first-born (their mothers have previous births):

$$\widehat{y_i} = 3{,}506 - 12.1x_i \qquad R^2 = 0.039 \quad SSR = 63.5 \quad n = 380$$

$$(23) \qquad (3.1)$$

Find the most appropriate strategy to better understand the behaviour of the dependent variable (structural break?) and *interpret* your results.

10 We have the following information about the behavior of consumption:

$$C_t = 500 + 0.9Income_t + 0.3Assets_t \;\; t = 1940 - 2003 \;\; SST = 1{,}000 \;\; SSE = 300$$

$$C_t = 400 + 0.8Income_t + 0.2Assets_t \;\; t = 1940 - 1979 \;\; SST = 1{,}250 \;\; SSE = 900$$

$$C_t = 600 + 0.95Income_t + 0.35Assets_t \; t = 1980 - 2003 \;\; SST = 1{,}200 \;\; SSE = 950$$

Test the null hypothesis that the regression coefficients are the same in the two sampled time sub-periods knowing that T=64 and *interpret* your result.

11 We have the following estimated regression model that explains the behavior of salaries:

$$wage_i = 300 + 25edu_i + 37exp_i + 14male_i - 22black_i - 50south_i + 45urban_i$$

Such that *wage* is the weekly salary in dollars, *edu* is years of education, *exp* is years of professional experience, *male* is a gender dummy variable with a value of 1 if the sampled individual is a male, *black* is a race dummy variable with a value of 1 if the sampled individual is black, *south* is a dummy variable with a value of 1 if the sampled individual lives in the south of the country and *urban* is a dummy variable with a value of 1 if the sampled individual lives in an urban area.

a- Which would be the *predicted average salary* for a black female that lives in a rural area at the north of the country independently from *edu* and *exp*?

b- Taking two males from our sample with the same years of education and the same years of professional experience, which would be the *estimated average difference in their weekly salary* if we know that one of them is black and lives in a southern city of the country and the other one is white and lives in a northern city of the country?

12 We have the following regression model:

$$y_t = \beta_0 + \beta_1 Time_t + \sum_{i=1}^{3} \gamma_i D_{it} + u_t$$

Such that: $\widehat{\beta_0} = 0.2$; $\widehat{\beta_1} = 0.01$; $\widehat{\gamma_1} = 0.5$; $\widehat{\gamma_2} = 0.8$; $\widehat{\gamma_3} = 0.2$.

Time subscript at the end of the sampled period is $Time_t = T = 200$ and the last observation is the second quarter in 2010. Which are the *predicted values* of the dependent variable for the third quarter in 2010 and for the fourth quarter in 2010?

13 The variable *s* denotes the time invested in sleeping at night (minutes per week), *w* is the time invested in working (minutes per week), *e* (level of education) and *a* (age of the individual) are measured in years and *m* is a dummy variable with a value of 1 if the individual is a male. Sample size is 706 individuals.

$$\widehat{s_i} = 3{,}840.83 + 0.163w_i - 11.7e_i - 8.7a_i - 0.128a_i^2 + 87.75m_i \qquad R^2 = 0.117$$

(235.22) (0.018) (5.86) (11.21) (0.134) (29.33)

a- *Interpret* this estimated regression model.
b- Are the effects of the variable age statistically significant at 1% significance level?
c- Is there evidence to say that males sleep more than females?
d- Is the above model globally significant at 5% significance level?

14 The following model was fitted to data on 50 states:

$$Y = 13{,}472 + 547X_1 + 5.48X_2 + 493X_3 + 32.7X_4 + 5{,}793X_5 - 3{,}100X_6 \qquad R^2 = 0.54$$

(7,123.22) (124.3) (1.858) (208.9) (234) (2,897) (1,761)

Where:

Y= annual salary of the attorney general of the state, in thousand dollars.

X1= average annual salary of lawyers, in thousand dollars.

X2= number of bills enacted in previous legislative session.

X3= number of due process reviews by state courts that resulted in overturn of legislations in previous 40 years.

X4= length of term of the attorney general of the state.

X5= dummy variable taking a value 1 if justices of the state supreme court can be removed from office by the governor, judicial review board or majority vote of the supreme court and 0 otherwise.

X6= dummy variable taking value of 1 if Supreme Court justices are elected on partisan ballots and 0 otherwise.

a- *Interpret* the coefficients associated to the two dummy variables.
b- *Test*, at 5% significance level, whether the dummy variables are individually significant.

15 In a survey of 27 undergraduates at the University of Illinois the accompanying results were obtained with grade point averages(Y), the number of hours per week spent studying(x_1), the average number of hours spent preparing for tests(x_2), the number of hours per week spent in bars(x_3), whether students take notes or mark highlights when reading texts(x_4=1 if yes and, 0 if no), and the average number of credit hours taken per semester(x_5). The following regression was estimated by least squares:

$$\hat{Y} = 1.9968 + 0.0099x_1 + 0.0763x_2 - 0.1365x_3 + 0.0636x_4 + 0.1379x_5$$

$$R^2 = 0.2646$$

a- *Interpret* the coefficient of determination and use it to *test* the null hypothesis that, taken as a group, the five independent variables do not linearly influence the dependent variable.

b- *Interpret* the coefficients associated to x_3 and x_4.

16 We have the following estimated regression model that explains the behavior of salaries:

$$\log(wage_i) = 300 + 0.05edu_i + 0.07exp_i + 0.14male_i - 0.12black_i - 0.002south + 0.06urban_i + 0.08married$$

Such that *wage* is the weekly salary in dollars, *edu* is years of education, *exp* is years of professional experience, *male* is a gender dummy variable with a value of 1 if the sampled individual is a male, *black* is a race dummy variable with a value of 1 if the sampled individual is black, *south* is a dummy variable with a value of 1 if the sampled individual lives in the south of the country, *urban* is a dummy variable with a value of 1 if the sampled individual lives in an urban area and *married* is a dummy variable with a value of 1 if the sampled individual is a married individual.

a- Which would be the *predicted average salary difference* for a black single female that lives in a urban area at the north of the country respect the reference category and independently of *edu* and *exp*?

b- Taking two males from our sample with the same years of education and the same years of professional experience, which would be the *estimated average difference in their weekly salary* if we know that one of them is black, single and lives in a southern city of the country and the other one is white, married and lives in a northern city of the country?

17 Using the data of the mid-term exam results, the Econometrics teacher estimates the following regression model (strategy 1):

$$\widehat{exam}_i = -1.338 + 0.648part_i + 0.284ps_i \quad R^2 = 0.433 \quad SSR = 465.1 \quad n = 171$$

Strategy 2 consists on performing two different regressions. The first one only takes into account students that are foreign individuals:

$$\widehat{exam}_i = -0.485 + 0.521part_i + 0.0.326ps_i \quad R^2 = 0.432 \quad SSR = 175.4 \quad n = 73$$

And the second one only takes into account Spanish students:

$$\widehat{exam}_i = -1.961 + 0.794part_i + 0.198ps_i \quad R^2 = 0.442 \quad SSR = 266.1 \quad n = 98$$

a- *Find* the most appropriate strategy to better understand the behavior of the mid-term exam grades and interpret your results.

b- *Specify* a model in which you could test directly if there is a difference in the performance of the mid-term exam depending on whether the student is Spanish or foreigner, independently of other factors.

18 Using data in students' GPAs, the following equation is estimated:

$$\widehat{sat} = \underset{(6.29)}{1{,}028} + \underset{(3.83)}{19.30hsize} - \underset{(4.29)}{45.09female} - \underset{(12.71)}{169.81black} + \underset{(18.15)}{62.31female \cdot black}$$

$$n = 4{,}137 \qquad R^2 = 0.0858$$

The variable *sat* is the combined SAT score, *hsize* is the size of the student's high school graduating class, in hundreds, *female* is a gender dummy variable and *black* is a race dummy variable equal to 1 for blacks and 0 otherwise.

Note: summary statistics for SAT score: mean = 1,030; min = 540; max = 1,504

a- Holding *hsize* fixed, what is the estimated difference in SAT score between non-black females and non-black males? How statistically significant is this estimated difference?

b- Holding *hsize* fixed, what is the estimated difference in SAT score between non-black males and black males? Test the null hypothesis that there is no difference between their scores, against the alternative that there is a difference.

c- Holding *hsize* fixed, what is the estimated difference in SAT score between black females and non-black females? What would you need to do to test whether the difference is statistically significant?

19 We have the following estimated regression model:

$$\widehat{\log(w_i)} = 1.6 - 0.32fem_i + 0.16\log(size_i) + 0.05edu_i \quad R^2 = 0.31 \quad SSR = 359$$

(0.02) (0.02) (0.02) (0.002)

Such that w_i measures salaries in thousand dollars for each of our 2,000 sampled individuals, edu_i measures education in years, fem_i is a gender dummy variable with a value of 1 if the individual *i* is a female, and *size* is a variable measuring the number of workers working in the company.

a- *Interpret* the above estimation results.
b- Are gender and company size categorical factors statistically significant?

We estimate an alternative model changing the previous specification such that:

$$\widehat{\log(w_i)} = 1.6 - 0.26fem_i + 0.18\log(size_i) + 0.05edu_i - 0.16fem_i * \log(size_i)$$

$$R^2 = 0.32 \qquad SSR = 341$$

c- Do small companies discriminate against women more or less than larger firms? Is the discrimination statistically significant?

