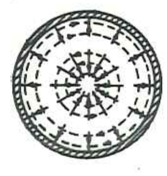
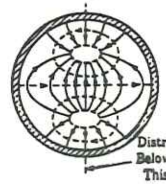
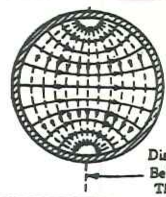
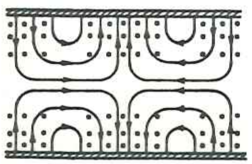
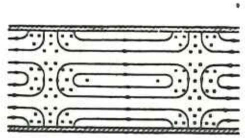
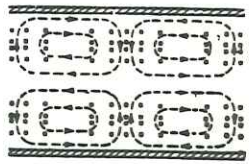
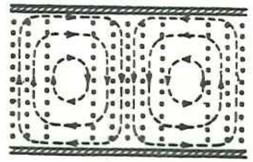
Fíjense en la dirección de las líneas de E.
BAJA ATENUACIÓN

Similar al TE_{10} en guías rectangulares

Table 8.9
Summary of Wave Types for Circular Guides[a]

Wave Type	TM_{01}	TM_{02}	TM_{11}	TE_{01}	TE_{11}
Field distributions in cross-sectional plane, at plane of maximum transverse fields			 Distributions Below Along This Plane		 Distributions Below Along This Plane
Field distributions along guide					
Field components present	E_z, E_r, H_ϕ	E_z, E_r, H_ϕ	$E_z, E_r, E_\phi, H_r, H_\phi$	H_z, H_r, E_ϕ	$H_z, H_r, H_\phi, E_r, E_\phi$
p_{nl} or p'_{nl}	2.405	5.52	3.83	3.83	1.84
$(k_c)_{nl}$	$\frac{2.405}{a}$	$\frac{5.52}{a}$	$\frac{3.83}{a}$	$\frac{3.83}{a}$	$\frac{1.84}{a}$
$(\lambda_c)_{nl}$	$2.61a$	$1.14a$	$1.64a$	$1.64a$	$3.41a$
$(f_c)_{nl}$	$\frac{0.383}{a\sqrt{\mu\varepsilon}}$	$\frac{0.877}{a\sqrt{\mu\varepsilon}}$	$\frac{0.609}{a\sqrt{\mu\varepsilon}}$	$\frac{0.609}{a\sqrt{\mu\varepsilon}}$	$\frac{0.293}{a\sqrt{\mu\varepsilon}}$
Attenuation due to imperfect conductors	$\frac{R_S}{a\eta}\frac{1}{\sqrt{1-(f_c/f)^2}}$	$\frac{R_S}{a\eta}\frac{1}{\sqrt{1-(f_c/f)^2}}$	$\frac{R_S}{a\eta}\frac{1}{\sqrt{1-(f_c/f)^2}}$	$\frac{R_S}{a\eta}\frac{(f_c/f)^2}{\sqrt{1-(f_c/f)^2}}$	$\frac{R_S}{a\eta}\frac{1}{\sqrt{1-(f_c/f)^2}}\left[\left(\frac{f_c}{f}\right)^2+0.420\right]$

[a] Electric field lines are shown solid and magnetic field lines are dashed.

TABLE 9-3
Summary of TE^z_{mn} and TM^z_{mn} mode characteristics of circular waveguide

