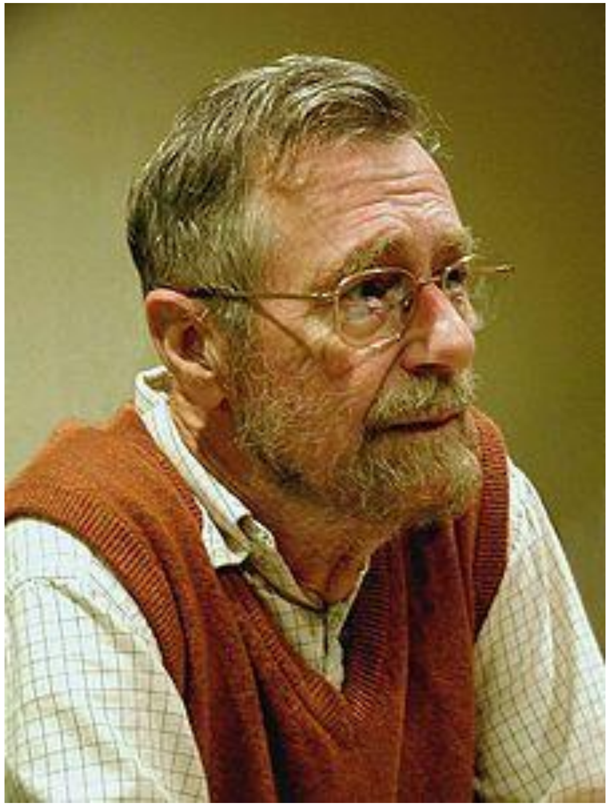
Dijkstra's algorithm

By Laksman Veeravagu and Luis Barrera

The author: Edsger Wybe Dijkstra



"Computer Science is no more about computers than astronomy is about telescopes."

http://www.cs.utexas.edu/~EWD/

Edsger Wybe Dijkstra

- May 11, 1930 – August 6, 2002

- Received the 1972 A. M. Turing Award, widely considered the most prestigious award in computer science.

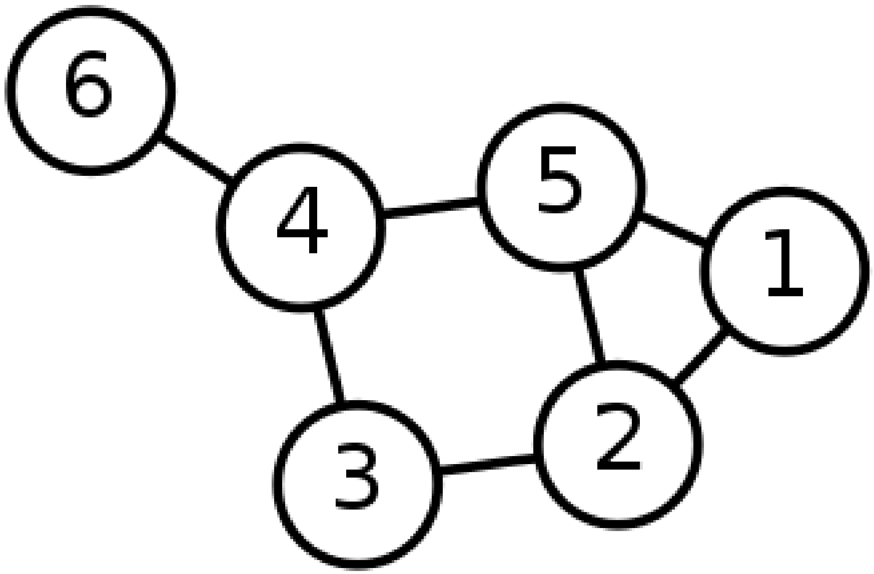
- The Schlumberger Centennial Chair of Computer Sciences at The University of Texas at Austin from 1984 until 2000

- Made a strong case against use of the GOTO statement in programming languages and helped lead to its deprecation.

- Known for his many essays on programming.

Single-Source Shortest Path Problem

Single-Source Shortest Path Problem - The problem of finding shortest paths from a source vertex *v* to all other vertices in the graph.



Dijkstra's algorithm

Dijkstra's algorithm - is a solution to the single-source shortest path problem in graph theory.

Works on both directed and undirected graphs. However, all edges must have nonnegative weights.

Approach: Greedy

Input: Weighted graph G={E,V} and source vertex *v*∈V, such that all edge weights are nonnegative

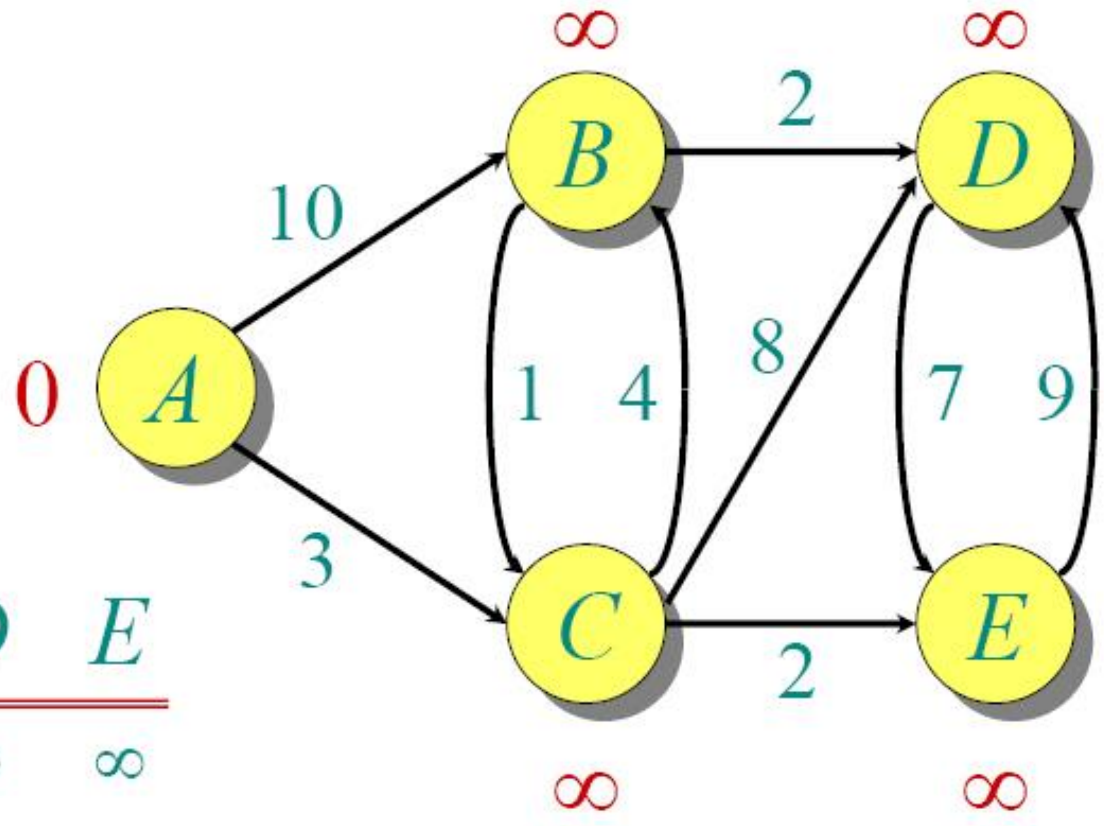
Output: Lengths of shortest paths (or the shortest paths themselves) from a given source vertex *v*∈V to all other vertices