20 The following model is regressed using data in quarterly form from 1990 to 2005 (64 observations) for Malaysian stock prices against output knowing that there was an economic crisis in 1997.

$$Y_t = \beta_0 + \beta_1 X_t + U_t$$

The first regression using all the data produced a SSR of 0.56. Then, two regressions were run. The first one on a subsample of the data from 1990-1997, giving a SSR of 0.23. The second one was on the simple from 1998 to 2005, producing a SSR of 0.17. *Test* whether the crisis in 1997 produced a significant shock in the behavior of Malaysian stock prices.

21 We have the following estimated regression model:

$$\hat{y}_i = 186.4 + 2.33x_i - 126D_i - 1.29D_i x_i \qquad R^2 = 0.5055 \quad n = 34$$

(45.67) (0.86) (37.01) (1.02)

Such that y_i measures annual expenditure on beer in dollars for each of our 34 sampled individuals, x_i measures individual annual income in thousand dollars and D_i is a dummy variable with a value of 1 if the individual *i* is a female and 0 if the individual is a male.

a- What will be the *difference* in consumption between a male and a female with the same annual income?
b- *Test* at 1% level the following: there are no differences in beer consumption across gender.
c- *Test* at 5% level the following: there are no differences in the marginal propensity to consume beer respect to income across gender.

22 A regression model was estimated using 350 students to compare performance of students taking a business statistics course either as a standard 14-weekcourse or as an intensive 3-week course:

	X1	X2	X3	X4	X5	X6	X7
Estimated coeff.	1.417	2.162	0.868	1.0845	0.4694	0.0038	0.0484
Std.Error	*0.4568*	*0.3287*	*0.4393*	*0.3766*	*0.0628*	*0.0094*	*0.0776*

Where:

Y= score on a standardised test of understanding of statistics after taking the course.

X1= dummy variable taking the value 1 if the 3-week course was taken and 0 if the 14-week course was taken.

X2= student´s grade point average.

X3= dummy variable taking the value 0 or 1, depending on which of two teachers had taught the course.

X4= dummy variable taking the value 1 if the student is a male and 0 if female.

X5= score on a standardised test of understanding mathematics before taking the course.

X6= number of semester credit hours the student had completed.

X7= student´s age.

Knowing that the value of the determination coefficient is 0.344, answer the following questions:

a- *Interpret* all the beta coefficients.
b- *Test* the individual significance of X1 at 1% significance level.

c- *Test* the overall significance of the model at 5% significance level.

23 We have obtained the following estimated model in a study carried out for 100 multinationals firms:

$$\widehat{E}_i = 2.3 + 0.05T_i - 2.4C_i + 1.9F_i$$

Where E is the number of employees (,00 employees), T has a value of 1 if the company applies the last technological improvements and 0 otherwise, C has a value of 1 if there are competitors located within 50 km distance and 0 otherwise and F has a value of 1 if there is a complementary company located within 50 km distance and 0 otherwise. *Explain* whether the following statements are true or false:

a- A company that applies the last technological improvements employs, on average, 5 employees more than a company without the last technological improvements.

b- For each competitor located within 50 km distance, a company employs, on average, 240 workers less than another company without any competitor located within 50 km distance.

24 We have a housing price model with the following variables: *price* (house prices), *sqrft* (house size), *bdrms* (number of bedrooms) and *colonial* (dummy variable equal to one if the house is of the colonial style. The estimation results are the following (sample size is 88 houses):

$$\widehat{\log(price_i)} = 5.56 + 0.707\log(sqrft_i) + 0.027bdrms_i + 0.054colonial_i$$

$$(0.65) \qquad (0.093) \qquad (0.029) \qquad (0.045)$$

a- *Interpret* this estimated regression model.

b- Is the effect of the variable *bdrms* statistically significant?

c- Is there a significant evidence to say that colonial houses are more expensive than the rest of the houses independently of the rest of the factors?

d- Is the above model globally significant knowing that $R^2 = 0.649$?

25 The following stock price model was regressed using monthly data from 1980m1 to 1989m12:

$$s_t = \beta_0 + \beta_1 y_t + u_t$$

It is believed there is a structural break at 1987m11, following a stock market crash. The regression using all the data produced a SSR of 0.97. Then two further regressions were run from 1980m1 to 1987m11, which produced a SSR of 0.58 and another regression from 1987m12 to 1989m12 produced a SSR of 0.32.

a- Do you think the stock market crash at 1987m11 was statistically significant?

b- Why are structural breaks a problem for financial econometrics? Give examples of some recent structural breaks.

26 We have the following estimated regression model:

$$\widehat{y_i} = 100 - 0.8x_i - 0.3D_i x_i \qquad R^2 = 0.47 \quad n = 55$$

(27) (0.05) (0.01)

Such that y_i measures profits in thousand dollars for each of our 55 sampled companies, x_i measures production costs in thousand dollars and D_i is a dummy variable with a value of 1 if the company *i* is a manufacturing firm and 0 if the company is a services firm.

a- *Interpret* the estimated regression model.
b- Is $D_i x_i$ a significant explanatory variable? Why? *Explain* your answer.
c- *Find* the predicted profits for a manufacturing company with 55.000 dollars as production costs.

27 Let´s consider the following regression model using a sample of annual data from 1970 until 2001 (both included) for the Castilla-León economy:

$$\widehat{y_t} = 134.6 + 10.89x_t + 21.6D_t + 3.91D_t x_t \qquad R^2 = 0.921$$

(56.2) (2.06) (3.99) (0.91)

Such that y_t are annual regional exports, x_t is the annual exchange rate (pts/$) and D_t is a dummy variable equals to 1 if $t \leq 1985$ and equals to 0 if $t > 1985$ (Spain being a European Union member).

a- *Interpret* the above regression model.
b- *Test* the validity of the additive and multiplicative dummy effects in the above model at 1% significance level.
c- *Test* the overall fit of the model at 5% significance level.

28 The following model was estimated to examine the short run interest rate:

$$\hat{y}_t = 5.5 + 0.93x_t - 0.38x_{t-1} + 0.5y_{t-1} - 0.05D_{1t} + 0.08D_{2t} + 0.06D_{3t}$$

Such that x_t is the interest rate for the Treasury bills with a maturity of 90 days and D_{it} are seasonal dummy variables where i corresponds to the first, second and third year quarter respectively.

Interpret the above estimated regression model.

29 The following wage equations have been estimated using data on workers from Vietnam:

$$\widehat{\log(salary)} = \underset{}{1.25} + \underset{(0.35)}{0.15}gender + \underset{(0.03)}{0.02}exp \qquad (0.004)$$

$$\widehat{\log(salary)} = \underset{}{1.55} + \underset{(0.48)}{0.10}gender + \underset{(0.05)}{0.015}exp - \underset{(0.005)}{0.0005}gender * exp \qquad (0.002)$$

Where salary is measured in US dollars and gender is a dummy variable taking the value of 1 if the worker is a male and 0 if the worker is a female, exp measures the years of work experience.

- **a-** *Why* the coefficients associated to gender and experience are lower in the second than in the first model?
- **b-** What is the estimated average *difference* between a man´s salary with 5 years work experience and that of a woman´s with 10 years work experience according to the first model?
- **c-** What is the estimated average *difference* between a man´s salary with 5 years work experience and that of a woman´s with 10 years work experience according to the second model?
- **d-** *Test* that the salary difference between men and women does not depend on experience.

30 To see whether people living in urban areas spend more on fish than people living in rural areas, we get the following estimation results:

OLS Estimation results

Dependent: log(expenditure in fish)					
Explanatory Variable	OLS Coefficient	t-statistic	Degrees of Freedom	Significance level	t-critical
Intercept	6.375		36	0.01	-
log(income)	1.313	5.328	36	0.01	2.719
gender	-0.055	-1.378	36	0.01	-2.719
urban	0.143	10.311	36	0.01	2.719
Sample size (n)	40	F-statistic	Degrees of Freedom	Significance level	F-critical
R-squared	0.750	36	3(n),36(d)	0.01	4.40

Where the dependent variable is expenditure in fish (with log), *income* is disposable income (with log), *gender* is a gender dummy with 1 if male and 0 if female and *urban* is another dummy which takes the value 1 if person lives in an urban area. Please, answer the following three questions:

a- *Interpret* the above estimations results (only the value of the OLS coefficient for each of the explanatory variables).
b- Is the variable *gender* individually significant to explain the behavior of fish expenditures (at 1% significance level)? *Explain*.
c- Is the model globally significant at 1% significance level? *Explain*.

31 A group of researchers in the field of environmental economics has conducted a survey to investigate patterns of apples' consumption, both regular and Eco labeled. The sample contains the responses of 660 individuals. The following information is available:

Variable	Description
regq	Quantity demanded regular apples, lbs
ecoq	Quantity demanded Eco labeled apples, lbs
regp	Price of regular apples, pounds
ecop	Price of Eco labeled apples, pounds
educ	Years of schooling
age	Age in years
hhsize	Household size
faminc	Family income, thousands
male	=1 if the individual is a male

Three different models have been estimated using ecoq as dependent variable**.** The results are presented next. (Standard errors in parenthesis)

	Model 1	**Model 2**	**Model 3**
const	1.965	2.007	1.137
	(0.380)	(0.387)	(0.911)
ecop	-2.926	-2.962	-2.889
	(0.588)	(0.592)	(0.596)
regp	3.029	3.063	3.034
	(0.711)	(0.714)	(0.716)
male		-0.126	-0.101
		(0.221)	(0.227)
educ			0.034
			(0.045)
age			0.0008
			(0.007)
faminc			0.002
			(0.003)
hhsize			0.057
			(0.069)
n	660	660	660
R^2	0.036	0.036	0.040
SSR	4051.05	4049.05	4033.81

Note: Set α equal to 5% when needed.

a- *Interpret* the coefficients on the price variables from Model 1 and *comment* on their signs and magnitudes. Are regular apples and eco-labeled apples substitute goods?

b- *Report* the individual *t*-tests from Model 1. At the individual level, are the price variables statistically significant?

c- Is there a gender difference in the quantity demand for eco-labeled apples? If so, is the difference statistically significant? *Justify* your answer.

d- *Compare* the goodness of fit between Model 1 and Model 2.

e- *Explain* with your own words how would you extend Model 3 to allow a different effect of education on apples' consumption by gender.

f- Model 3 adds the variables faminc, hhsize, educ and age to the regression from part (b). *Test* whether these four variables are jointly significant.

