

- 11.1 Define a composite material with respect to a materials system.

A composite material is a materials system composed of a mixture or combination of two or more micro- or macroconstituents that differ in form and chemical composition and are essentially insoluble in each other.

- 11.2 What are the three main types of synthetic fibers used to produce fiber-reinforced-plastic composite materials.

The three main types of synthetic fibers used to produce fiber-reinforced-plastic composite materials are glass, aramid, and carbon.

- 11.3 What are some of the advantages of glass-fiber-reinforced plastics?

Advantages of glass-fiber-reinforced plastics are: high strength-to-weight ratio; good dimensional stability; good resistance to heat, cold, moisture, and corrosion; good electrical insulation properties; ease of fabrication; and relatively low cost.

- 11.4 What are the differences in the compositions of E and S glasses? Which is the strongest and the most costly?

Compared to S glass, E glass has higher percentages of SiO_2 (65 percent versus 52 to 56 percent) and Al_2O_3 (25 percent versus 12 to 16 percent). Also, E glass contains about 10 percent MgO as an additional compound whereas S glass contains 16 to 25 percent CaO and 8 to 13 percent B_2O_3 . S glass is both stronger and more costly than E glass.

- 11.5 How are glass fibers produced? What is a glass-fiber roving?

Glass fibers are produced by first drawing molten glass from a furnace as monofilaments and gathering large numbers of the filaments into glass fiber strands. The strands are then used to create glass-fiber yarns or rovings. A glass-fiber roving is a collection of bundles of continuous filaments which may be manufactured as continuous strands or as a woven material.

- 11.6 What properties make carbon fibers important for reinforced plastics?

Carbon fibers are important for reinforced plastics because they possess a combination of properties which are highly desired in reinforcement fibers – low weight, very high strength, and high stiffness.

- 11.7 What are two materials used as precursors for carbon fibers?

Polyacrylonitrile (PAN) and pitch are the precursors of carbon fibers.

- 11.8 What are the processing steps for the production of carbon fibers from polyacrylonitrile (PAN)? What reactions take place at each step?

Carbon fibers are produced from polyacrylonitrile (PAN) precursor fibers through a three step process: (1) stabilization, (2) carbonization, and (3) graphitization. During the stabilization stage, stretched PAN fibers are held in tension and oxidized in air at about 200 to 220°C. Next, the fibers are subjected to a carbonizing heat treatment in which the PAN-based fibers become transformed into carbon fibers through the elimination of O, H, and N from the precursor fiber. Finally, an optional graphitization treatment increases the fibers' modulus of elasticity by increasing the preferred orientation of the graphitelike crystallites within each fiber.

- 11.9 What is a tow of carbon fibers?

A tow of carbon fibers is a bundle containing a large number of fibers, on the order of one thousand.

- 11.10 What processing steps are carried out if a very-high-strength type of carbon fiber is desired? If a very-high-modulus type carbon fiber is desired, what processing steps are carried out?

If a very-high-strength type of carbon fiber is desired, only the first two stages – stabilization and carbonization – are performed since graphitization increases elasticity at the expense of strength. Thus, if a very-high-modulus type carbon fiber is desired, all three steps are completed.

- 11.11 What is an aramid fiber? What are two types of commercially available aramid fibers?

An aramid fiber is an aromatic polyamide fiber which has high strength due to covalent bonding and high rigidity due to the aromatic ring structure. Two commercially available types of aramid fibers are Kevlar 29 and Kevlar 49.

- 11.12 What type of chemical bonding takes place within the aramid fibers? What type of chemical bonding takes place between the aramid fibers?

Covalent chemical bonds occur within the aramid fibers while hydrogen bonding exists between the aramid fibers.

- 11.13 How does the chemical bonding within and between the aramid fibers affect their mechanical strength properties?

Within the aramid fibers, covalent bonding provides high strength in the longitudinal direction of the fibers whereas weak hydrogen bonding bonds the polymer chains together in the transverse direction.

- 11.14 Compare the tensile strength, tensile modulus of elasticity, elongation, and density properties of glass, carbon, and aramid fibers (Table 11.1 and Fig. 11.8).

The Kevlar 49 aramid fibers have the best combination of high strength, high modulus, low density, and elongation. The carbon fibers have the best tensile modulus and strength, however, their elongation at break is approximately one-third that of glass and 57 percent lower than Kevlar while their density is about 25 percent higher than the density of Kevlar 49. In general, the glass fibers have lower strengths and moduli and higher densities but superior elongation.

- 11.15 Define specific tensile strength and specific tensile modulus. What type of reinforcing fibers of those shown in Fig. 11.9 has the highest specific modulus and what type has the highest specific tensile strength?

Specific tensile strength is the ratio of tensile strength to density. Specific tensile modulus is the ratio of tensile modulus to density. Graphite and boron fibers have high specific tensile moduli while Kevlar 49 has the highest specific tensile strength.

- 11.16 What are two of the most important matrix plastics for fiber-reinforced plastics? What are some advantages of each type?

Two of the most important matrix plastics for fiber-reinforced plastics are unsaturated polyester and epoxy resins. The polyester resins are lower in cost while the epoxy resins have higher strengths and lower shrinkage after curing.

- 11.17 How does the amount and arrangement of the glass fibers in fiberglass-reinforced plastics affect their strength?

In general, the strength of fiberglass-reinforced plastics increases with increased glass fiber content and with an increased percentage of parallel strands of glass.

- 11.18 What are the main property contributions of the carbon fibers in carbon-fiber-reinforced plastics? What are the main property contributions of the matrix plastic?

Carbon fibers contribute high tensile properties for rigidity and strength while the matrix provides for the alignment of fibers and contributes some impact strength.

- 11.19 Why are some carbon-fiber-epoxy composite laminates designed with the carbon fibers of different layers oriented at different angles to each other?

Some carbon-fiber-epoxy composite laminates are designed with the carbon fibers of different layers oriented at different angles to each other in order to tailor the strength to the specific engineering requirements. Since a laminate's strength lies primarily in the direction of the fibers, different levels of strength are achieved by varying the angular orientation of the layers in different repeating patterns.

- 11.20 A unidirectional carbon-fiber-epoxy-resin composite contains 68 percent by volume of carbon fiber and 32 percent epoxy resin. The density of the carbon fiber is 1.79 g/cm^3 and that of the epoxy resin is 1.20 g/cm^3 . (a) What are the weight percentages of carbon fibers and epoxy resin in the composite? (b) What is the average density of the composite?

Applying a basis of 1 cm^3 of composite material, we have 0.68 cm^3 of carbon fiber and 0.32 cm^3 of epoxy resin.