DIJKSTRA'S ALGORITHM - PSEUDOCODE

```
dist[s] ←0                                    (distance to source vertex is zero)
for  all v ∈ V–{s}
        do  dist[v] ←∞                        (set all other distances to infinity)
S←∅                                           (S, the set of visited vertices is initially empty)
Q←V                                           (Q, the queue initially contains all vertices)
while Q ≠∅                                    (while the queue is not empty)
do   u ← mindistance(Q,dist)                  (select the element of Q with the min. distance)
      S←S∪{u}                                 (add u to list of visited vertices)
       for all v ∈ neighbors[u]
              do  if   dist[v] > dist[u] + w(u, v)          (if new shortest path found)
                         then      d[v] ←d[u] + w(u, v)     (set new value of shortest path)
                              (if desired, add traceback code)
return dist
```

DIJKSTRA ANIMATED EXAMPLE

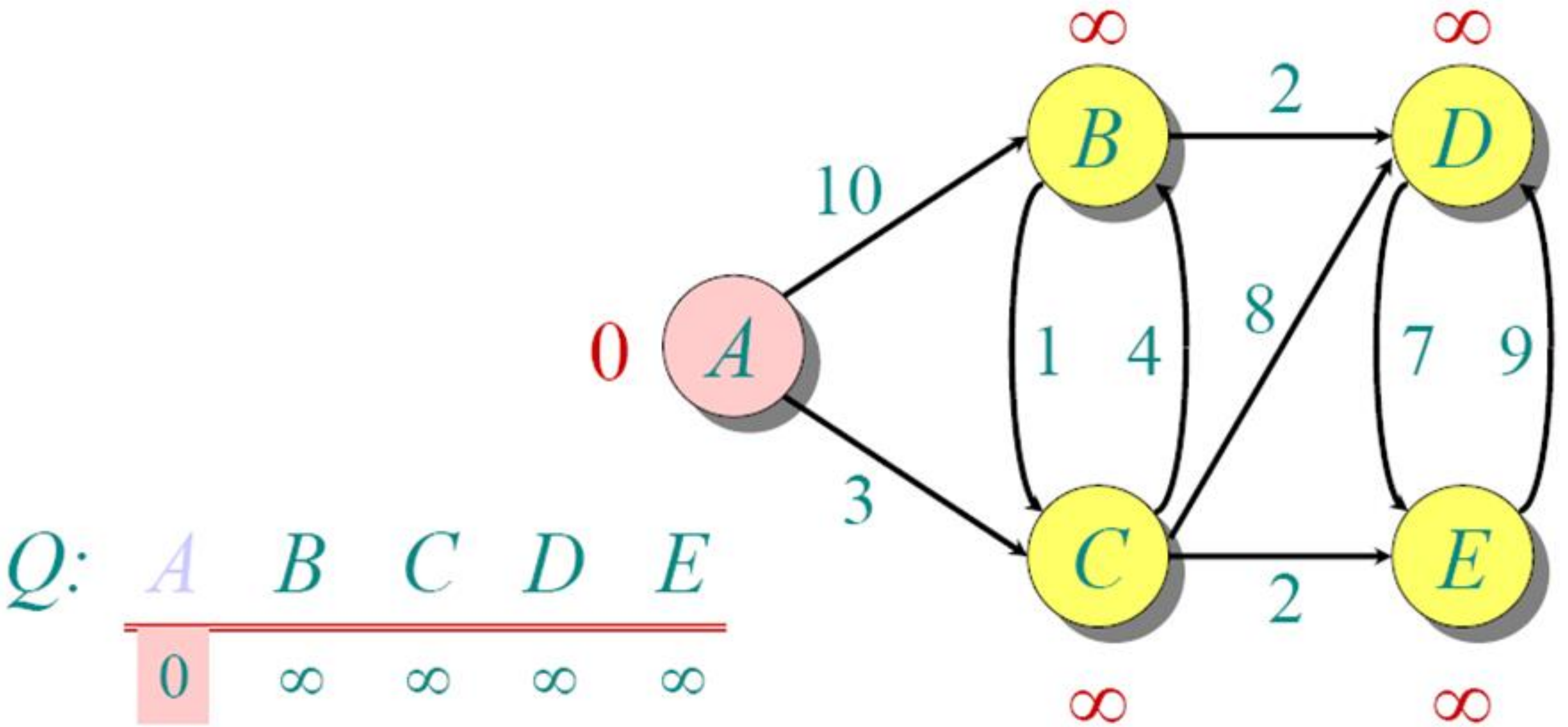
Initialize:



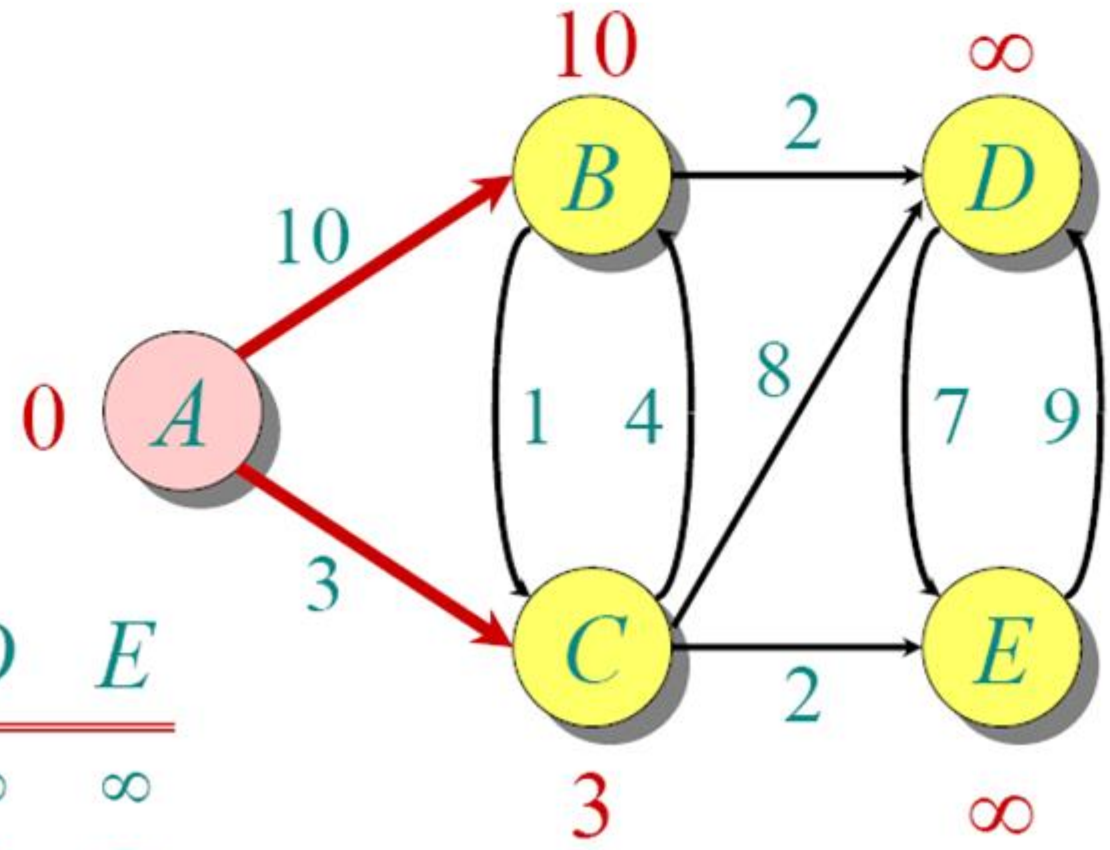
Q:	A	B	C	D	E
	0	∞	∞	∞	∞

S: {}

Dijkstra Animated Example



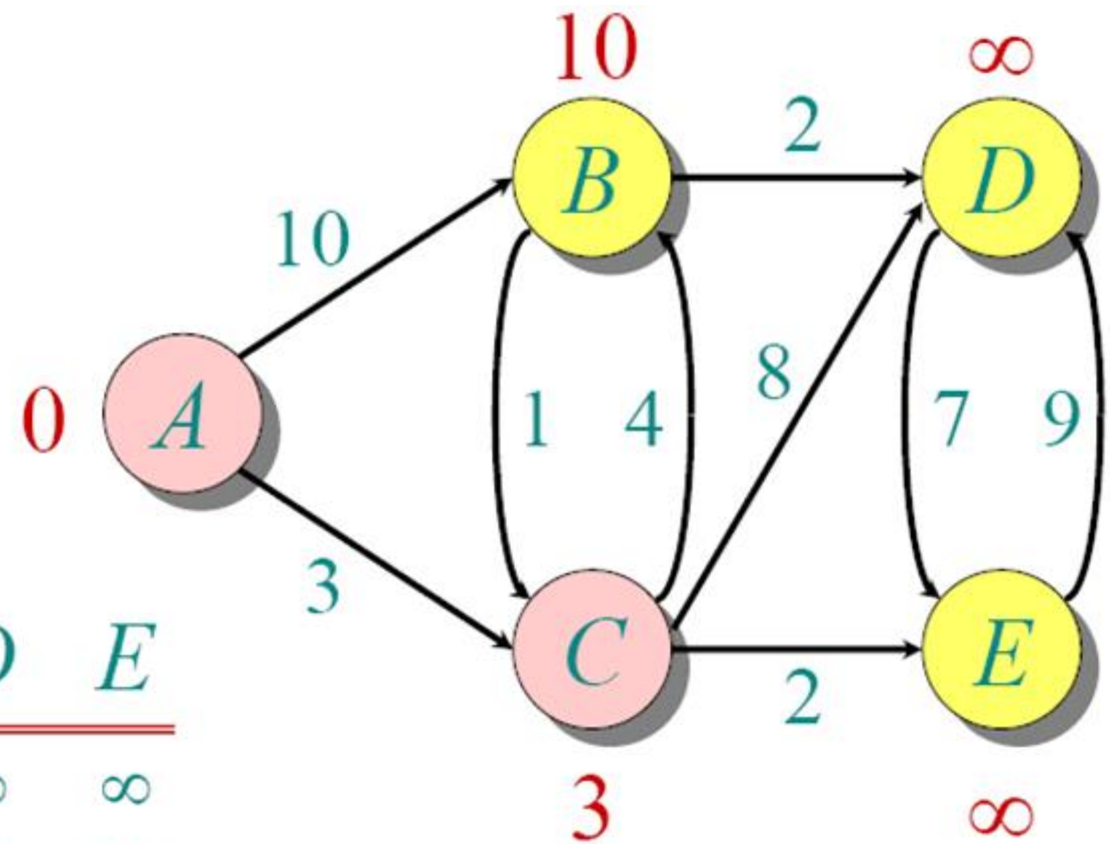
DIJKSTRA ANIMATED EXAMPLE



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞

S: { A }