32 Gathering data for Michigan manufacturing firms in 2010, we obtain the following estimation results using a log transformation:

OLS Estimation results

Dependent: log(training per employee)					
Explanatory Variable	OLS Coefficient	t-statistic	Degrees of Freedom	Significance level	t-critical
Intercept	46.67		101	0.01	-
log(sales)	0.987	7.559	101	0.01	2.626
log(employees)	-0.555	-10.378	101	0.01	-2.626
grant	0.125	3.111	101	0.01	2.626
Sample size (n)	105	F-statistic	Degrees of Freedom	Significance level	F-critical
R-squared	0.237	10.456	3(n),101(d)	0.01	3.98

Such that, the dependent variable is hours of training per employee, the variable *sales* represents annual sales, *employees* is the number of employees and *grant* variable is a dummy equals to one if the firm received a job training grant for 2010 and zero otherwise. Please, answer the following three questions:

a- *Interpret* the above estimations results (only the value of the OLS coefficient for each of the explanatory variables).
b- Is the variable *grant* individually significant to explain the dependent variable (at 1% significance level)? *Explain*.
c- Is the model globally significant at 1% significance level? *Explain*.

33 A group of researchers has conducted a survey that contains information on smoking behavior and other variables for a random sample of 807 single adults from the United States. The following information is available:

Variable	Description
cigs	Average number of cigarettes smoked per day
cigprice	State cigarette price, cents per pack
educ	Years of schooling
age	Age in years
income	Annual income, dollars
white	=1 if the individual is white
restaurn	=1 if state restaurant smoking restrictions

Three different models have been estimated using cigs as dependent variable**.** The results are presented next. (*Notes*: Standard errors in parenthesis; l_ stands for natural logarithm)

	Model 1	**Model 2**	**Model 3**
const	-0.983 (8.685)	-3.765 (8.898)	-2.011 (8.964)
cigprice	-0.048 (0.102)	-0.012 (0.103)	-0.005 (0.103)
l_income	1.429 (0.6793)	1.433 (0.679)	1.891 (0.713)
white		0.0009 (1.483)	-0.036 (1.479)
restaurn		-2.961 (1.136)	-2.949 (1.134)
educ			-0.377 (0.168)
age			-0.045 (0.029)
n	807	807	807
R^2	0.005	0.013	0.021
SSR	151052.1	149770.7	148568.0

Note: Set α equal to 5% when needed.

a- *Interpret* the coefficients on the variables from Model 1 and *comment* on their signs and magnitudes. Is the income effect statistically significant?
b- *Interpret* the coefficient on the variable restaurn.
c- Is there a race difference in the quantity demanded for cigarettes? If so, is the difference statistically significant? *Justify* your answer.
d- *Explain* with your own words how you would extend Model 3 to allow a different effect of education on smoking habits by race.
e- Model 3 adds the variables age and educ to the regression from part (b). *Test* whether these two variables are jointly significant.

34 An insurance company finds that the probability of having a home insurance or not can be described by the following linear relationship:

$$\hat{y}_i = 0.002inc_i + 0.004age_i$$

Knowing that *inc* denotes annual income (in thousand Euros) of the individual and *age* the age of the individual (in years):

a- *Find* the probability of having a home insurance for an individual with 400,000 Euros income and being 30 years old.

b- *Find* the increment in the probability of having a home insurance if the individual´s income increases in 20,000 Euros.

35 A researcher investigates the possibility of a family having home ownership or not. He uses the following explanatory variables: (1) household income (in thousand Euros), (2) gender which is a dummy with a 1 if individual is a male and 0 if female, (3) being employed which is a dummy variable with a 1 if the individual is employed and 0 if unemployed and (4) age (years). The following logit model is estimated:

$$\widehat{y_i} = 0.009inc_i + 0.16gdr_i + 0.02work_i + 0.002age_i$$

a- *Find* the probability of having home ownership for a female individual with 80,000 Euros income, being 46 years old and having a job.
b- What is the *difference* in the probability of having home ownership between a female individual and a male individual with the same characteristics.

36 In 1985, neither Florida nor Georgia had laws banning open alcohol containers in vehicle compartments. By 1990, Florida had passed such a law, but Georgia had not.

a- Suppose you can collect random samples of the driving age population in both states, for 1985 and 1990. Let arrest be a binary variable equal to unity if a person was arrested for drunk driving during the year. Without controlling for any factors, *specify* a linear probability model that allows you to test whether the open container law reduced the probability of being arrested for drunk driving. Which coeffcient in your model measures the e_ect of the law?
b- Why might you want to control for other factors in the model? What other factors might you want to include? *Explain* your answer.

37 Suppose that you want to explain the behavior of a binary variable (*approve*) which is equal to one if a mortgage loan to an individual was approved. The key explanatory variable is *white*, a dummy variable equal to one if the applicant was White. The other applicants in the dataset are black and hispanic. To test for discrimination in the mortgage loan market, a linear probability model can be used:

$$approve_i = \beta_0 + \beta_1 white_i + otherfactors_i$$

a- If there is discrimination against minorities, and the appropriate factors have been controlled for, what is the sign of β_1? *Explain* your answer.
b- Regressing approve against white we obtain the following estimation results:

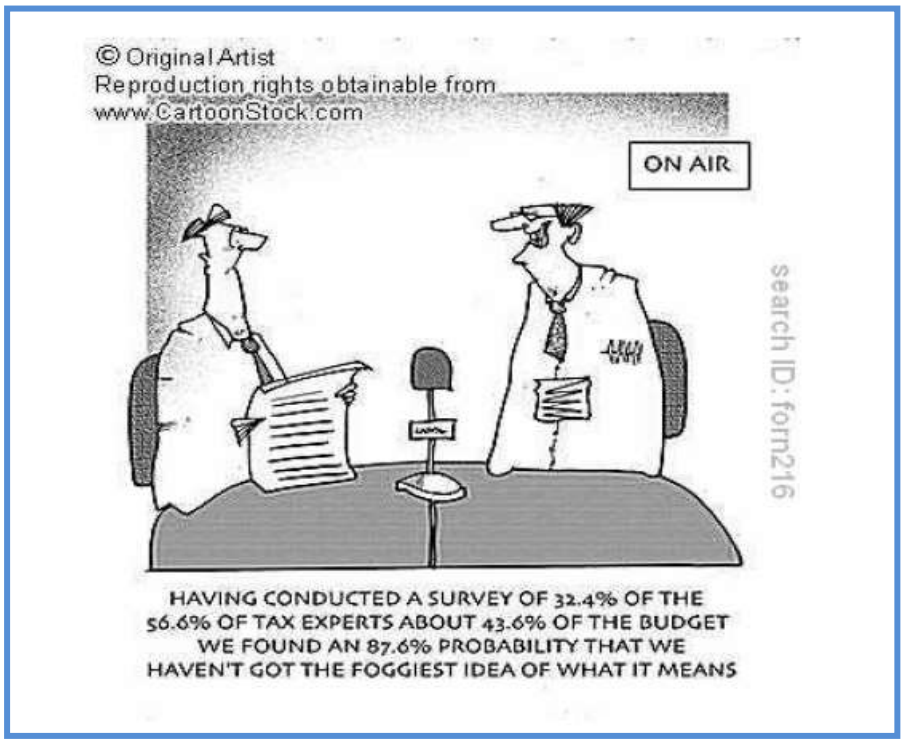
$$\widehat{approve}_i = 0.707 + 0.201white_i$$
$$(0.0182) \qquad (0.0198)$$
$$n = 1{,}989 \qquad R^2 = 0.048$$

Interpret the coefficient on White. Is it statistically significant? Is it practically large?

As controls, we add several explanatory variables such as percentage of total income in housing expenditures, percentage of total income in other obligations, whether the individual is a male, married and if he/she is unemployed. We re-estimate the model obtaining an estimate for $\beta_1 = 0.187$ and with $se = 0.019$.

c- *Interpret* the new beta one coefficient. What happens to the coefficient on white variable? Is there still evidence of discrimination against non-whites? *Explain* your answer.

d- *Justify* whether the following statement is true or false: "all the fitted values of the coefficients for the rest of variables in the second model are strictly between zero and one"



PS5
Estimation
Problems

COURSE CONTENT

-**Chapter 7**: Estimation Problems
-**Chapter 8**: Time Series and Autocorrelation

An econometrician is an expert who will know tomorrow why the things he predicted yesterday did not happen today.