	$TE^z_{mn}\begin{pmatrix} m = 0, 1, 2, \ldots, \\ n = 1, 2, 3, \ldots \end{pmatrix}$	$TM^z_{mn}\begin{pmatrix} m = 0, 1, 2, 3, \ldots, \\ n = 1, 2, 3, 4, \ldots \end{pmatrix}$
E^+_ρ	$-A_{mn}\dfrac{m}{\varepsilon\rho}J_m(\beta_\rho\rho)[-C_2\sin(m\phi) + D_2\cos(m\phi)]e^{-j\beta_z z}$	$-B_{mn}\dfrac{\beta_\rho\beta_z}{\omega\mu\varepsilon}J'_m(\beta_\rho\rho)[C_2\cos(m\phi) + D_2\sin(m\phi)]e^{-j\beta_z z}$
E^+_ϕ	$A_{mn}\dfrac{\beta_\rho}{\varepsilon}J'_m(\beta_\rho\rho)[C_2\cos(m\phi) + D_2\sin(m\phi)]e^{-j\beta_z z}$	$-B_{mn}\dfrac{m\beta_z}{\omega\mu\varepsilon}\dfrac{1}{\rho}J_m(\beta_\rho\rho)[-C_2\sin(m\phi) + D_2\cos(m\phi)]e^{-j\beta_z z}$
E^+_z	0	$-jB_{mn}\dfrac{\beta_\rho^2}{\omega\mu\varepsilon}J_m(\beta_\rho\rho)[C_2\cos(m\phi) + D_2\sin(m\phi)]e^{-j\beta_z z}$
H^+_ρ	$-A_{mn}\dfrac{\beta_\rho\beta_z}{\omega\mu\varepsilon}J'_m(\beta_\rho\rho)[C_2\cos(m\phi) + D_2\sin(m\phi)]e^{-j\beta_z z}$	$B_{mn}\dfrac{m}{\mu}\dfrac{1}{\rho}J_m(\beta_\rho\rho)[-C_2\sin(m\phi) + D_2\cos(m\phi)]e^{-j\beta_z z}$
H^+_ϕ	$-A_{mn}\dfrac{m\beta_z}{\omega\mu\varepsilon}\dfrac{1}{\rho}J_m(\beta_\rho\rho)[-C_2\sin(m\phi) + D_2\cos(m\phi)]e^{-j\beta_z z}$	$-B_{mn}\dfrac{\beta_\rho}{\mu}J'_m(\beta_\rho\rho)[-C_2\cos(m\phi) + D_2\sin(m\phi)]e^{-j\beta_z z}$
H^+_z	$-jA_{mn}\dfrac{\beta_\rho^2}{\omega\mu\varepsilon}J_m(\beta_\rho\rho)[C_2\cos(m\phi) + D_2\sin(m\phi)]e^{-j\beta_z z}$	0
$'$	$\dfrac{\partial}{\partial(\beta_\rho\rho)}$	
$\beta_c = \beta_\rho$	$\dfrac{\chi'_{mn}}{a}$	$\dfrac{\chi_{mn}}{a}$
f_c	$\dfrac{\chi'_{mn}}{2\pi a\sqrt{\mu\varepsilon}}$	$\dfrac{\chi_{mn}}{2\pi a\sqrt{\mu\varepsilon}}$
λ_c	$\dfrac{2\pi a}{\chi'_{mn}}$	$\dfrac{2\pi a}{\chi_{mn}}$

TABLE 9-3 *(Continued).*

	$TE^z_{mn}\begin{pmatrix} m = 0, 1, 2, \ldots, \\ n = 1, 2, 3, \ldots \end{pmatrix}$	$TM^z_{mn}\begin{pmatrix} m = 0, 1, 2, 3, \ldots, \\ n = 1, 2, 3, 4, \ldots \end{pmatrix}$
$\beta_z\ (f \geq f_c)$	$\beta\sqrt{1-\left(\dfrac{f_c}{f}\right)^2} = \beta\sqrt{1-\left(\dfrac{\lambda}{\lambda_c}\right)^2}$	
$\lambda_g\ (f \geq f_c)$	$\dfrac{\lambda}{\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}} = \dfrac{\lambda}{\sqrt{1-\left(\dfrac{\lambda}{\lambda_c}\right)^2}}$	
$v_p\ (f \geq f_c)$	$\dfrac{v}{\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}} = \dfrac{v}{\sqrt{1-\left(\dfrac{\lambda}{\lambda_c}\right)^2}}$	
$Z_w\ (f \geq f_c)$	$\dfrac{\eta}{\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}} = \dfrac{\eta}{\sqrt{1-\left(\dfrac{\lambda}{\lambda_c}\right)^2}}$	$\eta\sqrt{1-\left(\dfrac{f_c}{f}\right)^2} = \eta\sqrt{1-\left(\dfrac{\lambda}{\lambda_c}\right)^2}$
$Z_w\ (f \leq f_c)$	$j\dfrac{\eta}{\sqrt{\left(\dfrac{f_c}{f}\right)^2 - 1}} = j\dfrac{\eta}{\sqrt{\left(\dfrac{\lambda}{\lambda_c}\right)^2 - 1}}$	$-j\eta\sqrt{\left(\dfrac{f_c}{f}\right)^2 - 1} = -j\eta\sqrt{\left(\dfrac{\lambda}{\lambda_c}\right)^2 - 1}$
α_c	$\dfrac{R_s}{a\eta\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}}\left[\left(\dfrac{f_c}{f}\right)^2 + \dfrac{m^2}{(\chi'_{mn})^2 - m^2}\right]$	$\dfrac{R_s}{a\eta}\dfrac{1}{\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}}$