- (a) The weight percentages are based upon the individual and total mass of the components.

$$\text{Mass of carbon fiber} = m_{cf} = \rho_{cf}V_{cf} = (1.79 \text{ g/cm}^3)(0.68 \text{ cm}^3) = 1.217 \text{ g}$$

$$\text{Mass of epoxy resin} = m_{er} = \rho_{er}V_{er} = (1.20 \text{ g/cm}^3)(0.32 \text{ cm}^3) = 0.384 \text{ g}$$

$$\text{Total mass} = 1.601 \text{ g}$$

Since we are dealing with a ratio of weights, the gravity terms cancel. Thus,

$$\text{Wt \% carbon fibers} = \frac{(1.217 \text{ g})}{(1.601 \text{ g})} \times 100\% = \mathbf{76.0\%}$$

$$\text{Wt \% epoxy resin} = \frac{(0.384 \text{ g})}{(1.601 \text{ g})} \times 100\% = \mathbf{24.0\%}$$

- (b) The average density of the composite is:

$$\rho_c = \frac{m}{V} = \frac{1.601 \text{ g}}{1 \text{ cm}^3} = \mathbf{1.60 \text{ g/cm}^3}$$

- 11.21 The average density of a carbon-fiber-epoxy composite is 1.615 g/cm^3 . The density of the epoxy resin is 1.21 g/cm^3 and that of the carbon fibers is 1.74 g/cm^3 . (a) What is the volume percentage of carbon fibers in the composite? (b) What are the weight percentages of epoxy resin and carbon fibers in the composite?

Using a basis of 1 cm^3 , the total mass of the composite is $m = \rho V = (1.615 \text{ g/cm}^3)(1 \text{ cm}^3) = 1.615 \text{ g}$.

- (a) The volume percentages can be derived from the density equation.

$$\rho_c = \frac{\rho_{cf}V_{cf} + \rho_{er}V_{er}}{V_{cf} + V_{er}}$$

$$1.615 \text{ g/cm}^3 = \frac{(1.74 \text{ g/cm}^3)V_{cf} + (1.21 \text{ g/cm}^3)V_{er}}{1 \text{ cm}^3}$$

$$1.615 \text{ g} = (1.74 \text{ g/cm}^3)V_{cf} + (1.21 \text{ g/cm}^3)V_{er}$$

But $V_{cf} = 1 - V_{er}$. Substituting,

$$\begin{aligned} 1.615 \text{ g} &= (1.74 \text{ g/cm}^3)(1 - V_{er}) + (1.21 \text{ g/cm}^3)V_{er} \\ 0.125 &= 0.53V_{er} \\ V_{er} &= \mathbf{0.236} \text{ and } V_{cf} = \mathbf{0.764} \end{aligned}$$

(b) The weight percentages are calculated based upon the masses.

$$\text{Mass of carbon fiber} = m_{cf} = \rho_{cf}V_{cf} = (1.74 \text{ g/cm}^3)(0.764 \text{ cm}^3) = 1.329 \text{ g}$$

$$\text{Mass of epoxy resin} = m - m_{cf} = 1.615 \text{ g} - 1.329 \text{ g} = 0.286 \text{ g}$$

$$\text{Wt \% carbon fibers} = \frac{(1.329 \text{ g})}{(1.615 \text{ g})} \times 100\% = \mathbf{82.3\%}$$

$$\text{Wt \% epoxy resin} = \frac{(0.286 \text{ g})}{(1.615 \text{ g})} \times 100\% = \mathbf{17.7\%}$$

11.22 Derive an equation relating the elastic modulus of a layered composite of unidirectional fibers and a plastic matrix that is loaded under isostrain conditions.

The load on the composite is equal to the sum of the loads on the fiber layers and the matrix layers or $P_c = P_f + P_m$. Since $P = \sigma A$, we obtain

$$\sigma_c A_c = \sigma_f A_f + \sigma_m A_m$$

But the lengths of the fiber and matrix layers are equal, so the areas can be replaced by the respective volumes.

$$\sigma_c V_c = \sigma_f V_f + \sigma_m V_m$$

For isostrain conditions, all strains are equal ($\epsilon_c = \epsilon_f = \epsilon_m$). Thus we can divide each term by its corresponding strain.

$$\frac{\sigma_c V_c}{\epsilon_c} = \frac{\sigma_f V_f}{\epsilon_f} + \frac{\sigma_m V_m}{\epsilon_m}$$

Substituting the modulus of elasticity for σ/ϵ , and noting that V_c is equal to unity,

$$\boxed{E_c = E_f V_f + E_m V_m}$$

- 11.23 Calculate the tensile modulus of elasticity of a unidirectional carbon-fiber-reinforced-plastic composite material that contains 64 percent by volume of carbon fibers and is stressed under isostrain conditions. The carbon fibers have a tensile modulus of elasticity of 54.0×10^6 psi and the epoxy matrix a tensile modulus of elasticity of 0.530×10^6 psi.

$$E_c = E_f V_f + E_m V_m = (54.0 \times 10^6 \text{ psi})(0.64) + (0.53 \times 10^6 \text{ psi})(0.36)$$

$$= \mathbf{34.75 \times 10^6 \text{ psi} = 239.6 \text{ GPa}}$$

- 11.24 If the tensile strength of the carbon fibers of the 64 percent carbon-fiber-epoxy composite material of Prob. 11.23 is 0.31×10^6 psi and that of the epoxy resin 9.20×10^3 psi, calculate the strength of the composite material in psi. What fraction of the load is carried by the carbon fibers?

The strength of the composite material, using $V_c = 1$ as a basis, is

$$\sigma_c V_c = \sigma_f V_f + \sigma_m V_m$$

$$\sigma_c (1) = (0.31 \times 10^6 \text{ psi})(0.64) + (9.20 \times 10^3 \text{ psi})(0.36)$$

$$\sigma_c = \mathbf{2.02 \times 10^5 \text{ psi} = 1.39 \text{ GPa}}$$

The fraction of the load carried by the carbon fibers is:

$$\frac{P_f}{P_c} = \frac{E_f V_f}{E_c V_c} = \frac{(54.0 \times 10^6 \text{ psi})(0.64)}{(34.75 \times 10^6 \text{ psi})(1.0)} = \mathbf{0.995}$$

- 11.25 Calculate the tensile modulus of elasticity of a unidirectional Kevlar 49-fiber-epoxy composite material that contains 63 percent by volume of Kevlar 49 fibers and is stressed under isostrain conditions. The Kevlar 49 fibers have a tensile modulus of elasticity of 27.5×10^6 psi and the epoxy matrix has a tensile modulus of elasticity of 0.550×10^6 psi.

$$E_c = E_f V_f + E_m V_m = (27.5 \times 10^6 \text{ psi})(0.63) + (0.55 \times 10^6 \text{ psi})(0.37)$$

$$= \mathbf{17.53 \times 10^6 \text{ psi} = 120.9 \text{ GPa}}$$

- 11.26 If the tensile strength of the Kevlar 49 fibers is 0.550×10^6 psi and that of the epoxy resin is 11.0×10^6 psi, calculate the strength of the composite material of Prob. 11.25. What fraction of the load is carried by the Kevlar 49 fibers?