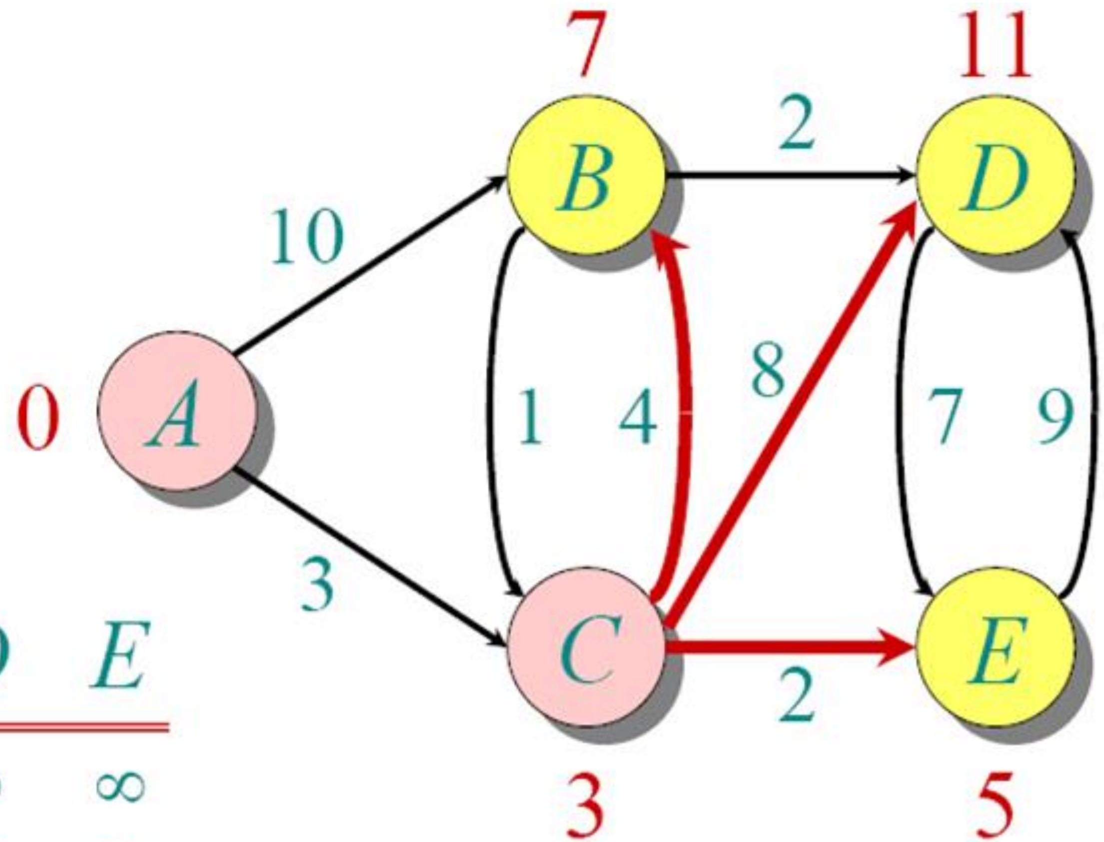
DIJKSTRA ANIMATED EXAMPLE



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞

S: { *A*, *C* }

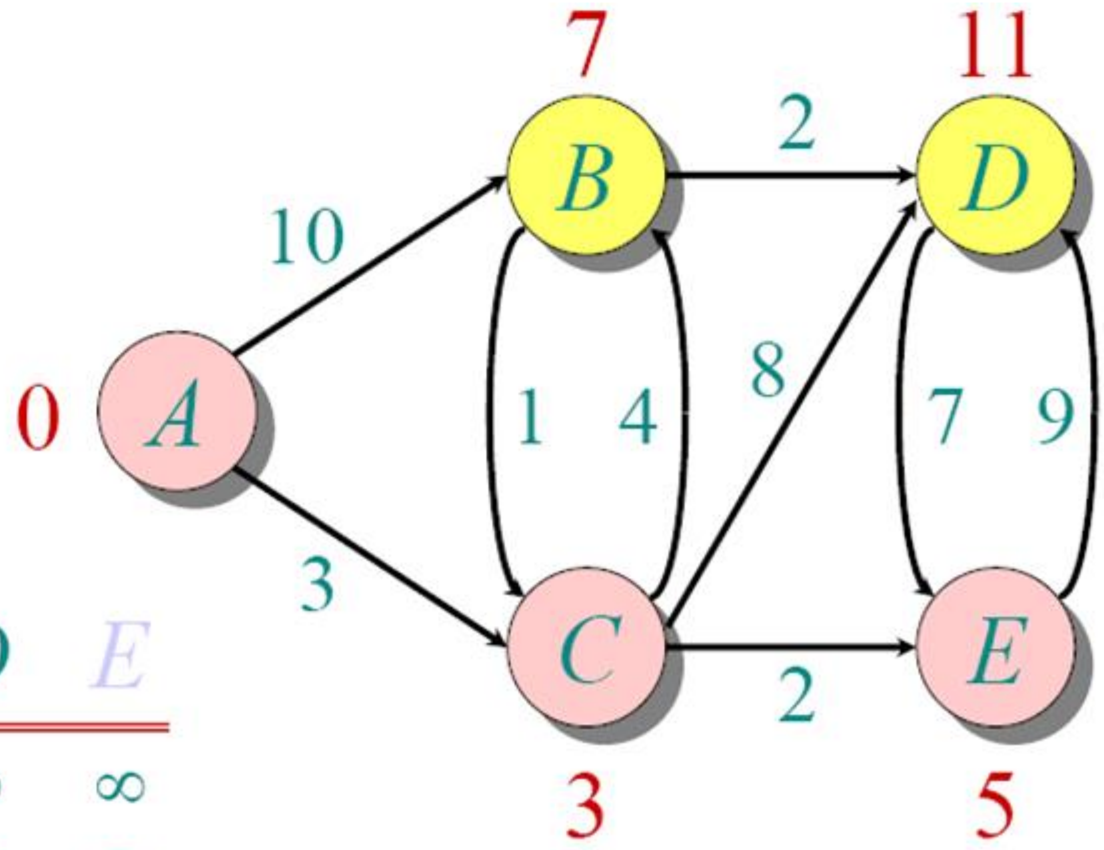
DIJKSTRA ANIMATED EXAMPLE



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5

S: { A, C }