1 *Answer* to the following three questions:

a- The first empirical studies aimed at measuring the impact of class size on education performance were based on data comparing the grades in comprehensive tests achieved by students from different schools and different class sizes. If we aimed at measuring the relationship between class size and academic performance with such data, could we infer that size has a causal effect on performance? *Justify*.

b- The presence of more policemen to fight crime is a matter of controversy. Suppose that we have data for all the capital cities in France about crime incidence per 10,000 inhabitants and number of police units per 10,000 inhabitants. With such data, could we obtain the causal effect of police surveillance on crime incidence? *Explain*.

c- Suppose that there is a positive and strong correlation between the amount of children´s books within a home and the academic performance of the children at that home. Could you say that the number of children´s book at home has a positive causal effect on the academic performance of children at such home. *Justify*.

2 Suppose you are interested in estimating the effect of hours spent in a SAT preparation course (hours) on total SAT score (sat). The population is all college-bound high school seniors for a particular year.

a- Suppose you are given a grant to run a controlled experiment. *Explain* how you would structure the experiment in order to estimate the causal effect of hours on sat.

b- Consider the more realistic case where students choose how much time to in a preparation course, and you can only randomly simple sat and hours from the population. Write the population model as:

$$sat_i = \beta_0 + \beta_1 hours_i + u_i$$

List, at least, two factors contained in the random perturbance term. Are these likely to have positive or negative correlation with hours? *Explain*.

3 The following equation describes the number of hours of television watched per week by a child as a function of his age, his education, his mother´s education, his father´s education and the number of siblings:

$$tvhours^* = \beta_0 + \beta_1 age + \beta_2 age^2 + \beta_3 mothedu + \beta_4 fathedu + \beta_5 sibs + u$$

We suspect the dependent variable contains a certain error of measurement. *Explain* the consequences in your estimation results.

4 We have the following variables:

Y: Food expenditure in USA.

X: Family income.

P: Price index.

Two different regressions are estimated with the following estimation results (standard errors are in brackets and sample size is 500):

Regression	Coefficient for X	Coefficient for P	Adjusted Determination coefficient
Y / P		2.462	0.614
		(0.407)	
Y / X; P	0.112	-0.739	0.978
	(0.003)	(0.114)	

Find and discuss the specifation error the first model is suffering. *Explain* it using the estimation results of the above table.

5 There is an econometric study at IE University, which relates the average grade in Econometrics with the time students employ in different activities during the week. Some students are asked about how many hours they employ in four different activities: study, sleep, work and leisure. Any activity must be included in one of these four categories such that the time spent in the four activities is 168 hours for each student.

The model is the following:

$$AGE = \beta_0 + \beta_1 study + \beta_2 sleep + \beta_3 work + \beta_4 leisure + u$$

a- *Find* the assumption that does not hold in this model and explain why.
b- How would you *rewrite* the model in order to solve the problem?

6 We have estimated a SLRM explaining office rental prices in the city of Madrid (*Y*) with the information contained in distance to the city center (*X*). The following two graphs: Figure 1(Y versus *X*) and Figure 2 (*residuals* versus *fitted values of Y*) are related to the above model.

Figure 1

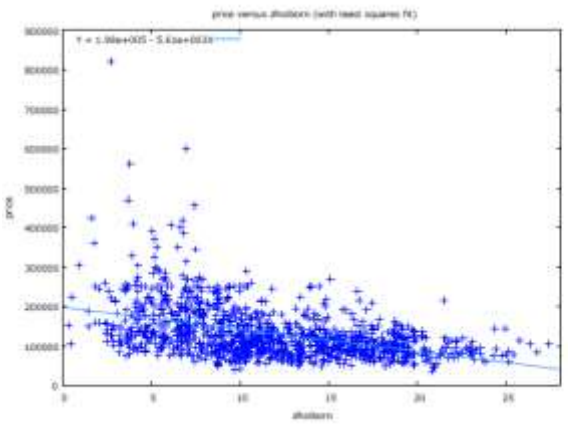
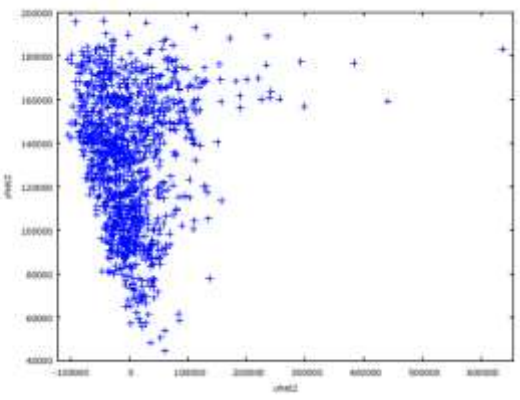


Figure 2



a- *Discus* according to the two graphs if the model may suffer a non-linearity problem
b- *Provide* an economic reason explaining the possible non-linearity in the above relationship.
c- How should Figure 2 be if the relationship between office rental prices and distance was a linear relationship?

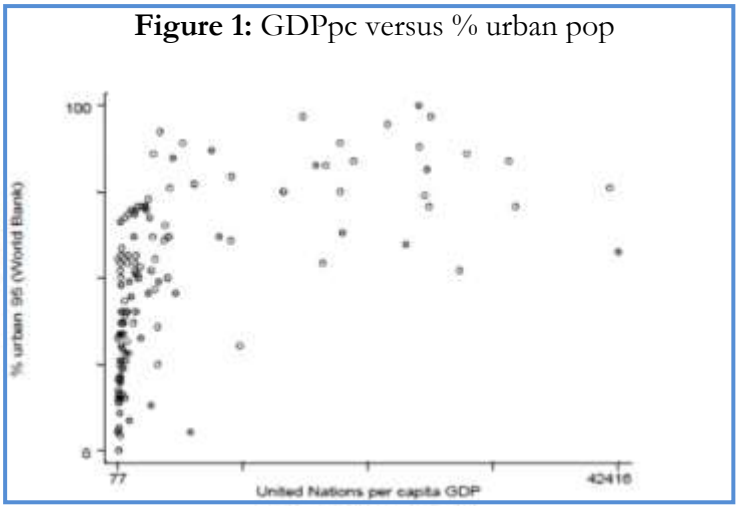
7 The following table shows two different samples with two explanatory variables each of them in order to study the behavior of Y (dependent variable):

		Sample 1		**Sample 2**	
Observation	Y	X1	X2	Z1	Z2
1	1	2	4	2	4
2	4	6	12	6	12
3	2	4	11	4	8

a- Can you *detect* a multicollinearity problem in any of the two samples?
b- If yes, please *explain* the consequences in your OLS estimations in each sample.
c- If yes, please *explain* the strategies that you would use in order to solve the problem in each sample.

8 Consider the regression of country level GDP per capita on percentage urban population in several countries (1995) obtaining a determination coefficient of 0.457 and obtaining the following graph when plotting the data (Figure 1):

Figure 1: GDPpc versus % urban pop



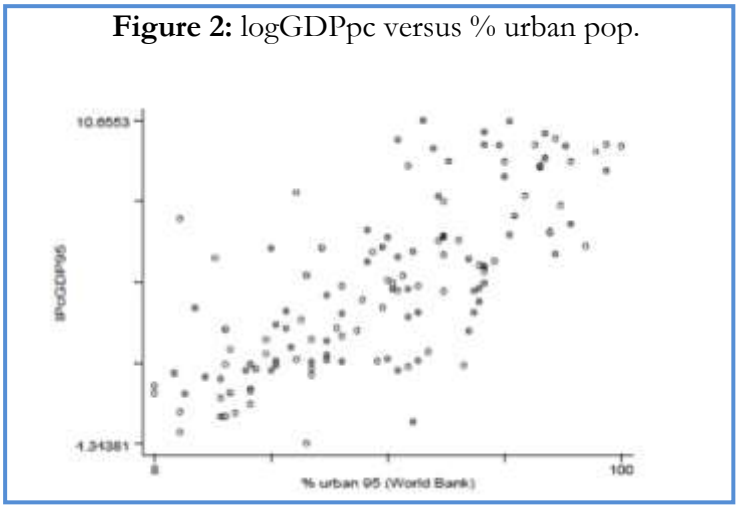
a- Can you *detect* a non-linear relationship between the two variables? Why?
b- Can you explain *solutions* to be implemented in order to solve the non-linearity problem?

Suppose now that we estimate the same model but using a semilog transformation obtaining the following estimation results:

$$\widehat{\log(GDPpc)}_i = 4.631 + 0.052urban_i \quad R^2 = 0.549$$

and obtaining Figure 2 when plotting the data:

Figure 2: logGDPpc versus % urban pop.



c- *Compare* the determination coefficients and the graphs between the two models. Do you think the semilog transformation might be a good solution for the nonlinearity problem? *Explain* your answer.

9 We have data for a sample of high schools in Vietnam where the variable *math* denotes the percentage of students who passed a math test. We want to estimate the effect that spending per student has on the outcomes of this test and propose the following model:

$$math = \beta_0 + \beta_1 \log(spend) + \beta_2 \log(enroll) + \beta_3 poverty + u$$

Where *poverty* describes the percentage of students living below the poverty line, *spend* denotes spending per student and *enroll* is the number of students enrolled in the high school.

a- We do not have data for *poverty* variable but the variable *lnchprg* describes the percentage of students eligible for a programme subsidising school lunches. *Why* is this variable a sensible proxy variable for *poverty*?

b- The table below shows the OLS estimates with and without the inclusion of *lnchprg* as an explanatory variable:

Explanatory variables	**(1)**	**(2)**
log(spend)	11.13	7.75
	(3.30)	(3.04)
log(enroll)	0.022	-1.26
	(0.615)	(0.58)
lnchprg	-	-0.324
		(0.036)
intercept	-69.24	-23.14
	(26.74)	(24.99)
n	408	408
Determination coefficient	0.0293	0.1893

Explain why the effect of spending and enrol are greater in the first model than in the second one? What about if we compare standard errors between the two models?

c- What *conclusions* can you derive when comparing both models?