TE

TABLE 9-1
Zeroes χ'_{mn} of derivative $J'_m(\chi'_{mn}) = 0$ ($n = 1, 2, 3, \ldots$) of the Bessel function $J_m(x)$

	$m = 0$	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$	$m = 6$	$m = 7$	$m = 8$	$m = 9$	$m = 10$	$m = 11$
$n = 1$	3.8318	1.8412	3.0542	4.2012	5.3175	6.4155	7.5013	8.5777	9.6474	10.7114	11.7708	12.826
$n = 2$	7.0156	5.3315	6.7062	8.0153	9.2824	10.5199	11.7349	12.9324	14.1155	15.2867	16.4479	17.600
$n = 3$	10.1735	8.5363	9.9695	11.3459	12.6819	13.9872	15.2682	16.5294	17.7740	19.0046	20.2230	21.430
$n = 4$	13.3237	11.7060	13.1704	14.5859	15.9641	17.3129	18.6375	19.9419	21.2291	22.5014	23.7607	25.008
$n = 5$	16.4706	14.8636	16.3475	17.7888	19.1960	20.5755	21.9317	23.2681	24.5872	25.8913	27.1820	28.460

TM

TABLE 9-2
Zeroes χ_{mn} of $J_m(\chi_{mn}) = 0$ ($n = 1, 2, 3, \ldots$) of Bessel function $J_m(x)$

	$m = 0$	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$	$m = 6$	$m = 7$	$m = 8$	$m = 9$	$m = 10$	$m = 11$
$n = 1$	2.4049	3.8318	5.1357	6.3802	7.5884	8.7715	9.9361	11.0864	12.2251	13.3543	14.4755	15.5898
$n = 2$	5.5201	7.0156	8.4173	9.7610	11.0647	12.3386	13.5893	14.8213	16.0378	17.2412	18.4335	19.6160
$n = 3$	8.6537	10.1735	11.6199	13.0152	14.3726	15.7002	17.0038	18.2876	19.5545	20.8071	22.0470	23.2759
$n = 4$	11.7915	13.3237	14.7960	16.2235	17.6160	18.9801	20.3208	21.6415	22.9452	24.2339	25.5095	26.7733
$n = 5$	14.9309	16.4706	17.9598	19.4094	20.8269	22.2178	23.5861	24.9349	26.2668	27.5838	28.8874	30.1791