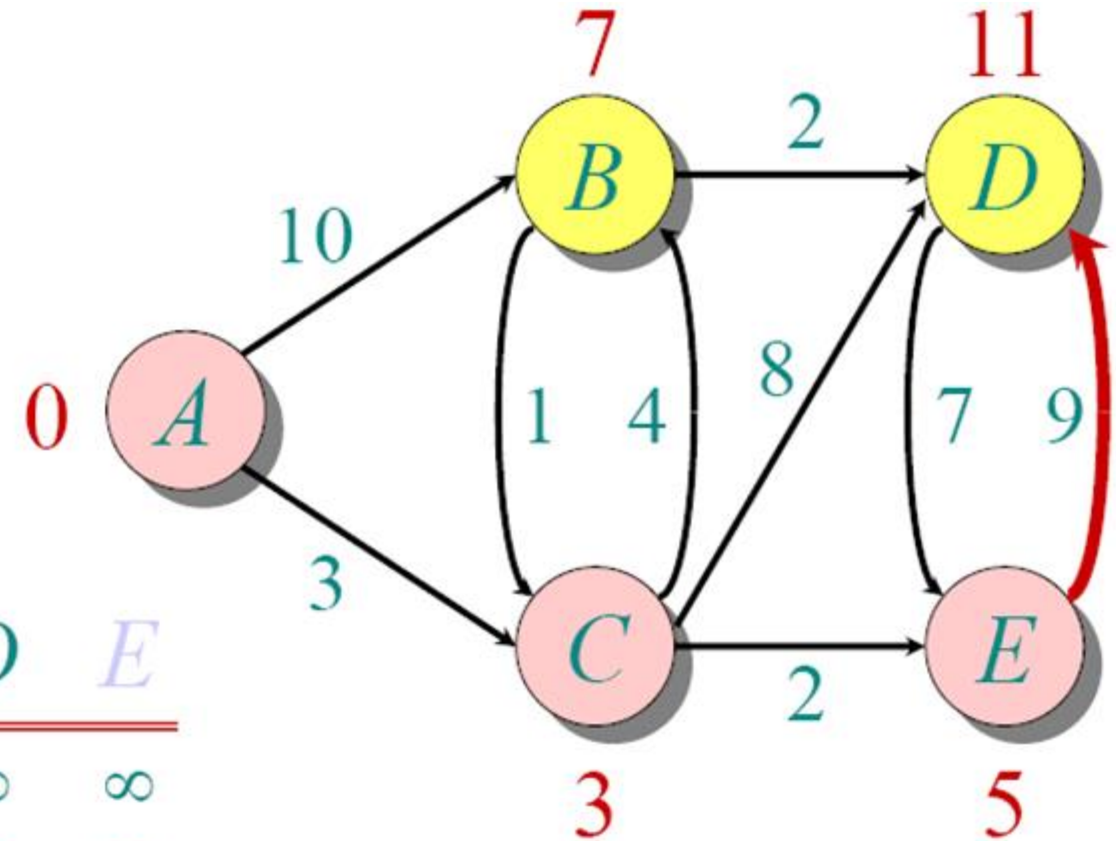
DIJKSTRA ANIMATED EXAMPLE



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5

S: { A, C, E }

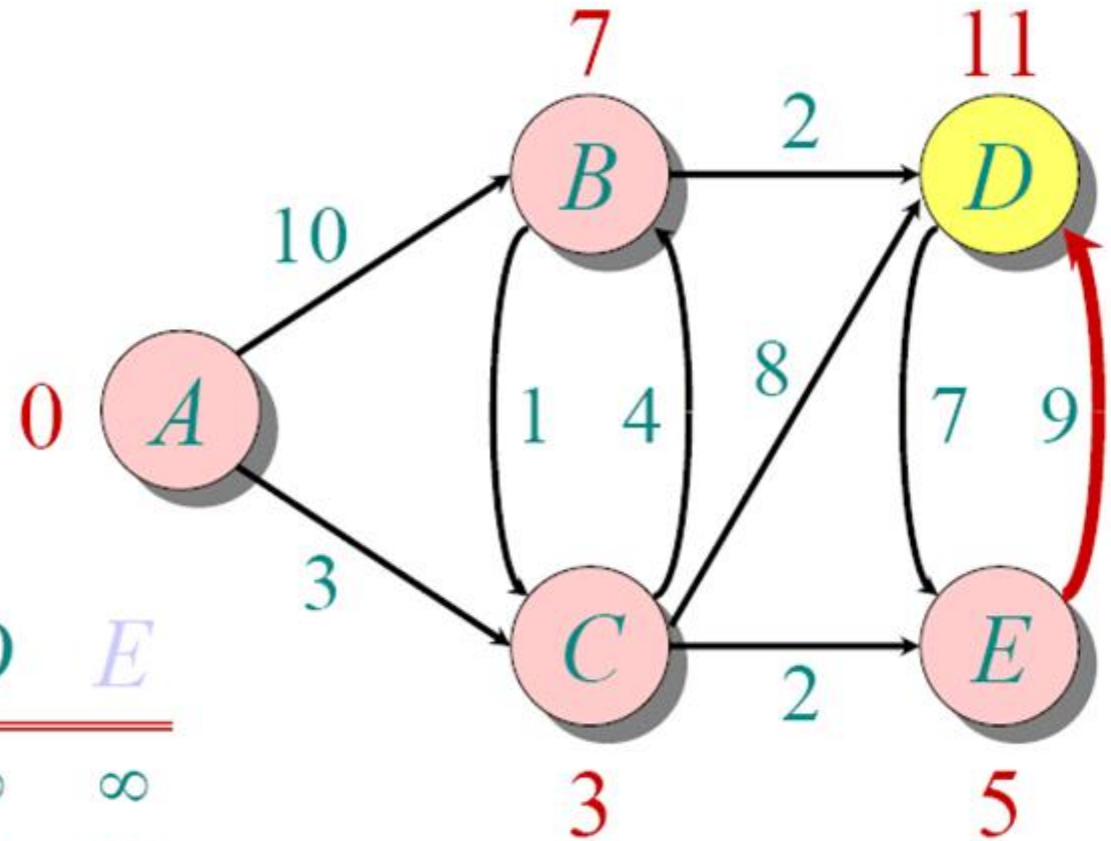
Dijkstra Animated Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