The strength of the composite material, using $V_c = 1$ as a basis, is

$$\sigma_c V_c = \sigma_f V_f + \sigma_m V_m$$

$$\sigma_c (1) = (0.55 \times 10^6 \text{ psi})(0.63) + (11.0 \times 10^6 \text{ psi})(0.37)$$

$$\sigma_c = \mathbf{4.42 \times 10^6 \text{ psi} = 30.48 \text{ GPa}}$$

The fraction of the load carried by the Kevlar 49 fibers is

$$\frac{P_f}{P_c} = \frac{E_f V_f}{E_c V_c} = \frac{(27.5 \times 10^6 \text{ psi})(0.63)}{(17.53 \times 10^6 \text{ psi})(1.0)} = \mathbf{0.988}$$

- 11.27 Derive an equation relating the elastic modulus of a layered composite of unidirectional fibers and a plastic matrix that is stressed under isostress conditions.

For isostress conditions, $\sigma_c = \sigma_f = \sigma_m = \sigma$, and the total strain experienced by the composite is equal to the sum of the strains in the fiber and matrix layers.

$$\epsilon_c = \epsilon_f + \epsilon_m$$

Assuming that the strain is unidirectional in the direction of the applied stress and assuming unit length for the composite after being stressed,

$$\epsilon_c = \epsilon_f V_f + \epsilon_m V_m$$

Assuming Hooke's law is valid under loading,

$$\frac{\sigma}{E_c} = \frac{\sigma V_f}{E_f} = \frac{\sigma V_m}{E_m}$$

Dividing by σ and developing a common denominator,

$$\frac{1}{E_c} = \frac{V_f E_m + V_m E_f}{E_f E_m} \quad \text{or} \quad E_c = \frac{E_f E_m}{V_f E_m + V_m E_f}$$

- 11.28 Calculate the tensile modulus of elasticity for a laminated composite consisting of 62 percent by volume of unidirectional carbon fibers and an epoxy matrix under isostress conditions. The tensile modulus of elasticity of the carbon fibers is 340 GPa and that of the epoxy is 4.50×10^3 MPa.

$$E_c = \frac{E_f E_m}{V_f E_m + V_m E_f} = \frac{(340 \text{ GPa})(4.5 \text{ GPa})}{(0.62)(4.5 \text{ GPa}) + (0.38)(340 \text{ GPa})} = \mathbf{11.59 \text{ GPa}}$$

- 11.29 Calculate the tensile modulus of elasticity of a laminate composite consisting of 62 percent by volume of unidirectional Kevlar 49 fibers and an epoxy matrix stressed under isostress conditions. The tensile modulus of elasticity of the Kevlar 49 fibers is 170 GPa and that of the epoxy is 3.70×10^3 MPa.

$$E_c = \frac{E_f E_m}{V_f E_m + V_m E_f} = \frac{(170 \text{ GPa})(3.7 \text{ GPa})}{(0.62)(3.7 \text{ GPa}) + (0.38)(170 \text{ GPa})} = \mathbf{9.40 \text{ GPa}}$$

- 11.30 Describe the hand lay-up process for producing a fiberglass-reinforced part. What are some advantages and disadvantages of this method?

In the hand lay-up process, a gel coat is first applied to the open mold. Fiberglass reinforcement, typically in the form of a cloth or mat, is then manually placed in the mold. A mixture of the base resin, catalysts and accelerators is then applied by pouring, brushing or spraying. Next, rollers or squeegees are used to thoroughly wet the reinforcement material with the resin and to remove entrapped air. The wall thickness may be increased by applying additional layers of fiberglass mat or woven roving and resin. The advantages of this method are low cost and the ability to make large parts. Disadvantages include the extensive time and labor required.

- 11.31 Describe the spray-up process for producing a fiberglass-reinforced part. What are some advantages and disadvantages of this method?

The spray-up process is used to develop fiberglass-reinforced parts by first feeding continuous-strand roving through a combination chopper and spray gun. The chopped roving and catalyzed resin are simultaneously deposited into the mold and then compressed with a roller or squeegee to remove air and facilitate impregnation of the resin into the fibers. Additional layers may be laid to achieve a desired thickness. Curing is performed at room temperature or using a moderate amount of heat. Advantages of this process are the ability to create complex shapes and the automation provided by the gun. A disadvantage is the higher costs associated with the method.

- 11.32 Describe the vacuum bag-autoclave process for producing a carbon-fiber-reinforced-epoxy part for an aircraft.

In the vacuum bag-autoclave process, a long, thin sheet of prepreg material, consisting of unidirectional long carbon fibers in a partially cured epoxy matrix, is laid out on a table. Pieces of the prepreg sheet are cut out and placed on top of each other on a shaped tool so that their orientations will provide the desired material strength. The tooling and attached laminate are then vacuum-bagged, to remove entrapped air. Next, the vacuum bag enclosing the laminate and tooling is placed in an autoclave for curing of the epoxy at a temperature and pressure of approximately 190°C and 100 psi, respectively. Finally, the assembly is removed from the autoclave and the composite part is stripped from its tooling.

- 11.33 Describe the filament-winding process. What is a distinct advantage of this process from an engineering design standpoint?

In the open-mold filament-winding process, the fiber reinforcement is fed through a resin bath and then wound on a mandrel. Once a sufficient number of layers have been applied, the wound mandrel is cured at room temperature or at an elevated temperature in an oven.

The molded part is then stripped from the mandrel. From an engineering design standpoint, a distinct advantage of this process is that the high degree of fiber orientation and the high fiber loading allow hollow cylinders to be produced with extremely high tensile strengths.

- 11.34 Describe the sheet-molding compound manufacturing process. What are some of the advantages and disadvantages of this process?

