Análisis II - 2013 - 1ª Semana

1) ejer. 14 (secc. 8.3)

$$\left.\begin{aligned} x &= t-\cos t = \frac{\pi}{4}-\frac{1}{\sqrt2} \\ \frac{dx}{dt} &= 1+\sin t = 1+\frac{1}{\sqrt2} \\ y &= 1-\sin t = 1-\frac{1}{\sqrt2} \\ \frac{dy}{dt} &= -\cos t = -\frac{1}{\sqrt2} \end{aligned}\right\} t=\pi/4$$

La recta tangente tiene de ecuaciones paramétricas

$$y=\left(\frac{\pi}{4}-\frac{1}{\sqrt2}\right)+\left(1+\frac{1}{\sqrt2}\right)t$$

$$x=\left(1-\frac{1}{\sqrt2}\right)-\frac{t}{\sqrt2}$$

2) ejemplo 5, pág 701

3) ejer 13 (secc. 12.8)

$u=u(x,y)$

$v=v(x,y)$

$$\left.\begin{aligned} x &= u^3+v^3 \\ y &= uv-v^2 \end{aligned}\right| \quad \begin{aligned} 1 &= 3u^2\frac{\partial u}{\partial x}+3v^2\frac{\partial v}{\partial x} \\ 0 &= v\frac{\partial u}{\partial x}+(u-2v)\frac{\partial v}{\partial x} \end{aligned} \quad \Big|\ u=v=1 \ \Big\{ \begin{aligned} 1 &= 3\frac{\partial u}{\partial x}+3\frac{\partial v}{\partial x} \\ 0 &= \frac{\partial u}{\partial x}-\frac{\partial v}{\partial x} \end{aligned} \Big\} \Rightarrow \frac{\partial u}{\partial x}=\frac{\partial v}{\partial x}=1/6$$

Derivando respecto de y haciendo $u=v=1$, se tiene

$$\left\{ 0=3\frac{\partial u}{\partial y}+3\frac{\partial v}{\partial y},\ 1=\frac{\partial u}{\partial y}-\frac{\partial v}{\partial y} \right\} \Rightarrow \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial y}=1/2$$

$$\frac{\partial(u,v)}{\partial(x,y)}=\begin{vmatrix} 1/6 & 1/6 \\ 1/2 & -1/2 \end{vmatrix}=\boxed{-\frac{1}{6}}$$

4) ejer. 12 (secc. 15.4)

$$\vec F=(y^2\cos x+z^3)\vec i+(2y\sin x-4)\vec j+(3xz^2+2)\vec k=$$

$$=\nabla(\underbrace{y^2\sin x+xz^3-4y+2z}_{\phi(x,y,z)})$$

$$\mathcal{C}\left\{\begin{aligned} x &= \sin^{-1}t \\ y &= 1-2t \\ z &= 3t-1 \end{aligned}\right| (0\le t\le 1)$$

extr. $(0,1,-1)$ y $(\pi/2,-1,2)$

$$W=\int_{\mathcal{C}}\vec F\cdot d\vec r=\Phi_2-\Phi_1=(y^2\sin x+xz^3-4y+2z)\Big|_{(0,1,-1)}^{(\pi/2,-1,2)}$$

$$=\boxed{15+4\pi}$$

Análisis II – 2013 – 2ª semana

1) ejer 15 (secc. 8.3) $\begin{cases} x = t^3 - t \\ y = t^2 \end{cases}$ $(0,1)$ $\begin{cases} t = -1 \\ t = 1 \end{cases}$

Puesto que $\dfrac{dy}{dx} = \dfrac{2t}{3t^2-1} = \dfrac{\pm 2}{2} = \pm 1$

Las tangentes en $(0,1)$ en $t = \pm 1$ tienen pendientes $\boxed{\pm 1}$

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2) ejemplo 3, pág. 715

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3) ejercc. 14 (secc 14.4) $\displaystyle\iint_{x^2+y^2\le 1} \ln(x^2+y^2)\,dA = \int_0^{2\pi} d\theta \int_0^1 (\ln r^2)\, r\,dr =$

$\displaystyle = 4\pi \int_0^1 r(\ln r)\,dr = 4\pi \left[\frac{r^2}{2}\ln r\right]_0^1 - \frac{1}{2}\int_0^1 r\,dr = 4\pi\left[0 - 0 - \tfrac{1}{4}\right] = \boxed{-\pi}$

$u = \ln r \;|\; dv = r\,dr$

$du = \dfrac{dr}{r} \;|\; v = \dfrac{r^2}{2}$

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4) ejer. repaso 6 (Capítulo 15.6) – pág. 992

$\hat{N}\,dS = \dfrac{-\vec{i} - 2\vec{j} - 3\vec{k}}{3}\,dx\,dy$. Si S es la parte del plano en el primer octante, entonces, la proyección de S sobre xy es el triángulo $0 \le y \le 3$, $0 \le x \le 6 - 2y$, luego:

$\displaystyle\iint_S (x\vec{i} + y\vec{j} + z\vec{k}) \cdot \hat{N}\,dS = -\frac{1}{3}\int_0^3 dy \int_0^{6-2y} (x + 2y + 6 - x - 2y)\,dx =$

$\displaystyle = -2\int_0^3 (6-2y) = -36 + 18 = \boxed{-18}$