S: { A, C, E }

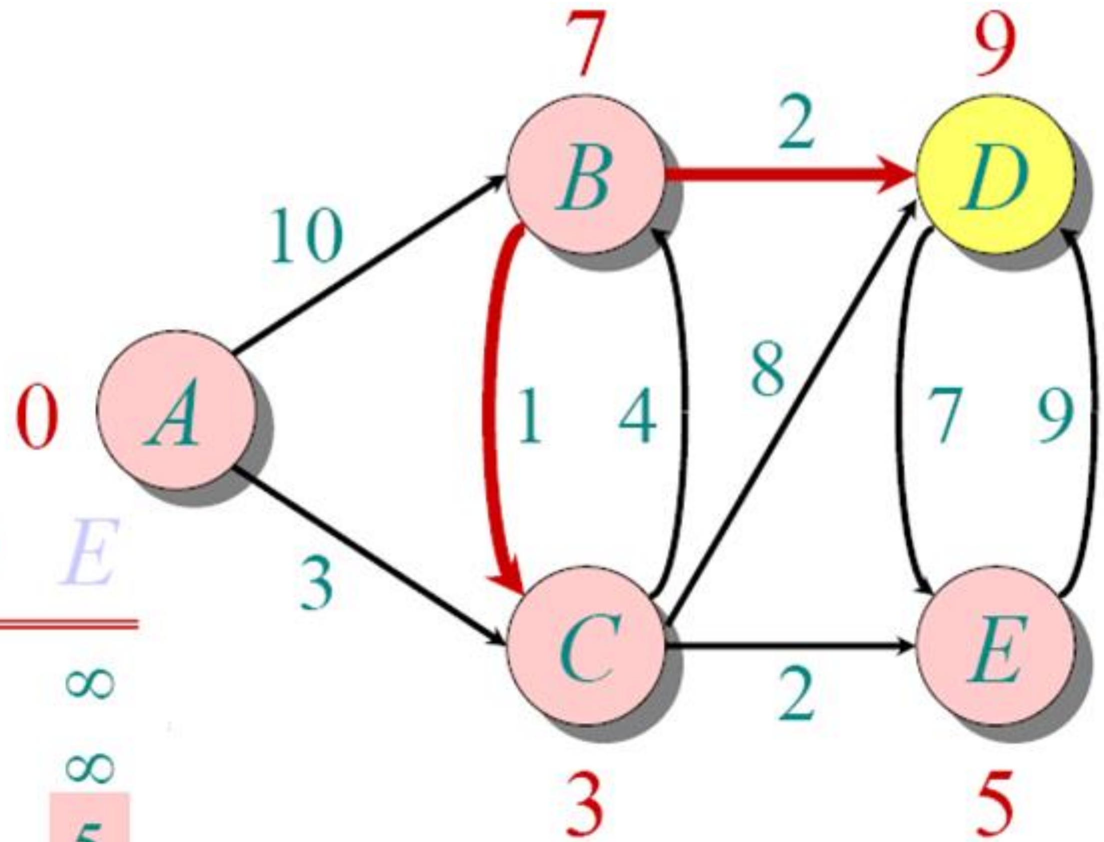
Dijkstra Animated Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