The sheet-molding compound (SMC) manufacturing process is a largely automated process. Continuous-strand fiberglass roving is chopped into lengths of about 2 inches and deposited on a layer of resin-filler paste as it travels on a polyethylene film. A second layer of resin-filler paste is subsequently deposited over the first to form a continuous sandwich of the fiberglass and resin filler. The layered composite is then compacted, rolled into package-sized rolls, and stored in a maturation room for one to four days. The SMC rolls are then cut into the proper charge pattern for a specific part, placed in a hot matched metal mold (149°C), and pressurized to about 1000 psi in a press. These conditions allow the SMC to flow uniformly throughout the mold. Advantages of the SMC process are efficient high-volume production, improved surface quality, and uniformity of product. A disadvantage is the large investment required for equipment.

- 11.35 Describe the pultrusion process for the manufacture of fiber-reinforced plastics. What are some advantages of this process?

The continuous pultrusion is used to manufacture fiber-reinforced plastics of constant cross section. In this process, continuous-strand fibers are impregnated in a resin bath and then drawn through a heated die that forms the stock shape. This process is advantageous because it is continuous and because it generates high strength stock by concentrating fibers parallel to stock length.

- 11.36 What advantages and disadvantages does concrete offer as a composite material?

Advantages offered by concrete are flexibility in design, low cost, durability, fire resistance, ability to be cast on site, and aesthetic appearance. Disadvantages include low tensile strength, low ductility, and some shrinkage.

- 11.37 What are the principal components of most concretes?

The principal components of most concretes are an aggregate made of coarse granular material, and a cement paste typically made from portland cement and water.

- 11.38 What are the basic raw materials for portland cement? Why is portland cement so called?

The basic raw materials for portland cement are lime (CaO), silica (SiO₂), alumina (Al₂O₃), and iron oxide (Fe₂O₃). This cement derives its name from a small peninsula on the south coast of England where the limestone is similar to portland cement.

11.39 How is portland cement made? Why is a small amount of gypsum added to portland cement?

Portland cement is made by crushing, grinding, and proportioning the raw materials for the desired composition and then blending them. Next, the mixture is heated to 1400 to 1650°C in a rotary kiln and thereby converted to cement clinker. After being cooled and pulverized, the clinker is mixed with a small amount of gypsum to control the cement's setting time.

11.40 What are the names, chemical formulas, and abbreviations for the four principal compounds of portland cement?

The names, chemical formulas, and abbreviations for the four principal compounds of portland cement are summarized below in the table.

Compound	Chemical Formula	Abbreviation
Tricalcium silicate	$3\text{CaO} \cdot \text{SiO}_2$	C_3S
Dicalcium silicate	$2\text{CaO} \cdot \text{SiO}_2$	C_2S
Tricalcium aluminate	$3\text{CaO} \cdot \text{Al}_2\text{O}_3$	C_3A
Tetracalcium aluminoferrite	$4\text{CaO} \cdot \text{Al}_2\text{O}_3 \cdot \text{Fe}_2\text{O}_3$	C_4AF

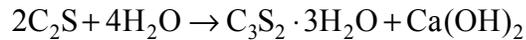
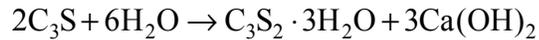
11.41 List the five main ASTM types of portland cement and give the general conditions for which each is used and their applications.

ASTM C150 Designation	General Conditions for Cement Use	Applications
I	Ordinary	sidewalks, reinforced concrete buildings, bridges, culverts, tanks, reservoirs
II	Moderate heat of hydration; Moderate sulfate resistance	Used for large structures poured in hot weather such as large piers and heavy retaining walls.
III	Rapid hardening	Used when concrete forms must be removed early from a structure and quickly put to use.
IV	Low heat of hydration	Used for massive concrete structures such as large gravity dams where the heat generated by the setting cement is a critical factor.
V	Sulfate-resistant	Used for applications where concrete is exposed to soils and groundwater containing a high sulfate content.

11.42 What type of chemical reaction occurs during the hardening of portland cement?

During the hardening of portland cement, hydration reactions occur.

11.43 Write the chemical reactions for C_3S and C_2S with water.



11.44 Which component of portland cement hardens rapidly and is mostly responsible for early strength?

Tricalcium silicate (C_3S) hardens rapidly and is mostly responsible for early strength.

11.45 Which component of portland cement reacts slowly and is mainly responsible for the strengthening after about one week?

Dicalcium silicate (C_2S) reacts slowly and is mainly responsible for the strengthening after about one week.

11.46 Which compound is kept to a low level for sulfate-resisting portland cements?

Tricalcium aluminate (C_3A) is kept to a low level for sulfate-resisting portland cements.

11.47 Why is C_4AF added to portland cement?

Tetracalcium aluminoferrite is added to portland cement to reduce the clinkering temperature and thus reduce manufacturing costs.

11.48 Why is it important that during the first few days of the curing of concrete the evaporation of water from its surface be prevented or reduced?

It is important to prevent or reduce rapid evaporation of water from the surface of concrete during the first few days of the curing so that the desired strength and durability is achieved. The compressive strength of concretes made with ASTM-type cements continues to significantly increase for time periods on the order of a month or a year.

11.49 What is air-entrained concrete? What is an advantage of air-entrained concrete?

Air-entrained concrete is a class of concretes which contain extremely small air bubbles – 90 percent are less than $100 \mu\text{m}$. Advantages of air-entrained concrete are improved resistance to freezing and improved workability of the concrete.

11.50 What method is used to make air-entrained concrete? What volume percent of air is used in the concrete for frost protection?

Air-entrained concretes are made by adding surface-active agents that lower the surface tension at the water-air interface so that air bubbles may form. To protect against frost, 4 to 8 percent air by volume must be generated.

- 11.51 How does the water-cement ratio (by weight) affect the compressive strength of concrete? What ratio gives a compressive strength of about 5500 psi to normal concrete? What is the disadvantage of too high a water-cement ratio? Of too low a water-cement ratio?

As the water-cement ratio by weight increases, the compressive strength of concrete decreases significantly. In order to achieve a compressive strength of about 5500 psi in normal concrete, a ratio of approximately 0.4 is required. If the water-cement ratio is increased to a high value, for example 0.7, the compressive strength is reduced dramatically; for normal concrete, the strength becomes approximately 2700 psi. While low ratio values increase the strength, too low of a value makes it difficult to work the concrete and to fill the forms completely.

- 11.52 What major factors should be taken into account in the design of concrete mixtures?

In designing concrete mixtures, three important factors should be considered: workability, strength and durability, and economy of production.

- 11.53 What are the absolute volume percent ranges for the major components of normal concrete?

For normal concrete, the ranges by absolute volume of the major components are: 7 to 15 percent cement; 25 to 30 percent fine aggregate; 31 to 51 percent coarse aggregate; and 16 to 21 percent water.