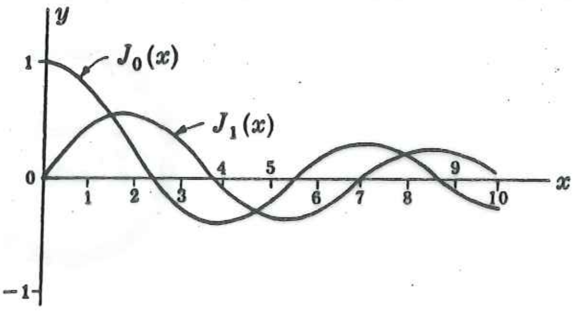


Fig. 24-1

funciones de Neumann

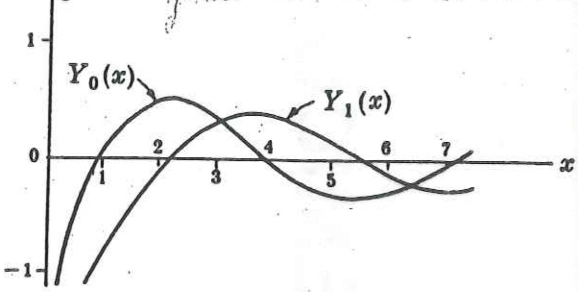


Fig. 24-2

funcion de Bessel modificada

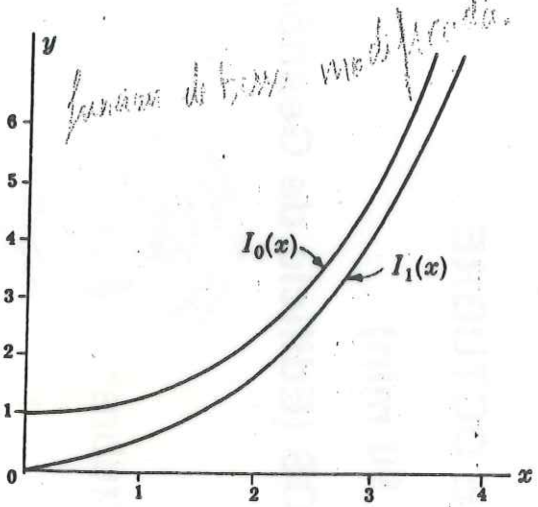


Fig. 24-3

funciones de Bessel ...
(2ª solución de E.B.M.
... entera)