10 We want to estimate a regression model explaining the behavior of property prices in the city of Barcelona in 2015 (cross sectional analysis). We are provided with a dataset containing information about property, neighborhood and buyer´s characteristics that can be used as explanatory variables. The following table describes those variables:

VARIABLE NAME	***DESCRIPTION***
advance	Loan amount when buying the property
age	Age of property
bathroom	Number of bathroms
bedroom	Number of bedrooms
buyage	Age of main buyer
chnone	No central heating (dummy)
dcitycenter	Distance to city center (km)
floorm2	Floor area of dwelling (m^2)
ftbuyer	First time buyer (dummy)
lagood	Dwelling is in neighborhood with higher-status social housing
labad	Dwelling is in neighborhood with lower-status social housing
pflat	Flat/maisonnette dwelling (dummy)
psemi	Semi-detached dwelling (dummy)
pdetach	Detached dwelling (dummy)
pterrace	Terraced dwelling (dummy)

a- In order to avoid specification errors, which variables would you keep in your analysis according to practical significance? *Justify* your choices.
b- *Explain*, the process you would follow in order to specify your final model and to choose the final variables in your model.
c- *Explain* the difference between practical and statistical significance.

11 We have the following information for the annual growth rates (%) in different countries about stock prices (Y) and in consumer prices (X):

Country	Stock prices (Y)	Consumer prices (X)	Predicted Y	Estimation Residuals
Australia	5	4.3		
Austria	11.1	4.6		
Belgium	3.2	2.4		
Canada	7.9	2.4		
Denmark	3.8	4.2		
Finland	11.1	5.5		
France	9.9	4.7		
Germany	13.5	2.2		
India	1.5	4		
Ireland	6.4	4		
Israel	8.9	8.4		
Italy	8.1	3.3		
Japan	13.5	4.7		
Mexico	4.7	5.2		
Netherlands	7.5	3.6		
New Zealand	4.7	3.6		
Sweden	8	4		
UK	7.5	3.9		
USA	9	2.1		

Knowing that: $\widehat{y_i} = 6.83 + 0.201x_i$

Answer to the following questions:

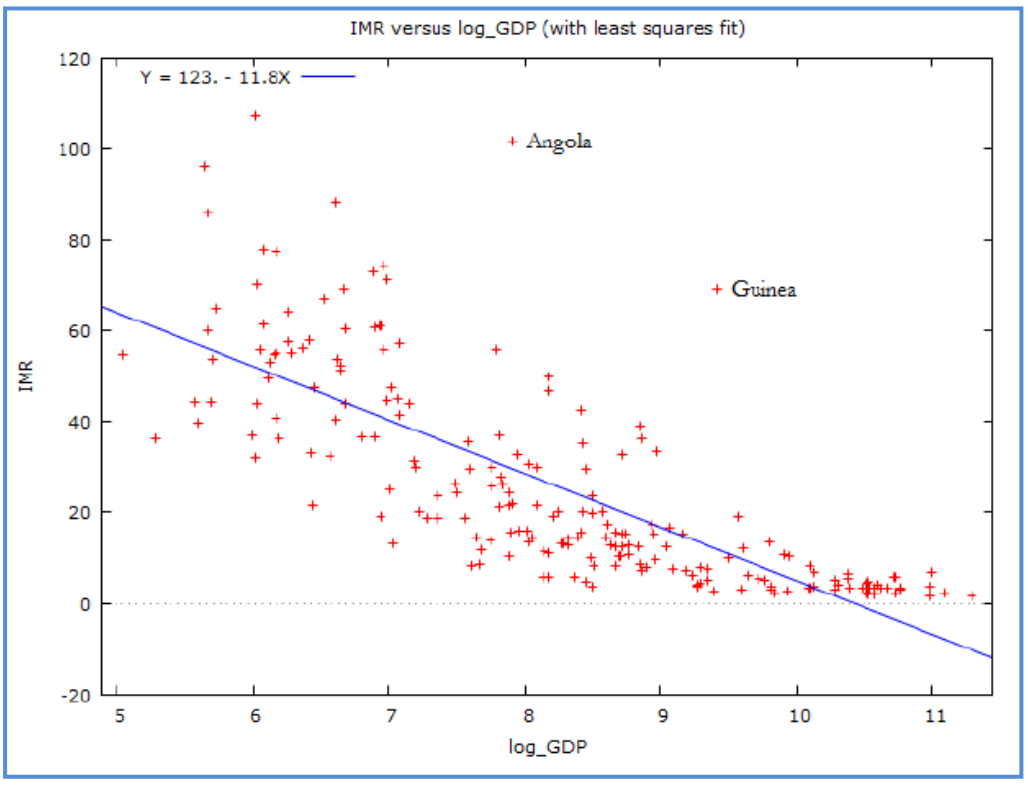
- **a-** *Complete* the missing values in the above table.
- **b-** *Show both graphically and formally* if the above data suffers from an outlier problem.
- **c-** If the answer to b is positive, please *explain* any strategy you would perform in order to solve the problem.

12 Imagine that you are interested in analyzing the determinants of infant mortality rates worldwide. Using the Development Reports from the World Bank in 2013, you get the following information for 248 countries:

IMR	Infant Mortality rate - is the number of deaths of infants per 1,000 live births.
GDP	GDP per capita (constant 2005 US$)

Source: World Bank Development Reports, 2013.

And construct the following figure:



a- Have a look at the graph above, why Angola and Guinea might be considered as outliers in this regression model? *Comment* on the implications of the inclusion of these two countries in the analysis.

b- Angola presents one of the highest infant mortality rates in this sample (103 per 1,000 live births). *Compute* the residual for this country given that our model predicts for Angola an infant mortality rate of 28.6 per 1,000 live births.

c- Knowing that the standard deviation of the estimation residuals (using all the observations) is 26.22, is Angola a significant outlier?

d- What about Guinea? Note that the estimation residual associated to Guinea observation is 52.

13 We have representative data for 30 years old for the US. Levine, Gustafson and Velenchik (1997) estimated a wage equation using the following variables:

Y = log(wage)

F = a dummy variable that takes a value of 1 for smokers and 0, otherwise

ED = years of education

Two specifications are considered:

MODEL 1: Y = -0.176F → omitting education

(se=0.031)

Coefficient of determination = 0.35

MODEL 2: Y = -0.080F + 0.070ED → including education

(se=0.021) (se=0.0004)

Coefficient of determination = 0.68

Compare the two fitted models and *explain* what happens when we omit one relevant variable (in this case, years of education).

14 Consider the following regression model with 41 observations (countries):

$$\log(y_i) = \beta_0 + \beta_1 \log(x_{1i}) + \beta_2 \log(x_{2i}) + u_i$$

Such that the dependent variable is the ratio of trade taxes (imports and export taxes) to total government revenues, the first explanatory variable is the ratio of exports plus imports to GDP and the second explanatory variable is GDP per capita. We estimate this model using OLS and obtain the residuals of the above regression. Then we do the following auxiliary regression:

$$\widehat{\log(y_i)} = -5.8 + 2.5 \log(x_{1i}) + 0.69 \log(x_{2i}) - 0.4[\log(x_{1i})]^2$$

$$-0.04[\log(x_{2i})]^2 + 0.002 \log(x_{1i}) \log(x_{2i})$$

Knowing that the determination coefficient for the auxiliary equation is 0.1148. Could you *compute* the White statistic? What is your *conclusion* about heteroscedasticity in your regression model?

15 We have the following data for 17 countries:

Country	M	G	Country	M	G
Belgium	849	2652	Luxembourg	1368	3108
Canada	778	3888	Netherlands	704	2429
Denmark	853	3159	Norway	634	1881
France	1000	2777	Portugal	215	718
Germany	1331	3095	Spain	239	957
Greece	185	1091	Sweden	1025	4101
Ireland	399	1331	U.K.	609	2174
Italy	554	1731	U.S.A.	1248	4799
Japan	679	1887			

A researcher estimates a regression using the above data and obtains that:

$$\widehat{M} = 74.2 + 0.27G \qquad R^2 = 0.6$$

$$(128.1) \quad (0.05)$$

a- *Draw* a scatter plot using M and G in each of the axes and *explain* why the researcher should expect that there is a problem of heteroscedasticity.

b- *Explain* the consequences of heteroscedasticity on the properties of the estimated coefficients.

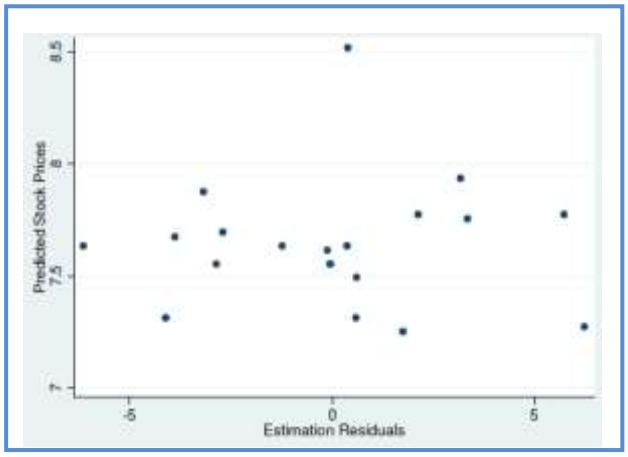
Due to the fact that the previous model has a heteroscedasticity problem, the researcher performs the following two regressions:

$$\frac{\widehat{M}}{G} = 0.32 - 39.4Z \quad R^2 = 0.23$$

$$\widehat{logM} = -1.66 + 1.05logG \quad R^2 = 0.84$$

c- Knowing that the determination coefficient for the auxiliary equation in the first model is 0.25 and in the second one 0.61, which solution is solving the heteroscedasticity problem? Work at 1% significance level.