- 11.54 We want to produce 100 ft³ of concrete with a ratio of 1:1.9:3.8 (by weight) of cement, sand, and gravel. What are the required amounts of the components if 5.5 gal of water per sack of cement is to be used? Assume the free moisture contents of the sand and gravel are 3 and 0 percent, respectively. The specific gravities of the cement, sand and gravel are 3.15, 2.65, and 2.65, respectively. (One sack of cement weighs 94 lb and 1 ft³ water = 7.48 gal.) Give answers for the cement in sacks, the sand and gravel in pounds, and the water in gallons.

First calculate the absolute volumes of the constituents per sack of cement:

Constituent	Ratio by weight	Weight (lb)	SSDD (lb/ft ³)	Absolute vol. per sack of cement
Cement	1.0	1.0 × 94 lb = 94.0 lb	197	94 lb/(197 lb/ft ³) = 0.477 ft ³
Sand	1.9	1.9 × 94 lb = 178.6 lb	165	178.6 lb/(165 lb/ft ³) = 1.082 ft ³
Gravel	3.8	3.8 × 94 lb = 357.2 lb	165	357.2 lb/(165 lb/ft ³) = 2.165 ft ³
Water	(5.5 gal)			5.5 gal/(7.48 gal/ft ³) = 0.735 ft ³
Total absolute vol. per sack of cement =				4.459 ft ³

Thus, 4.459 ft³ of concrete is produced per sack of cement on a dry basis. To produce 100 ft³ of concrete using this basis, the following amounts of constituents are required:

1. Required amount of cement = 100 ft³ / (4.459 ft³/sack of cement) = **22.43 sacks**
2. Required amount of sand = (22.43 sacks)(94 lb/sack)(1.9) = 4006 lb
3. Required amount of gravel = (22.43 sacks)(94 lb/sack)(3.8) = **8012 lb**
4. Required amount of water = (22.43 sacks)(5.5 gal/sack) = 123.4 gal

Since the sand and gravel calculations were performed on a dry basis, they must be corrected to account for the moisture they contain. However, since the gravel has 0 percent moisture, only the sand must be considered on a wet basis:

$$\text{Required weight of wet sand} = (4006 \text{ lb})(1.03) = \mathbf{4126 \text{ lb}}$$

$$\text{Weight of water in sand} = 4126 \text{ lb} - 4006 \text{ lb} = 120 \text{ lb}$$

$$\text{Weight of water in gravel} = 0 \text{ lb}$$

$$\begin{aligned} \text{Required weight of water} &= 123.4 \text{ gal} - (120 \text{ lb})(1 \text{ ft}^3/62.4 \text{ lb})(7.48 \text{ gal/ft}^3) \\ &= 123.4 \text{ gal} - 14.4 \text{ gal} \\ &= \mathbf{109 \text{ gal}} \end{aligned}$$

- 11.55 We want to produce 50 ft³ of concrete with a ratio of 1:1.9:3.2 (by weight) of cement, sand, and gravel. What are the required amounts of the components if 5.5 gal of water per sack of cement is to be used? Assume the free moisture contents of the sand and gravel are 4 and 0.5 percent, respectively. The specific gravities of the cement, sand, and gravel are 3.15, 2.65, and 2.65, respectively. (One sack of cement weighs 94 lb and 1 ft³ water = 7.48 gal.) Give answers for the cement in sacks, the sand and gravel in pounds, and the water in gallons.

First calculate the absolute volumes of the constituents per sack of cement:

Constituent	Ratio by weight	Weight (lb)	SSDD (lb/ft ³)	Absolute vol. per sack of cement
Cement	1.0	1.0 × 94 lb = 94.0 lb	197	94 lb/(197 lb/ft ³) = 0.477 ft ³
Sand	1.9	1.9 × 94 lb = 178.6 lb	165	178.6 lb/(165 lb/ft ³) = 1.082 ft ³
Gravel	3.8	3.2 × 94 lb = 300.8 lb	165	300.8 lb/(165 lb/ft ³) = 1.823 ft ³
Water	(5.5 gal)			5.5 gal/(7.48 gal/ft ³) = 0.735 ft ³
Total absolute vol. per sack of cement =				4.117 ft ³

Thus, 4.117 ft³ of concrete is produced per sack of cement on a dry basis. To produce 50 ft³ of concrete using this basis, the following amounts of constituents are required:

1. Required amount of cement = 50 ft³ / (4.117 ft³/sack of cement) = **12.14 sacks**
2. Required amount of sand = (12.14 sacks)(94 lb/sack)(1.9) = 2168 lb
3. Required amount of gravel = (12.14 sacks)(94 lb/sack)(3.2) = 3652 lb

4. Required amount of water = (12.14 sacks)(5.5 gal/sack) = 66.77 gal

Since the sand and gravel calculations were performed on a dry basis, they must be corrected to account for the moisture they contain. Thus, on a wet basis:

Required weight of wet sand = (2168 lb)(1.04) = **2255 lb**

Weight of water in sand = 2255 lb – 2168 lb = 87 lb

Required weight of gravel = (3652 lb)(1.005) = **3670 lb**

Weight of water in gravel = 3670 lb – 3652 lb = 18 lb

Required weight of water = 66.8 gal – (87 lb + 18 lb) $(1 \text{ ft}^3 / 62.4 \text{ lb}) (7.48 \text{ gal/ft}^3)$
= 66.8 gal – 12.6 gal
= **54.2 gal**

11.56 Why is concrete mainly used in compression in engineering design?

Concrete is mainly used in compression in engineering design because its tensile strength is 10 to 15 times lower than its compressive strength.

11.57 What is reinforced concrete? How is it made?

Reinforced concrete is a form of concrete which is cast with steel reinforcements in the form of rods, wires, wire mesh, etc.

11.58 What is the main advantage of prestressed concrete?

The main advantage of prestressed concrete is that applied tensile stresses must first overcome the compressive stresses inherent in the concrete due to tensioned steel tendon reinforcements.

11.59 Describe how compressive stresses are introduced in pretensioned prestressed concrete.

Compressive stresses are introduced into pretensioned prestressed concrete through tensioned steel tendons. The casting process is performed by pouring concrete over the tendons which are stretched by an adjustable jack. Once the concrete has attained the desired strength, the jack force is released but the tendons, bonded to the cement, retain their tensioned state. Consequently, the inability of the tendons to elastically restore their original height causes compressive stresses to develop.

