CEU Universidad San Pablo

UNIT I – Review of Statistics: Jointly normal variables

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Jointly Gaussian random variables

- *X and Y* are jointly Gaussian r.v., is they are normal and the joint p.d.f. is

$$f_{XY}(x,y)=\frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}e^{\frac{-1}{2(1-\rho^2)}\left(\frac{(x-\mu_X)^2}{\sigma_X^2}-\frac{2\rho(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y}+\frac{(y-\mu_Y)^2}{\sigma_Y^2}\right)},$$

$$-\infty<x<+\infty,\ -\infty<y<+\infty,\ |\rho|<1.$$

where ρ is the correlation coefficient of X and Y

Jointly Gaussian random variables

- If *X and Y* are uncorrelated, then ρ =0

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} e^{\frac{-1}{2(1-\rho^2)}\left(\frac{(x-\mu_X)^2}{\sigma_X^2} - \frac{2\rho(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y} + \frac{(y-\mu_Y)^2}{\sigma_Y^2}\right)}$$

$$= \frac{1}{2\pi\sigma_X\sigma_Y} e^{\frac{-1}{2}\left(\frac{(x-\mu_X)^2}{\sigma_X^2} + \frac{(y-\mu_Y)^2}{\sigma_Y^2}\right)}$$

X and Y are independent!

$$= \frac{1}{\sqrt{2\pi\sigma_X^2}\sqrt{2\pi\sigma_Y^2}} e^{\frac{-(x-\mu_X)^2}{2\sigma_X^2}} e^{\frac{-(y-\mu_Y)^2}{2\sigma_Y^2}} = f_X(x) f_Y(y)$$

- Two uncorrelated jointly Gaussian r.v. are **independent** (and vice versa)

Jointly Gaussian random variables

- Normal r.v. are completely defined by their **mean** and **variance**
- Two **uncorrelated** jointly gaussian r.v. are **independent**
- Any **linear transformation** of several normal r.v. leads to a normal r.v.
- Given a jointly Gaussian p.d.f., any **marginal** function will be normal