S: { A, C, E, B }

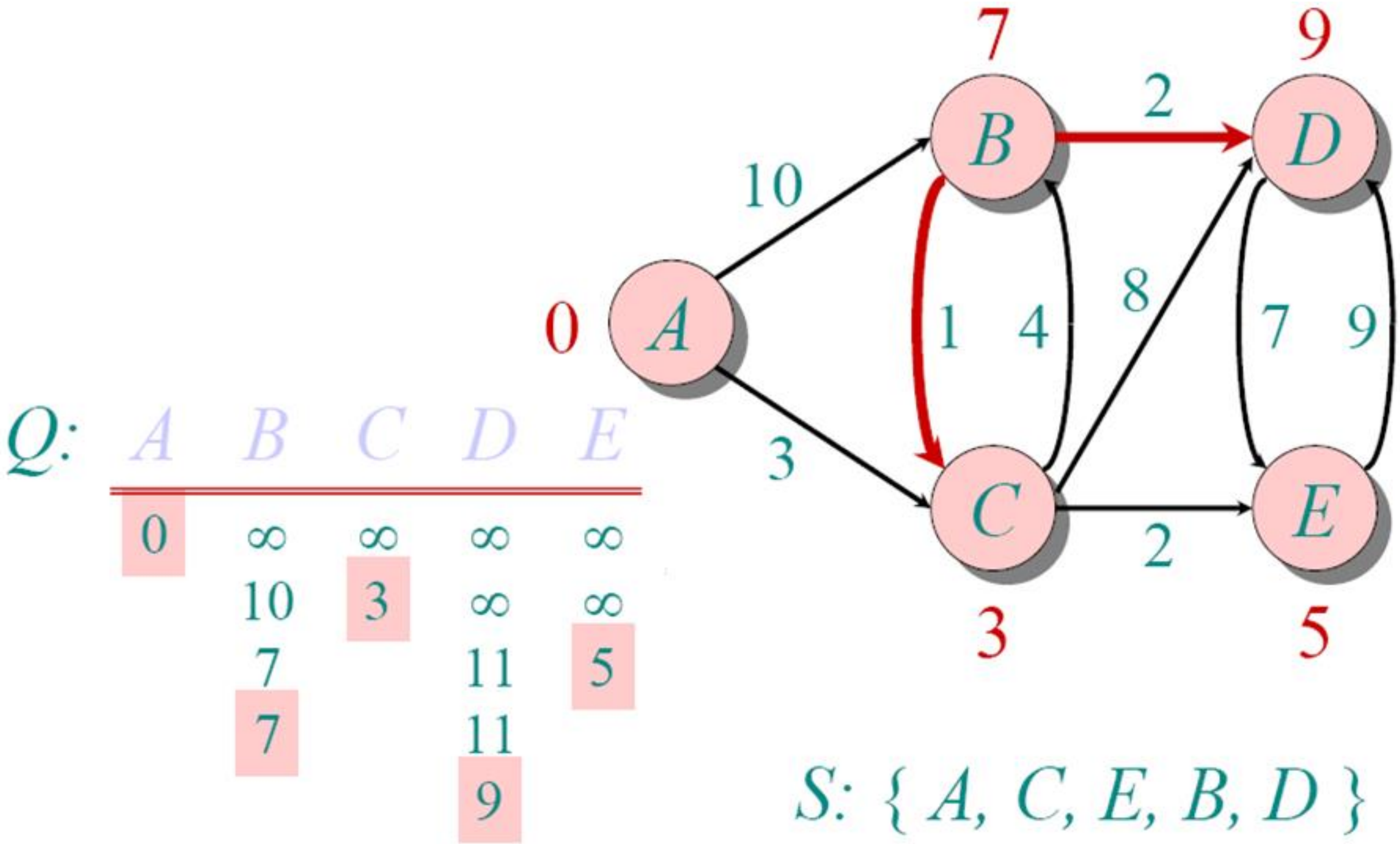
Dijkstra Animated Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	
				9	

S: { *A*, *C*, *E*, *B* }

DIJKSTRA ANIMATED EXAMPLE



Implementations and Running Times

The simplest implementation is to store vertices in an array or linked list. This will produce a running time of

$O(|V|^2 + |E|)$

For sparse graphs, or graphs with very few edges and many nodes, it can be implemented more efficiently storing the graph in an adjacency list using a binary heap or priority queue. This will produce a running time of

$O((|E|+|V|) \log |V|)$

Dijkstra's Algorithm - Why It Works

- As with all greedy algorithms, we need to make sure that it is a correct algorithm (e.g., it *always* returns the right solution if it is given correct input).

- A formal proof would take longer than this presentation, but we can understand how the argument works intuitively.

- If you can't sleep unless you see a proof, see the second reference or ask us where you can find it.

Dijkstra's Algorithm - Why It Works

- To understand how it works, we'll go over the previous example again. However, we need two mathematical results first:

- **Lemma 1**: Triangle inequality
  If δ(u,v) is the shortest path length between u and v,
  $\delta(u,v) \le \delta(u,x) + \delta(x,v)$
- **Lemma 2**:
  The subpath of any shortest path is itself a shortest path.

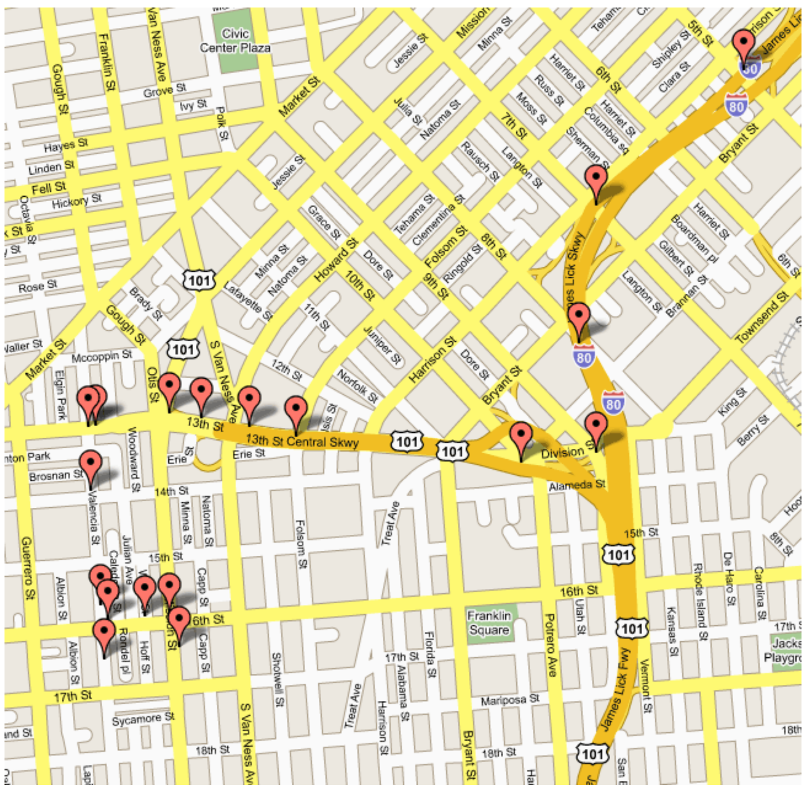
- The key is to understand why we can claim that anytime we put a new vertex in S, we can say that we already know the shortest path to it.
- Now, back to the example…

Dijkstra's Algorithm - Why use it?