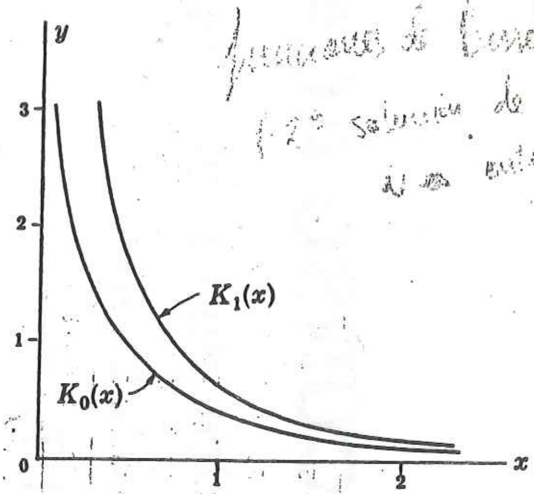


Fig. 24-4

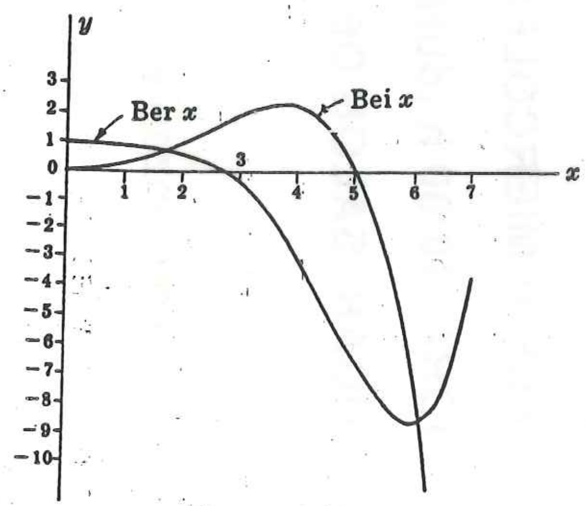


Fig. 24-5

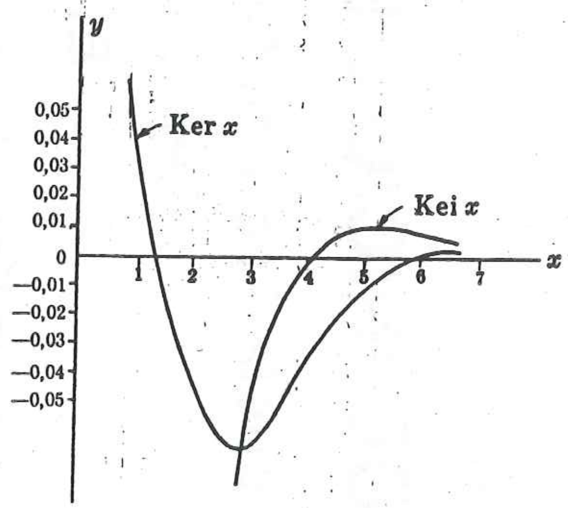
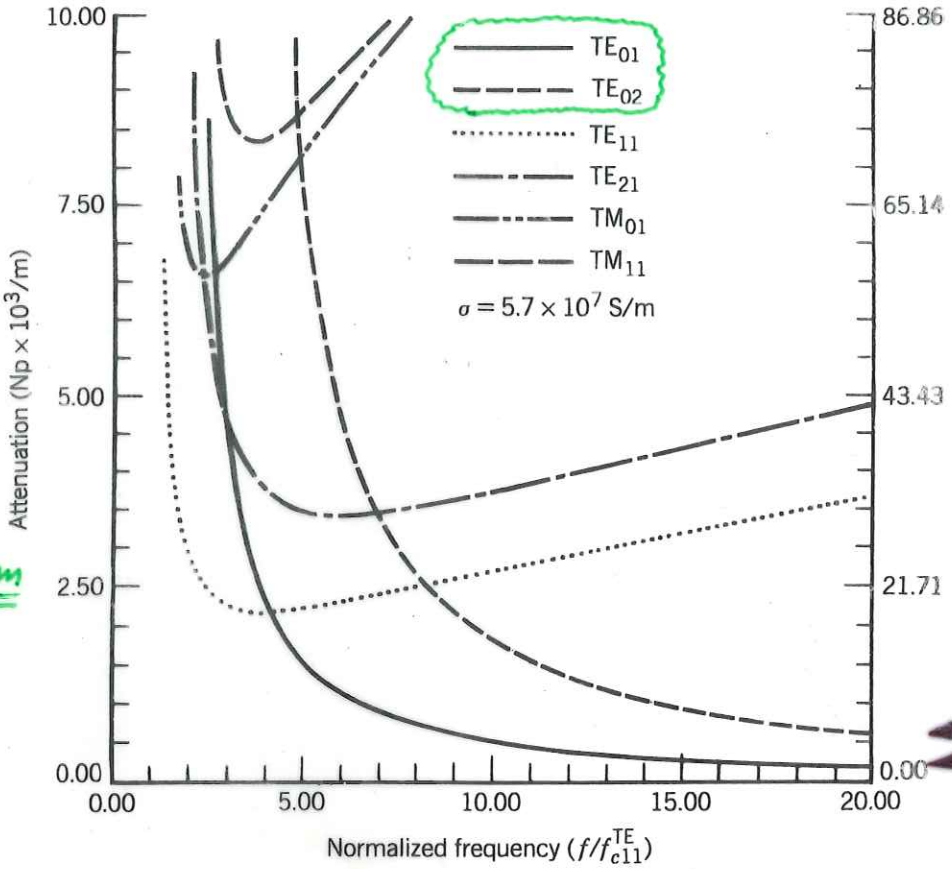