16 *Explain* the estimation problem that can be found in the following graph (predicted values of the dependent variable versus estimation residuals):



17 Consider the following regression model with 41 observations:

$$\log(Y_i) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + u_i$$

where Y = ratio of trade taxes (import and export taxes) to total government revenue, X_1 = ratio of the sum of exports plus imports to GNP, and X_2 = GNP per capita.

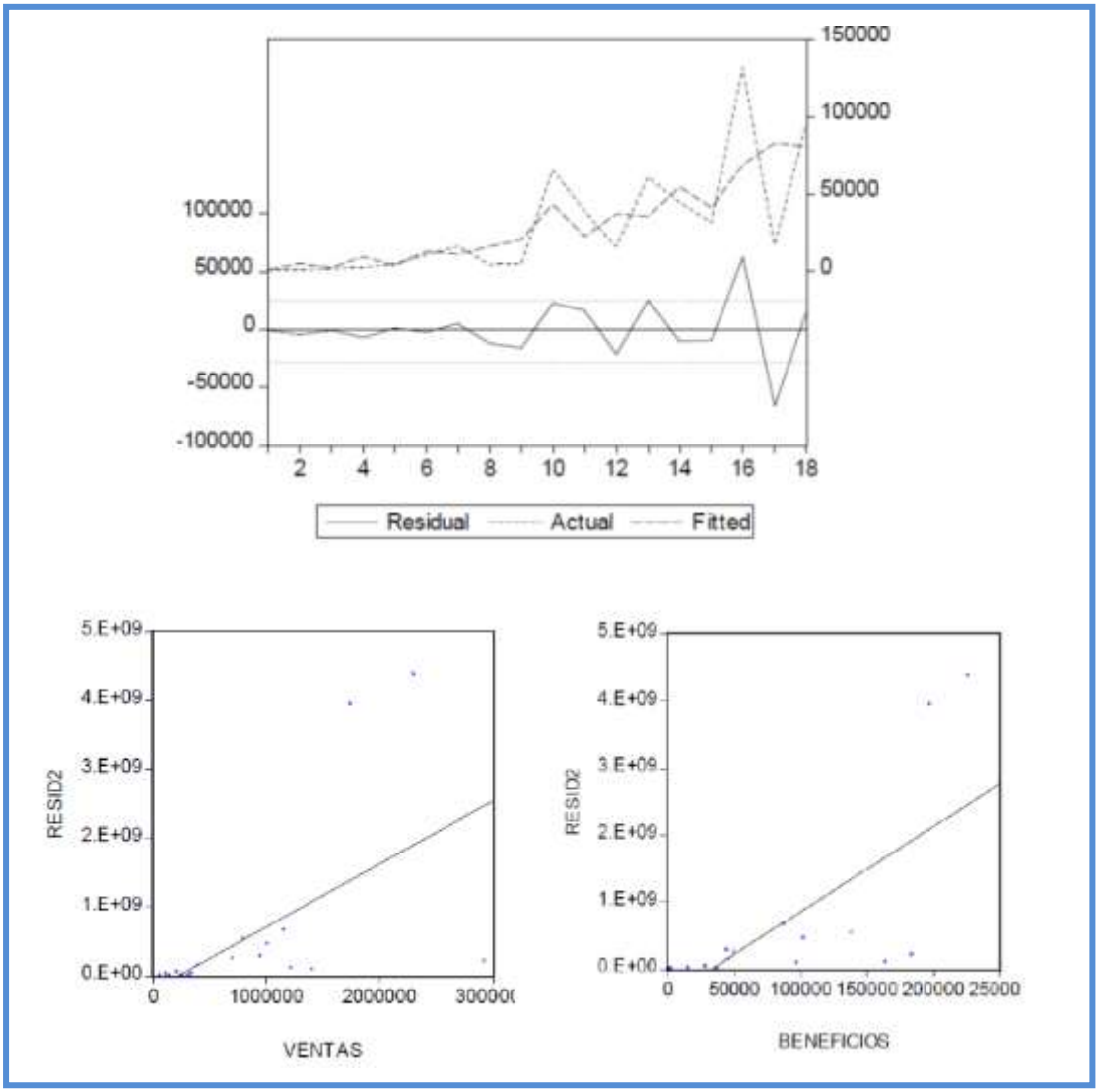
By computing the auxiliary regression, we compute its determination coefficient obtaining a value of 0.1148.

a- *Write* the theoretical specification of the auxiliary regression given the above model.
b- *Test* for heteroscedasticity at 5% significance level.

18 We have the following regression model explaining the behavior of R&D expenditures in 18 sectors of the economy using sales and profits as independent factors:

$$\widehat{RD}_i = -139.392 + 0.012 sales_i + 0.239 profits_i$$

a- *Look* at the following three graphs and *explain* why we should expect heteroscedasticity in the above regression model.



b- The following auxiliary regression was estimated:

$$\widehat{e_i^2} = 695{,}1942 + 1{,}349.7sales_i - 19{,}656.9profits_i - 0.0027sales_i^2$$
$$-0.1163profits_i^2 + 0.0501sales_i * profits_i \qquad R_s^2 = 0.889$$

Test for heteroscedasticity at 1% significance level.

19 We want to estimate a demand function for daily cigarette consumption. Since most people do not smoke, the dependent variable, cigs, is zero for most observations. The equation to be estimated uses the following explanatory variables: income (annual income in

Dollars), cigprc (state cigarette price cents per pack), educ (years of schooling), age (in years) and restaurn (dummy equal to one if there is state restaurant smoking restrictions). Using a simple of 807 individuals we obtain the following estimation results:

$$\widehat{cigs}_i = 0.375 + 0.00005 income_i + 0.00053 cigprc_i - 0.494 educ_i + 0.784 age_i$$

(6.874) (0.0000569) (0.1008) (0.168) (0.159)

$$-0.0091 age_i^2 - 2.845 restaurn_i$$

(0.0017) (1.112)

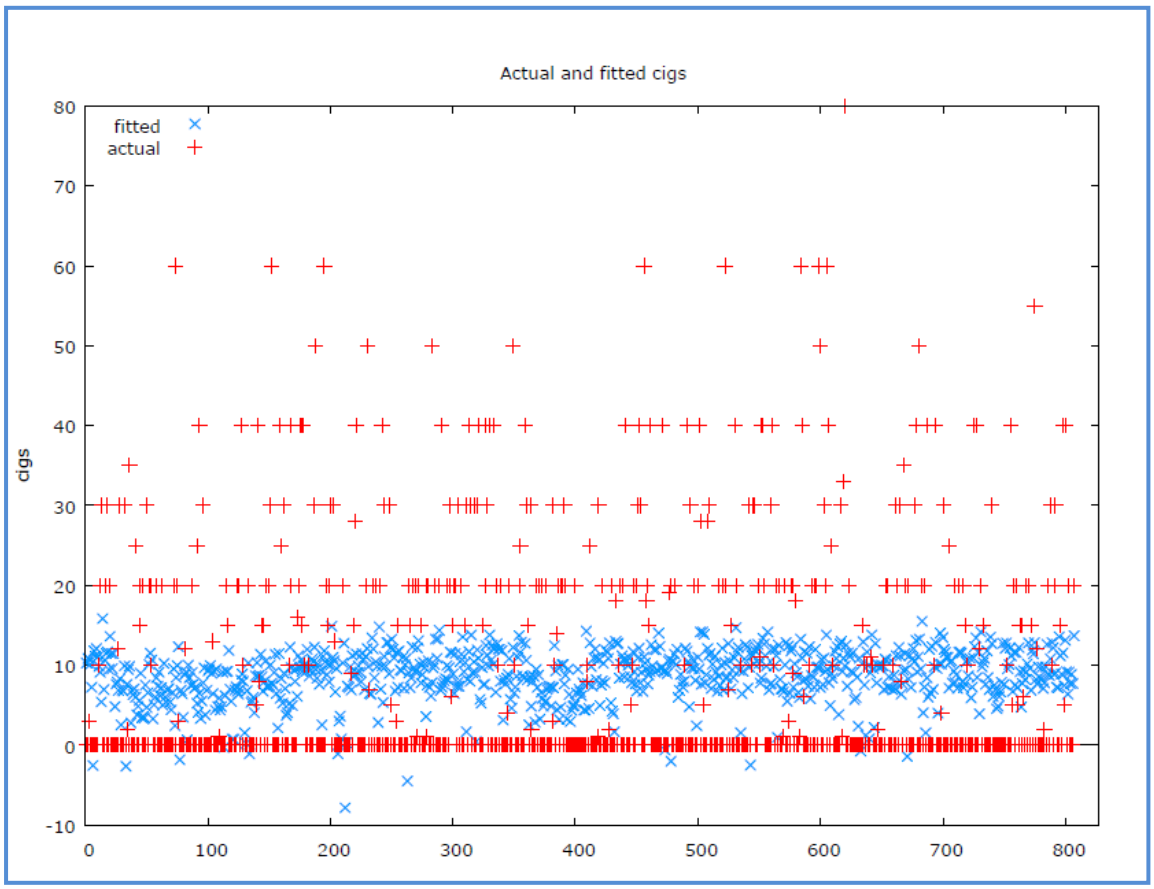
$$n = 807 \qquad R^2 = 0.052$$

a- *Interpret* the above estimation results. Are they realistic?

b- *Test* the individual significance of income and cigprc variables and *explain*.

c- Do policies, such as restaurant smoking restrictions, affect in a significant and expected way on smoking habits?