- As mentioned, Dijkstra's algorithm calculates the shortest path to every vertex.
- However, it is about as computationally expensive to calculate the shortest path from vertex u to every vertex using Dijkstra's as it is to calculate the shortest path to some particular vertex v.
- Therefore, anytime we want to know the optimal path to some other vertex from a determined origin, we can use Dijkstra's algorithm.

Applications of Dijkstra's Algorithm

- Traffic Information Systems are most prominent use
- Mapping (Map Quest, Google Maps)
- Routing Systems



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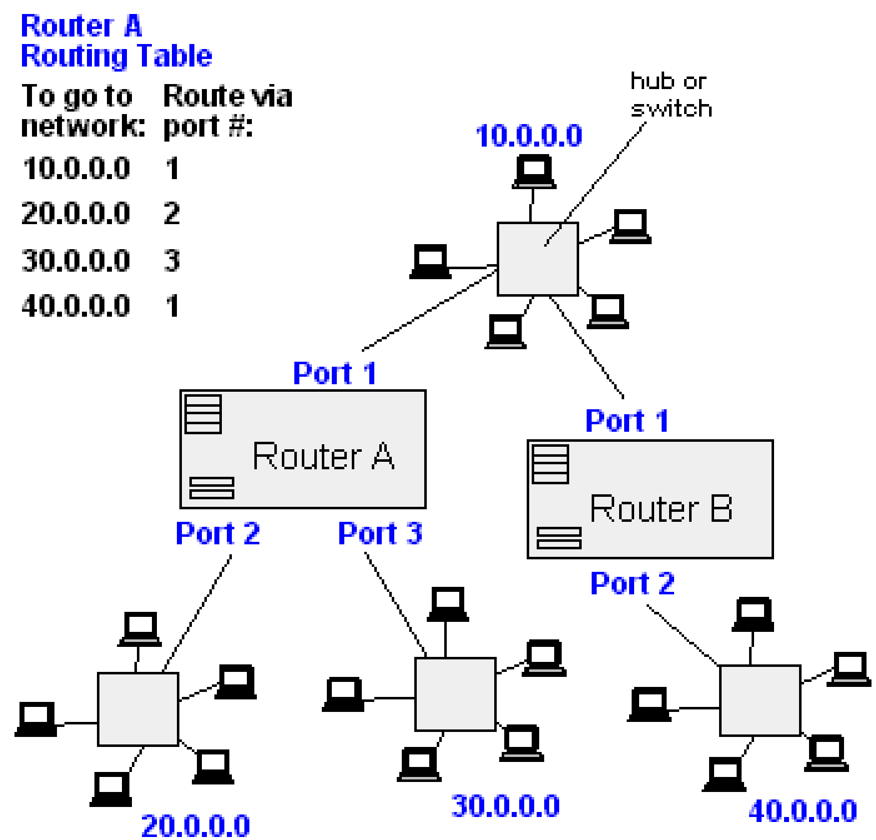
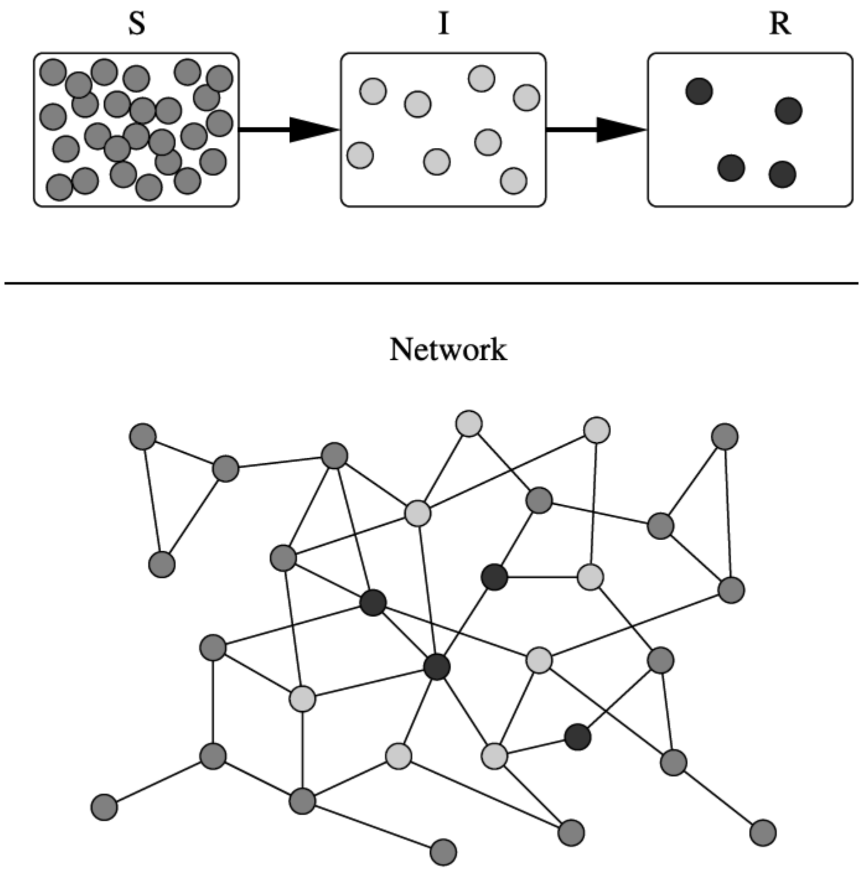


Applications of Dijkstra's Algorithm

- One particularly relevant this week: epidemiology
- Prof. Lauren Meyers (Biology Dept.) uses networks to model the spread of infectious diseases and design prevention and response strategies.
- Vertices represent individuals, and edges their possible contacts. It is useful to calculate how a particular individual is connected to others.
- Knowing the shortest path lengths to other individuals can be a relevant indicator of the potential of a particular individual to infect others.



References

- Dijkstra's original paper:
  E. W. Dijkstra. (1959) *A Note on Two Problems in Connection with Graphs*. Numerische Mathematik, 1. 269-271.
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