Fig. 24-6

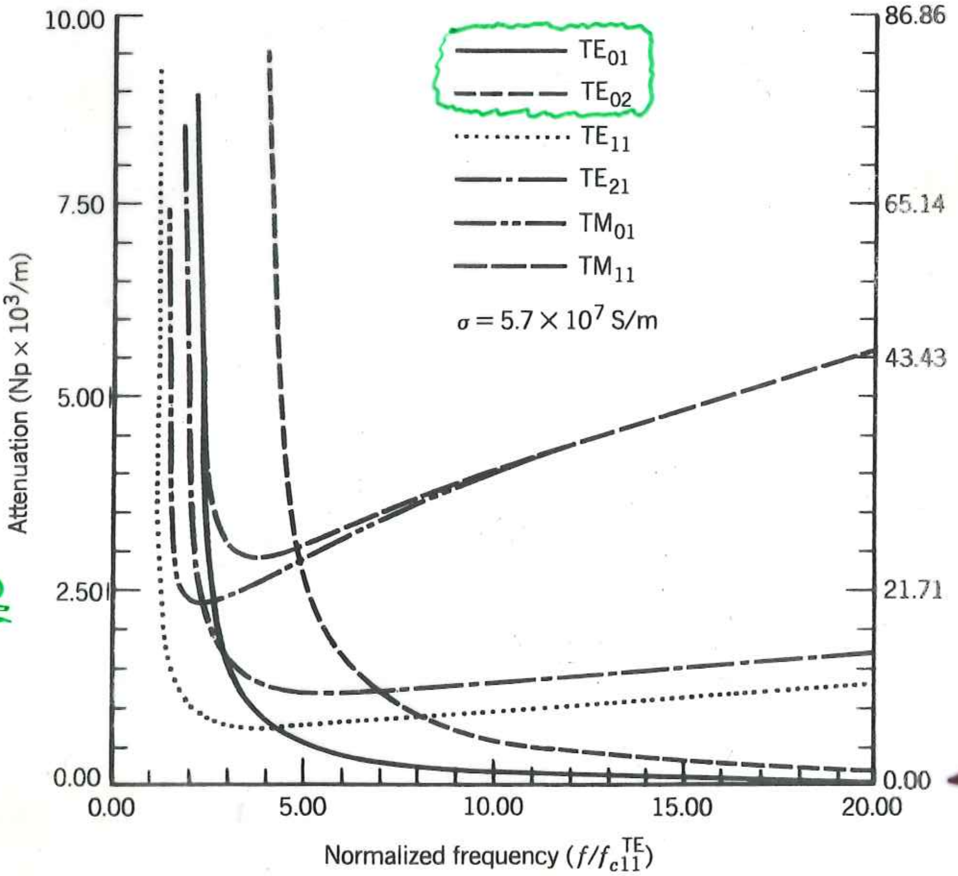
a = 1.5 cm



Observar esta tendencia a cero

(a)

a = 3 cm



(b)

FIGURE 9-4 Attenuation for TE^z_{mn} and TM^z_{mn} modes in a circular waveguide. (*a*) $a = 1.5$ cm. (*b*) $a = 3$ cm.