We plot actual (reds) and fitted (blues) values of cigs variable by observation number obtaining the following graph:



d- Looking at the actual values, do you think this model is linear? Why?

e- Some of the fitted values are negative values. Do you think this is realistic?

f- Do the errors underlying the above equation contain heteroscedasticity? *Test* for heteroscedasticity at 1% significance level knowing that the determination coefficient of the auxiliary regression is equal to 0.0649. Show both the F-test and the Chi-squared tests. Are your two results consistent?

20 We use data about 88 properties to test for heteroscedasticity in a simple housing price equation using the following three independent factors: lotsize (size of lot in square feet), sqrft (size of house in square feet) and bdrms (number of bedrooms). We estimate two different models: Model 1, which is the linear model, and model 2, which is the log model. The table below shows estimation results for both models.

a- *Interpret* the coefficient associated to bdrms in both models.

b- *Specify* the auxiliary regression to be able to perform the White test in Model 1 and *explain*.

c- Which model is homoscedastic at 1% significant level? *Explain* your answer.

OLS Estimation Results

Variable	**Model 1**	**Model 2**
constant	-21.77	5.61
	(29.48)	(0.65)
lotsize	0.00207	
	(0.00064)	
sqrft	0.123	
	(0.013)	
bdrms	13.85	0.037
	(9.01)	(0.028)
log(lotsize)		0.168
		(0.038)
log(sqrft)		0.71
		(0.093)
n	88	88
R squared	0.672	0.643
R squared auxiliary regression	0.383	0.108

21 Answer to the following five questions:

a- *Explain* the main differences between the trend and the irregular component in a time series.

b- *Explain* the main differences between a cyclical and a seasonal component in a time series.

c- *Explain* an econometric tool to take into account seasonal components in a time series.

d- *Explain* one possible econometric tool to detect irregular components in a time series.

e- *Explain* how the trend component can be identified in a SLRM.

22 The general fertility rate (gfr) is the number of children born to every 1,000 women of childbearing age. For years 1913 through 1984, the equation:

$$gfr_t = \beta_0 + \beta_1 pe_t + \beta_2 ww2_t + \beta_3 pill_t + u_t$$

explains gfr in term of the average real dollar value of the personal tax exemption (pe) and two dummy variables. The variable ww2 takes on the value unity during the years 1941 through 1945, when the United States was involved in World War II. The variable pill is unity from 1963 on, when the birth control pill was made available for contraception. Using a dataset, the following estimation results were obtained:

OLS Estimation Results

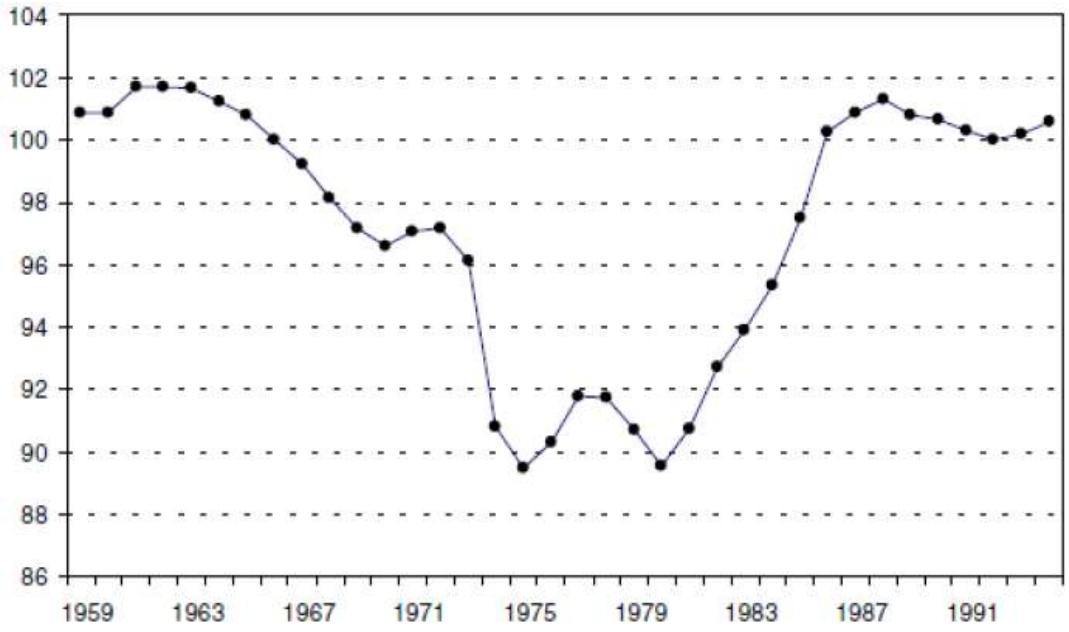
Variable	Model 1	Model 2
constant	98.68	95.87
	(3.21)	(3.28)
ww2	-24.24	-22.13
	(7.46)	(10.73)
pill	-31.59	-31.30
	(4.08)	(3.98)
pe(t)	0.083	0.073
	(0.03)	(0.126)
pe(t-1)		-0.0058
		(0.1557)
pe(t-2)		0.034
		(0.126)
n	72	70
SSR	14,664.27	13,032.64

R Squared	0.473	0.499
Adjusted R squared	0.45	0.459

Answer to the following questions using the above estimation results,

- **a-** *Interpret* and *validate* Model 1 at 1% significance level.
- **b-** Explain the reasoning behind estimating Model 2.
- **c-** Why Model 2 has sample size being 70 observations while in Model 1 sample size is 72? *Provide* a real reasoning for the specification of Model 2. What is the purpose of introducing lagged variables of pe?
- **d-** *Compute* and *compare* the impact propensity of pe on gfr with the long-run propensity of pe on gfr in Model 2.
- **e-** *Test* whether the introduction of the lags in Model 2 is a significant improvement over Model 1. Is the effect of pe on gft a contemporaneous effect? *Explain*.

23 We are interested in estimating a demand equation for housing services. We collect data for the United States and for the period 1959-2003 (45 years) of the following variables: HOUS (consumer expenditure on housing services in billion dollars), DPI (disposable personal income in billion dollars) and PRELHOUS (price index for housing that keeps track of whether housing is becoming more or less expensive relative to other type of services. In the following graph, we plot the relative Price index for housing in the period of analysis and in the following table you can find estimation results for different demand functions specifications.



OLS Estimation Results

Dependent variable: log(HOUS)			
Variable	Model 1	Model 2	Model 3
log(DPI)	1.03	0.33	0.25
	(0.01)	(0.15)	(0.14)
log(DPI)(-1)		0.58	0.20
		(0.15)	(0.20)
log(DPI)(-2)			0.49
			(0.13)
log(GPRHOU)	-0.48	-0.09	-0.28
	(0.04)	(0.17)	(0.17)
log(GPRHOU)(-1)		-0.36	0.23
		(0.17)	(0.30)
log(GPRHOU)(-2)			-0.38
			(0.18)
T	36	35	34
Adjusted R squared	0.998	0.999	0.999

a- *Interpret* the graph above.

b- *Interpret* estimation results of Model 1.

c- What is the impact elasticity of HOUS respect to DPI in Model 2? What about in Model 3? *Compare* them.

d- What is the long-run elasticity of HOUS respect to GPRHOU in Model 2? What about in Model 3? *Compare* them.

e- Compare the long-run elasticities of HOUS respect to DPI and respect to GPRHOU in Model 3.

f- *Explain* the strategy and econometric tools you would use in order to choose the best specification among the three models above.

24 A student (so-called A) has information about 30 observations for two variables, X and Y. He is told that Y depends on X and on a random term such that:

$$y_t = \beta_0 + \beta_1 x_t + u_t$$

He is asked to estimate β_1. He does not know that the true value of β_1 is 5, and he performs the following experiments:

1) He uses first OLS and estimate β_1 obtaining the following results: $\hat{\beta}_1$=4.64; se($\hat{\beta}_1$)=1.30.
2) Next, he is told that the random term follows a first order autoregressive model such that:

$$\widehat{e_t} = 0.7e_{t-1} + \varepsilon_t$$

Where ε_t satisfies G-M conditions.
Student A defines the following:

$$y_t^* = y_t - 0.7y_{t-1}$$
$$x_t^* = x_t - 0.7x_{t-1}$$

And he performs the regression using y_t^* as the dependent variables and x_t^* as the explanatory variable, obtaining the following results: $\hat{\beta}_1$=5.14; se($\hat{\beta}_1$)=0.75.

Nine different students (so-called B, C, D…J) perform the same two experiments but with different random terms. The results are shown in the following table:

Student	**Exp. 1**		**Exp. 2**	
	$\hat{\beta}_1$	**se($\hat{\beta}_1$)**	$\hat{\beta}_1$	**se($\hat{\beta}_1$)**
A	4.64	1.3	5.14	0.75
B	4.56	1.57	4.96	0.87
C	6.54	1.77	5.57	0.98
D	5.19	0.9	5.49	0.63
E	5.81	1.3	5.37	0.77
F	5.24	1.22	4.92	0.65
G	4.27	1.1	4.19	0.61
H	5.26	0.97	4.7	0.71
I	6.8	1.55	6.17	0.76
J	3.83	1.72	5.33	0.82

a- *Compare and explain* why students should not be satisfied with the results obtained in the first experiment.
b- *Explain* why students should be satisfied with the results in the second experiment when they were told that:

$$\widehat{e_t} = 0.7e_{t-1} + \varepsilon_t$$

c- *Test* for autocorrelation at 1% significance level knowing that the standard error associated to the 0.7 coefficient of the AR(1) process is equal to 0.11. *Explain* the type of autocorrelation the model is suffering.