11.60 Describe how compressive stresses are introduced in posttensioned prestressed concrete.

Compressive stresses are introduced in posttensioned prestressed concrete by steel tendons which are stretched subsequent to concrete pouring. The tendons, encased by hollow conduits, are subjected to tension via a jack after the poured concrete has attained

a specified strength. Prior to releasing the jack pressure, a fitting is installed to maintain the tensioned state of the tendons and thus provide compressive stresses.

11.61 What is asphalt? Where is asphalt obtained?

Asphalt is a bitumen – a hydrocarbon with some oxygen, sulfur and other impurities – which is obtained from petroleum refining or from the processing of bitumen-bearing rock and surface deposits.

11.62 What are chemical composition ranges for asphalts?

The chemical composition ranges for asphalts are 80 to 85 percent carbon, 9 to 10 percent hydrogen, 2 to 8 percent oxygen, 0.5 to 7 percent sulfur, and small amounts of nitrogen and other trace metals.

11.63 What does an asphalt mix consist of? What is the asphalt content of a type IV road-paving asphalt?

Asphalt mix consists of aggregates and asphalt. A type IV road-paving asphalt contains 3 to 7 percent asphalt and 35 to 50 percent aggregate. The resulting mix passes through a no. 8 sieve.

11.64 What characteristics are desirable for the aggregate for a road-paving asphalt?

Desirable characteristics of aggregate for road-paving asphalt are densely packed angular aggregate that does not polish easily, that interlocks, and that bonds well with the asphalt.

11.65 Describe the different layers in the cross section of a tree stem. Also, give the functions of each layer.

The cross section of a tree consists of layers having the following functions.

1. The *outer bark* layer is composed of dry, dead tissue and provides external protection for the tree.
2. The *inner bark* layer is moist and soft and carries food from the leaves to all of the growing parts of the tree.
3. The *cambium* layer is the tissue layer between the bark and the wood that forms the wood and bark cells.
4. The *sapwood* is the light-colored wood that forms the outer part of the tree stem. The sapwood contains some living cells that function for food storage and carry sap from the roots to the leaves of the tree.
5. The *heartwood* is the older inner region of the tree stem that is no longer living. The heartwood is usually darker than the sapwood and provides strength for the tree.
6. The *pith* is the soft tissue at the center of the tree around which the first growth of the tree takes place.

- 11.66 What is the difference between softwoods and hardwoods? Give several examples of both. Are all hardwoods physically hard?

From a botanical basis, trees are classified as softwood if the tree seed is exposed and as hardwood if the seed is covered. Another distinction is that softwood trees are, in general, evergreens which retain their leaves, while hardwoods are deciduous trees which shed their leaves annually. Examples of softwood trees are fir, spruce, pine and cedar. Hardwood trees include maple, birch, oak, elm, and cherry. Most hardwoods are physically hard, however, exceptions such as balsa wood are soft.

- 11.67 What are the subrings of the annual growth rings of trees?

The two subrings of the annual growth rings are *earlywood* (spring) and *latewood* (summer).

- 11.68 What axis is parallel to the annual rings? What axis is perpendicular to the annual ring?

The tangential axis is parallel to the annual rings while the radial axis is perpendicular to the annual rings.

- 11.69 Describe the microstructure of a softwood tree.

The microstructure of softwood trees consists mainly of long, thin-walled tubular cells called tracheids which measure 3 to 5 mm in length and 20 to 80 μm . At the center of the cell is a large open space called a lumen. The lumen transports water along the longitudinal axis while smaller holes at the ends of the cell allow water to flow between cells. Earlywood cells have a relatively large diameter, thin walls and large lumen while latewood cells have a small diameter, thick walls and small lumen.

- 11.70 What are the functions of the wood rays of a tree?

Wood rays consist of parenchyma cells that are used for food storage.

- 11.71 Describe the microstructure of a hardwood tree. What is the difference between ring-porous and diffuse-porous tree microstructures?

The microstructure of a hardwood tree contains vessels and fibers. The vessels, thin-walled structures made up of vessel elements, conduct fluid in the longitudinal direction of the tree stem. In ring-porous hardwoods, the vessels in earlywood are larger than those in latewood, whereas, in diffuse-porous hardwoods, the vessel diameters are essentially uniform throughout the growth ring. Fibers are elongated longitudinal cells that provide structural support. These cells are about 0.7 to 3 mm long and less than 20 μm in diameter.

- 11.72 Describe the cell-wall ultrastructure of a wood cell.

The cell-wall ultrastructure of a wood cell consists of a primary and a secondary wall. The primary wall is formed during the cell division of initial cell growth. Once this wall grows to its full size in the transverse and longitudinal directions, the secondary wall develops in concentric layers that grow into the cell center. These walls consist of microfibrils bonded together by a lignin cement which provides rigidity. The microfibrils, believed to consist of a cellulose crystalline core surrounded by an amorphous region of hemicellulose and lignin, are arranged in varying orientations from one layer of the cell wall to the next.

11.73 Describe the constituents of a wood cell.

The principal constituents of a wood cell are cellulose, hemicellulose and lignin. Cellulose, a linear polymer consisting of glucose units having a degree of polymerization of 5000 to 10,000, constitutes 45 to 50 percent of the solid wood material. The covalent bonding within and between these glucose units creates a straight, stiff molecule with high tensile strength. Lateral bonding also exists between the units due to hydrogen and permanent dipolar bonding. Hemicellulose is a branched amorphous molecule containing several types of sugars and makes up 20 to 25 percent of the solid wood. Lignin, which constitutes the remaining 20 to 30 percent by weight of the wood, are complex, cross-linked, three-dimensional polymeric materials formed from phenolic units.

11.74 A piece of wood containing moisture weighs 210 g, and after oven drying to a constant weight, it weighs 125 g. What is its percent moisture content?

The percent moisture content is equal to the difference of the wood's weight before and after oven drying.

$$\begin{aligned}\% \text{ Moisture content} &= \frac{\text{weight of wet wood} - \text{weight of dry wood}}{\text{weight of dry wood}} \times 100\% \\ &= \frac{210 \text{ g} - 125 \text{ g}}{125 \text{ g}} \times 100\% = \mathbf{68.0\%}\end{aligned}$$

11.75 A piece of wood contains 15 percent moisture. What must its weight have been before oven drying if it has a constant weight of 125 g after drying?

$$\begin{aligned}\% \text{ Moisture content} &= \frac{\text{weight of wet wood} - \text{weight of dry wood}}{\text{weight of dry wood}} \times 100\% \\ 15.0 &= \frac{x - 125 \text{ g}}{125 \text{ g}} \times 100\% \\ x &= \mathbf{143.8 \text{ g}}\end{aligned}$$

11.76 A piece of wood contains 45 percent moisture. What must its final weight be after oven drying if it weighed 165 g before drying?

$$\% \text{ Moisture content} = \frac{\text{weight of wet wood} - \text{weight of dry wood}}{\text{weight of dry wood}} \times 100\%$$

$$45.0 = \frac{165 \text{ g} - x}{x} \times 100\%$$

$$x = \mathbf{113.8 \text{ g}}$$

- 11.77 What is the reason for the relatively high strength of wood in the longitudinal direction of the tree stem as compared to the transverse direction?