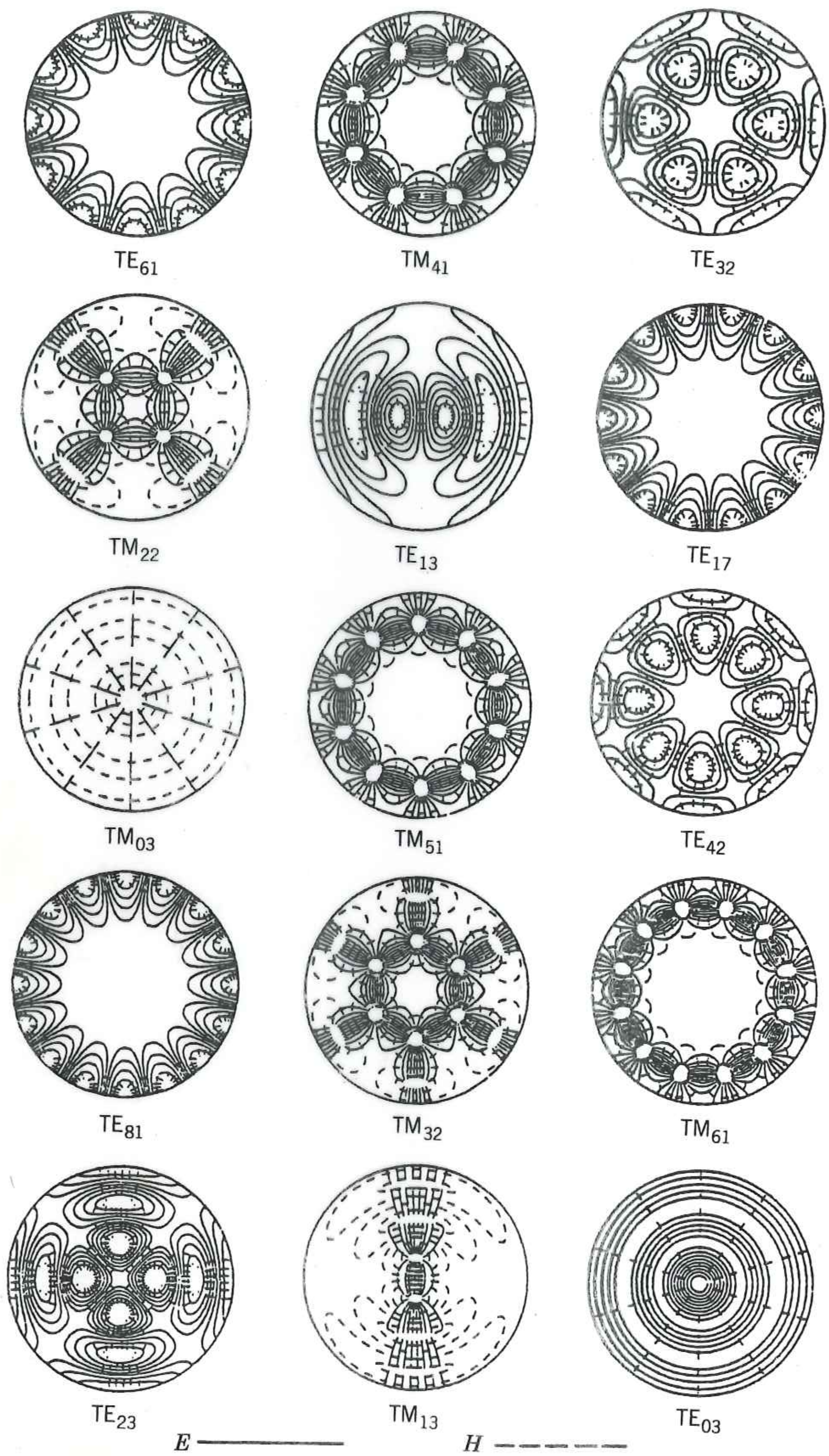


FIGURE 9-2 (*Continued*).