25 Are changes in consumer prices over time related to changes in manufacturing capacity utilization, changes in the money supply, and unemployment rates? We have information about the following variables for the 40 recent years. The variables to be investigated are:

ChCPI = change in the Consumer Price Index (all items)

CapUtil = change in the manufacturing capacity utilization rate

ChgM1 = change in the M1 component of the money supply

ChgM2 = change in the M2 component of the money supply

Unem = unemployment rate (percent)

Regression Analysis

R^2	0.310		
Adjusted R^2	0.231	n	40
R	0.556	k	4
Std. Error	2.751	Dep. Var.	**ChCPI**

ANOVA table

Source	*SS*	*df*	*MS*	*F*	*p-value*
Regressior	118.7398	4	29.6850	3.92	.0098
Residual	264.8899	35	7.5683		
Total	383.6298	39			

Regression output

Variables	*Coefficients*	*Std. error*	*t (df=35)*	*p-value*	*VIF*
Intercept	-39.4880	11.4973	-3.435	.0015	
CapUtil	0.4647	0.1249	3.720	.0007	1.566
ChgM		0.1082	-0.828	.4131	1.427
ChgM2	0.2102	0.1389	1.514	.1391	1.101
Unem	0.9857	0.4000	2.464	.0188	1.896

a- *Validate* the model both individually and globally at 1% significance level.

b- You are told the structure of the error terms in the above model follows the following structure:

$$\widehat{e_t} = -0.88e_{t-1} + \varepsilon_t$$

(0.22)

Test for autocorrelation at 5% significance level and *explain* your answer and the type of autocorrelation the model is suffering.

26 A money demand function is defined as follows:

$$log(M1_t) = \alpha + \beta_1 log(GDP_t) + \beta_2 RS_t + \beta_3 PR_t + u_t$$

Where M1 is the narrow money supply, GDP is real GDP, RS is the interest rate and PR is the rate of inflation.

The model was estimated using quarterly data for the United states over the period 1952:1-1992:4 (T=163 observations) and using two different specifications: Model 1 assumes the model suffers autocorrelation AR(1) and Model 2 assumes the structure in the error terms following AR(2) process. The following table shows estimation results:

OLS Results

Variable	Model 1	Model 2	Model 3
constant	-0.56	-1.54	2.33
	(0.013)	(0.09)	(1.01)
log(GDP)	0.997	0.888	0.971
	(0.082)	(0.091)	(0.077)
RS	-0.056	-0.044	-0.041
	(0.001)	(0.001)	(0.001)
PR	0.404	0.382	0.377
	(0.022)	(0.018)	(0.021)
e(-1)		0.921	0.765
		(0.222)	(0.098)
e(-2)			-0.329
			(0.056)
T	163	162	161
SSR	1,796.49	792.54	716.72
Adjusted R squared	0.738	0.739	0.812

Note that the dependent variable in Model 1 is log(M1) and the dependent variable in Models 2 and 3 is the estimation residuals of Model 1.

a- *Test* for autocorrelation using Model 2 at 1 % significance level.
b- *Test* for autocorrelation using Model 3 at 1% significance level.
c- *Explain* the conclusion you arrive at with regard to the serial correlation in our money demand function.

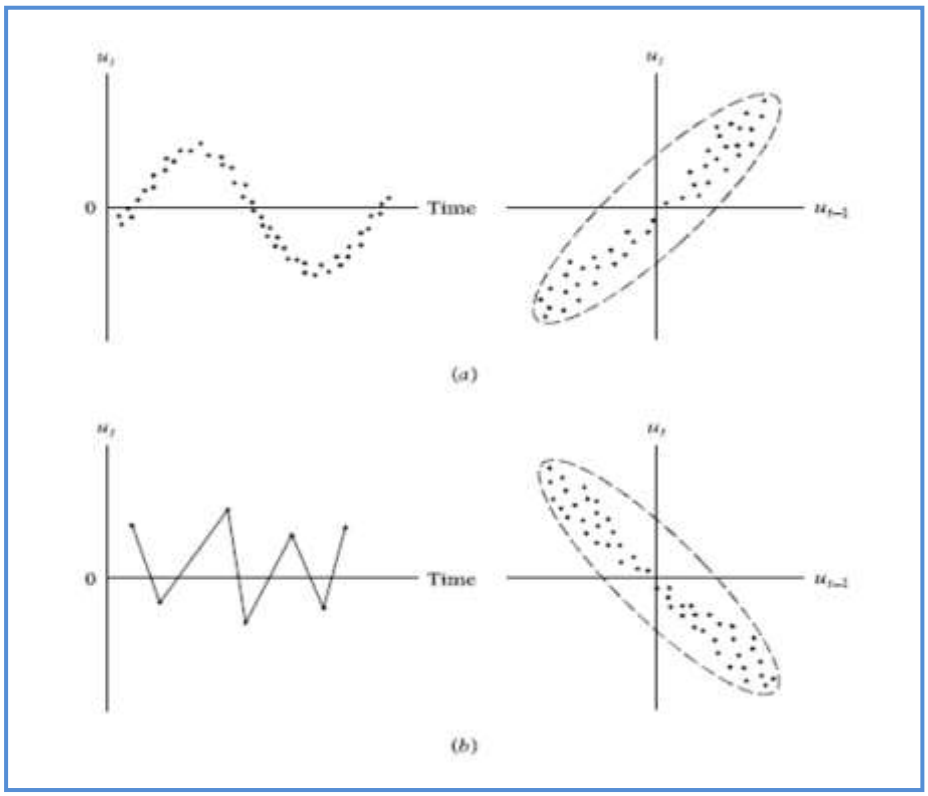
27 Consider the following dynamic linear regression model:

$$\widehat{y_t} = 0.05 + 0.95y_{t-1} + 0.97x_t \qquad T = 120 \quad R^2 = 0.95$$
$$(0.01) \quad (0.05) \qquad (0.11)$$

Knowing that the error term follows a second order autoregressive structure:

- **a-** *Specify* the auxiliary regression to be estimated in order to be able to test for autocorrelation.
- **b-** Knowing that the LM statistic (Breusch-Godfrey) is 12.30, *compute* the determination coefficient of the auxiliary regression.
- **c-** *Test* for autocorrelation at 1% significance level.

28 *Explain* the following two graphs: (a) upper two graphs and (b) lower two graphs in terms of autocorrelation.



29 We want to analyze quarterly new car sales on price, income, unemployment and population over 64 quarters and we obtain the following estimation results:

$$\widehat{NCS_t} = 25{,}531 + 50.116PRC_t + 630.491INC_t - 41.812UN_t - 150.679POP_t$$

$$T = 64 \quad R^2 = 0.441$$

However, you are told this model is naïve since you can allow for serial correlation of fourth order. Please, answer the following two questions:

- **a-** *Specify* the auxiliary regression to be estimated in order to be able to test for autocorrelation.
- **b-** Knowing that the LM statistic (Breusch-Godfrey) is 26.486, *test* for autocorrelation at 1% significance level.

30 *Answer* to the following multiple-choice questions about estimation problems:

A- Autocorrelation refers to a situation in which:

- **a-** Successive error terms derived from the application of regression analysis to time series data are correlated.
- **b-** There is a high degree of correlation between two or more of the independent variables included in a multiple regression model.
- **c-** The dependent variable is highly correlated with the independent variable(s) in a regression analysis.
- **d-** The application of a multiple regression model yields estimates that are nonlinear in form.

B- A situation in which measures of two or more variables are statistically related but are not in fact causally linked because the statistical relationship is caused by a third omitted variable is called:

- **a-** Partial correlation
- **b-** Linear correlation
- **c-** Spurious correlation
- **d-** Marginal correlation

C- Step-wise regression is the most widely used search procedure of developing the ……. regression model without examining all possible models.

a- worst
b- best
c- medium
d- least

D- If there is measurement error in both dependent and explanatory variables of your simple linear regression model, then

a- OLS is unbiased but inefficient.
b- OLS is unbiased but inconsistent.
c- OLS is biased and inefficient.
d- OLS is biased but efficient.

E- A non-formal way to detect a non-linearity problem is plotting your model fitted values versus the

a- Values of your independent variables
b- Values of your explanatory variables
c- Model residuals
d- Model predictions

F- Multicollinearity refers to a situation in which

a- The dependent variable is highly correlated with the explanatory variables included in the regression model.
b- There is a high degree of correlation between the explanatory variables included in a multiple regression model.
c- The application of a multiple regression model yields estimates that are nonlinear form.
d- None of the above.

G- If your dataset has heteroscedasticity, but you completely ignore the problem and use OLS, you will

a- Get biased estimates of the parameters.

b- Get parameter standard errors that could be either too large or too small.

c- Get *t*-statistics that make you too optimistic about your parameters being statistically different from zero.

d- Get *t*-statistics that make you too pessimistic about your parameters being statistically different from zero.

H- A useful graphical method for detecting the presence of heteroscedasticity is

a- Plot y against each x variable in turn

b- Plot the residuals from a preliminary regression against the x variables, each in turn

c- Plot the squared residuals from a preliminary regression against the x variables, each in turn

d- Plot the logarithm of the squared residuals from a preliminary regression against the x variables, each in turn