Wood has relatively high strength in the longitudinal direction of the tree stem as compared to the transverse direction primarily because of the strong covalent bonds of the cellulose microfibrils that are mainly oriented longitudinally. Perpendicular to the grain, the strength depends on weaker hydrogen bonds that bond the cellulose molecules laterally.

- 11.78 What is the green condition for wood? Why is wood much weaker in the green condition than in the kiln-dried condition?

The green condition for wood refers to wood having the moisture content of living trees. Such wood is much weaker than kiln-dried wood because the removal of water causes the wood to shrink and thus become denser and stronger. The water eliminated comes from the less-ordered regions of the microfibrils' cellulose and allows the cell molecular structure to compact and form internal bridges through hydrogen bonding.

- 11.79 Why does wood shrink much more in the transverse direction than in the longitudinal direction?

Wood shrinks much more in the transverse direction than in the longitudinal direction because the microfibrils length dimension is mainly oriented in the longitudinal direction of the tree stem.

- 11.80 A metal-matrix composite (MMC) is made of a 6061-Al alloy matrix and continuous boron fibers. The boron fibers are produced with an $11.5 \mu\text{m}$ diameter tungsten-wire core that is coated with boron to make a final $107 \mu\text{m}$ diameter fiber. A unidirectional composite is made with 51 vol % of the boron fibers in the Al 2024 matrix. Assuming the law of mixtures applies to isostrain conditions, calculate the tensile modulus of the composite in the direction of the fibers. Data are $E_B = 370 \text{ GPa}$, $E_W = 410 \text{ GPa}$, and $E_{Al} = 70.4 \text{ GPa}$.

$$E_{\text{comp}} = f_W E_W + f_B E_B + f_{Al} E_{Al} \quad f_{W+B} = 0.51, \quad f_{Al} = 1 - 0.51 = 0.49$$

where,

$$f_W = \frac{\text{area of W wire}}{\text{area of B fiber}} \times f_{W+B} = \frac{\pi(11.5 \mu\text{m})^2}{\pi(107 \mu\text{m})^2} \times 0.51 = 5.89 \times 10^{-3}$$

$$f_B = \frac{\text{area of B fiber} - \text{area of W wire}}{\text{area of B fiber}} \times f_{W+B} = \frac{\pi(107 \mu\text{m})^2 - \pi(11.5 \mu\text{m})^2}{\pi(107 \mu\text{m})^2} \times 0.51$$

$$= 0.504$$

Substituting,

$$E_{\text{comp}} = (5.89 \times 10^{-3})(410 \text{ GPa}) + (0.504)(370 \text{ GPa}) + (0.49)(70.4 \text{ GPa}) = \mathbf{223.4 \text{ GPa}}$$

- 11.81 A newly developed metal-matrix composite is made for the National Aerospace plane with a matrix of the intermetallic compound titanium aluminide (Ti_3Al) and continuous silicon carbide fibers. A unidirectional composite is made with the SiC continuous fibers all in one direction. If the modulus of the composite is 220 GPa and assuming isostrain conditions, what must the volume percent of SiC fibers in the composite be if $E(\text{SiC}) = 390 \text{ GPa}$ and $E(\text{Ti}_3\text{Al}) = 145 \text{ GPa}$?

$$E_{\text{comp}} = f_{\text{SiC}}E_{\text{SiC}} + f_{\text{Ti}_3\text{Al}}E_{\text{Ti}_3\text{Al}} \quad \text{where} \quad f_{\text{Ti}_3\text{Al}} = 1 - f_{\text{SiC}}$$

$$220 \text{ GPa} = f_{\text{SiC}}(390 \text{ GPa}) + (1 - f_{\text{SiC}})(145 \text{ GPa})$$

$$f_{\text{SiC}} = \mathbf{0.306}$$

- 11.82 A metal-matrix composite is made with a matrix of Al 6061 alloy and 47 vol % Al_2O_3 continuous fibers all in one direction. If isostrain conditions prevail, what is the tensile modulus of the composite in the direction of the fibers? Data are $E(\text{Al}_2\text{O}_3) = 395 \text{ GPa}$ and $E(\text{Al 6061}) = 68.9 \text{ GPa}$.

$$E_{\text{comp}} = f_{\text{Al}_2\text{O}_3}E_{\text{Al}_2\text{O}_3} + f_{\text{Al}}E_{\text{Al}} \quad \text{where} \quad f_{\text{Al}_2\text{O}_3} = 0.47, \quad f_{\text{Ti}_3\text{Al}} = 0.53$$

$$= (0.47)(395 \text{ GPa}) + (0.53)(68.9 \text{ GPa})$$

$$= \mathbf{222.2 \text{ GPa}}$$

- 11.83 An MMC is made with an Al 2024 alloy with 20 vol percent SiC whiskers. If the density of the composite is 2.90 g/cm^3 and that of the SiC fibers is 3.10 g/cm^3 , what must the density of the Al 2024 alloy be?

$$\rho_{\text{comp}} = f_{\text{SiC}}\rho_{\text{SiC}} + f_{\text{Al}}\rho_{\text{Al}} \quad \text{where} \quad f_{\text{SiC}} = 0.20, \quad f_{\text{Al}} = 0.80$$

$$2.90 \text{ g/cm}^3 = (0.20)(3.10 \text{ g/cm}^3) + 0.8\rho_{\text{Al}}$$

$$\rho_{\text{Al}} = \mathbf{2.85 \text{ g/cm}^3}$$

- 11.84 A ceramic-matrix composite (CMC) is made with continuous SiC fibers embedded in a reaction-bonded silicon nitride (RBSN) matrix with all the SiC fibers aligned in one direction. Assuming isostrain conditions, what is the volume fraction of the SiC fibers in

the composite if the composite has a tensile modulus of 250 GPa? Data are $E(\text{SiC}) = 395$ GPa and $E(\text{RBSN}) = 155$ GPa.

$$E_{\text{comp}} = f_{\text{SiC}}E_{\text{SiC}} + f_{\text{RBSN}}E_{\text{RBSN}} \quad \text{where} \quad f_{\text{RBSN}} = 1 - f_{\text{SiC}}$$

$$250 \text{ GPa} = f_{\text{SiC}}(395 \text{ GPa}) + (1 - f_{\text{SiC}})(155 \text{ GPa})$$

$$f_{\text{SiC}} = \mathbf{0.396}$$

- 11.85 The largest preexisting flaw in the reaction-bonded silicon nitride matrix of Prob. 11.84 is $6.0 \mu\text{m}$ in diameter, and the largest surface notch on the SiC fibers is $3.5 \mu\text{m}$ deep. Calculate the stress at which cracks first form in the composite when stress is slowly applied under isostrain conditions and in the direction of the fibers. Data are $K_{\text{IC}}(\text{RBSN}) = 3.5 \text{ MPa}\cdot\sqrt{\text{m}}$ and $K_{\text{IC}}(\text{SiC}) = 4.8 \text{ MPa}\cdot\sqrt{\text{m}}$.

For isostrain conditions, $\epsilon_{\text{comp}} = \epsilon_{\text{SiC}} = \epsilon_{\text{RBSN}}$ and we can write

$$\frac{\sigma_{\text{comp}}}{E_{\text{comp}}} = \frac{\sigma_{\text{SiC}}}{E_{\text{SiC}}} = \frac{\sigma_{\text{RBSN}}}{E_{\text{RBSN}}}$$

Considering the SiC fibers, $a = 3.5 \mu\text{m}$ for surface cracks. Assuming $Y = 1$,

$$\sigma_{\text{comp}} = \frac{E_{\text{comp}}}{E_{\text{SiC}}} \left(\frac{K_{\text{IC, SiC}}}{\sqrt{\pi a}} \right) = \left(\frac{250 \text{ GPa}}{395 \text{ GPa}} \right) \left[\frac{4.8 \text{ MPa}\cdot\sqrt{\text{m}}}{\sqrt{\pi(3.5 \times 10^{-6} \text{ m})}} \right] = 916 \text{ MPa}$$

Considering the RBSN matrix, a is based on the largest preexisting flaw of $6 \mu\text{m}$; $a = 6 \mu\text{m}/2 = 3 \mu\text{m}$. Thus,

$$\sigma_{\text{comp}} = \frac{E_{\text{comp}}}{E_{\text{RBSN}}} \left(\frac{K_{\text{IC, RBSN}}}{\sqrt{\pi a}} \right) = \left(\frac{250 \text{ GPa}}{155 \text{ GPa}} \right) \left[\frac{3.5 \text{ MPa}\cdot\sqrt{\text{m}}}{\sqrt{\pi(3.0 \times 10^{-6} \text{ m})}} \right] = 1839 \text{ MPa}$$

Thus the SiC fibers will begin to crack first at an applied stress of **916 MPa**.

- 11.86 A ceramic-matrix composite is made with an aluminum oxide (Al_2O_3) matrix and continuous silicon carbide fiber reinforcement with all the SiC fibers in one direction. The composite consists of 30 vol % of SiC fibers. If isostrain conditions exist, calculate the tensile modulus of the composite in the direction of the fibers. If a load of 8 MN is applied to the composite in the direction of the fibers, what is the elastic strain in the composite if the surface area over which the load is applied is 55 cm^2 ? Data are $E(\text{Al}_2\text{O}_3) = 350$ GPa and $E(\text{SiC}) = 340$ GPa.

The tensile modulus in the direction of the fibers is

$$\begin{aligned}
 E_{\text{comp}} &= f_{\text{Al}_2\text{O}_3} E_{\text{Al}_2\text{O}_3} + f_{\text{SiC}} E_{\text{SiC}} \quad \text{where } f_{\text{Al}_2\text{O}_3} = 0.70, f_{\text{SiC}} = 0.30 \\
 &= (0.70)(350 \text{ GPa}) + (0.30)(340 \text{ GPa}) \\
 &= \mathbf{347.0 \text{ GPa}}
 \end{aligned}$$

The elastic strain in the composite for a 8MN load applied over 55 cm² is

$$\epsilon_{\text{comp}} = \frac{\sigma_{\text{comp}}}{E_{\text{comp}}} = \frac{P_{\text{comp}} / A_{\text{comp}}}{E_{\text{comp}}} = \frac{(8 \text{ MN}) / [(55 \text{ cm}^2)(10^{-4} \text{ m}^2/\text{cm}^2)]}{347 \times 10^3 \text{ MPa}} = \mathbf{4.19 \times 10^{-3}}$$

- 11.87 For Prob. 11.86, the Al₂O₃ matrix has flaws up to 10 μm in diameter and the largest surface notch of the SiC fibers is 4.5 μm. (a) Will the matrix or the fibers crack first? (b) What stress on the composite in the direction of the fibers will cause the first crack to form? Data are $K_{\text{IC}}(\text{Al}_2\text{O}_3) = 3.8 \text{ MPa}\cdot\sqrt{\text{m}}$ and $K_{\text{IC}}(\text{SiC}) = 4.6 \text{ MPa}\cdot\sqrt{\text{m}}$.

(a) Considering the SiC fibers, $a = 4.5 \mu\text{m}$ for surface cracks. Assuming $Y = 1$,

$$\sigma_{\text{comp}} = \frac{E_{\text{comp}}}{E_{\text{SiC}}} \left(\frac{K_{\text{IC, SiC}}}{\sqrt{\pi a}} \right) = \left(\frac{347 \text{ GPa}}{340 \text{ GPa}} \right) \left[\frac{4.6 \text{ MPa}\cdot\sqrt{\text{m}}}{\sqrt{\pi(4.5 \times 10^{-6} \text{ m})}} \right] = 1249 \text{ MPa}$$

Considering the Al₂O₃ matrix, a is based on the largest preexisting flaw of 10 μm; $a = 10 \mu\text{m}/2 = 5 \mu\text{m}$. Thus,

$$\sigma_{\text{comp}} = \frac{E_{\text{comp}}}{E_{\text{Al}_2\text{O}_3}} \left(\frac{K_{\text{IC, Al}_2\text{O}_3}}{\sqrt{\pi a}} \right) = \left(\frac{347 \text{ GPa}}{350 \text{ GPa}} \right) \left[\frac{3.8 \text{ MPa}\cdot\sqrt{\text{m}}}{\sqrt{\pi(5.0 \times 10^{-6} \text{ m})}} \right] = 951 \text{ MPa}$$

Thus the Al₂O₃ matrix will begin to crack first at an applied stress of **951 MPa**.