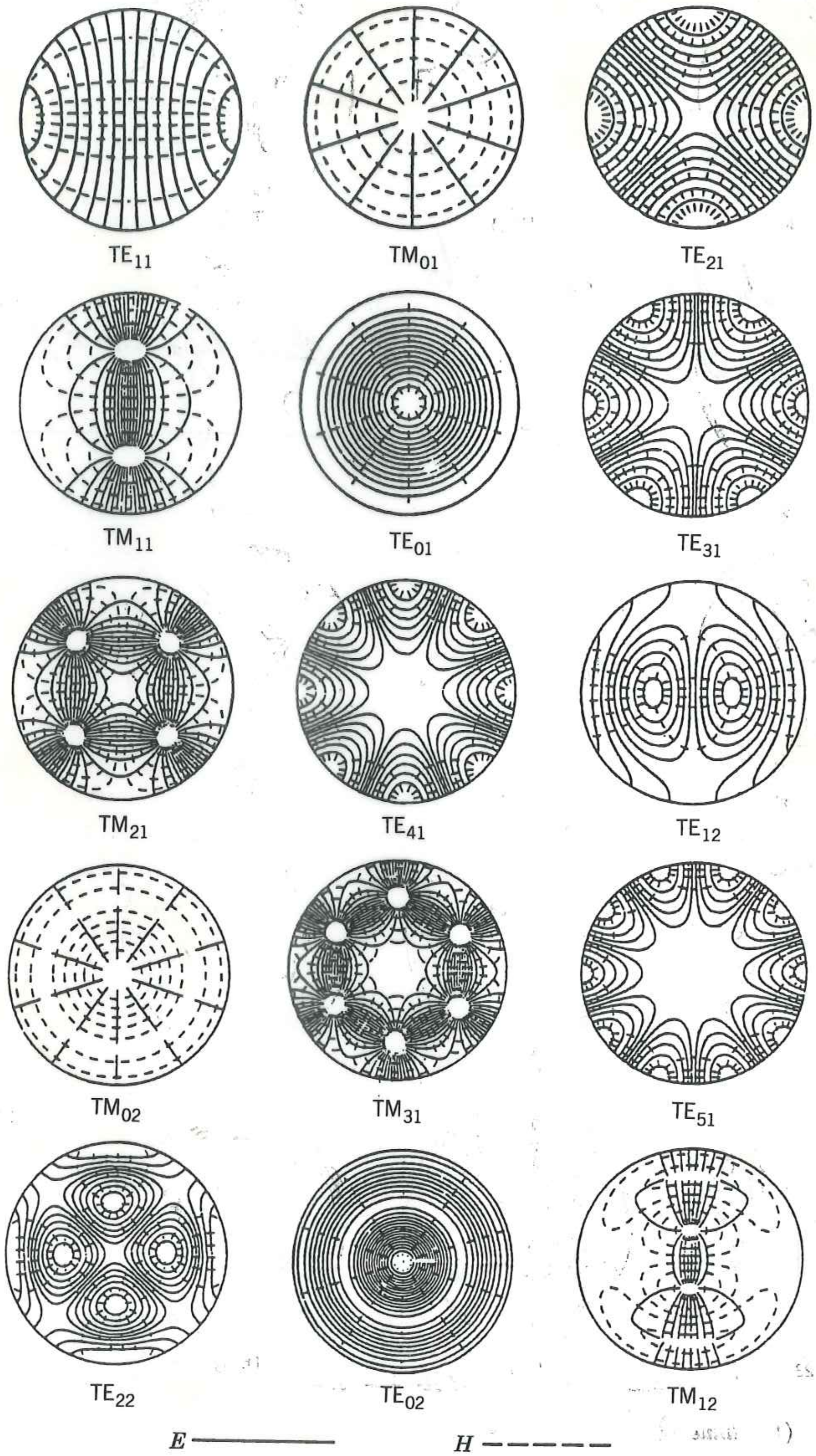


FIGURE 9-2 Field configurations of first 30 TE^z and/or TM^z modes in a circular waveguide. (*Source:* C. S. Lee, S. W. Lee, and S. L. Chuang, "Plot of modal field distribution in rectangular and circular waveguides," *IEEE Trans. Microwave Theory Tech.*, © 1966, IEEE.)

Ej. (Balanis Pag 475)

Una guía circular de radio $a = 3$ rellena de poliestireno $\varepsilon_r = 2.56$ se usa a una frecuencia de 3 GHz. Para el modo dominante determinar:

a) frecuencia de corte
b) longitud de onda de la guía
c) Constante de fase β (rad/cm)
d) Impedancia
e) Ancho de banda de operación de un único modo.
(Asumir solo modos TE)

a) El modo dominante es el TE_{11} y su frecuencia de corte es

$$f_{c\,TE_{11}} = \frac{1.8412}{2\pi a\sqrt{\mu\varepsilon}} = \frac{1.8412\,(30\times10^9)}{2\pi(3)\cdot\sqrt{2.56}} = 1.8315\ GHz$$

b)

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{9.375}{0.4017} = 23.34\ cm$$

long onda en el dieléctrico.

Comparar

$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}} = \frac{30\times10^9}{2\times10^9\sqrt{2.56}} = 9.375\ cm$$

c)

$$\beta_z = \beta\cdot\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{2\pi}{\lambda}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{2\pi}{9.375}(0.4017) = 0.2692\ rad/cm$$

$\varepsilon = \varepsilon_r\,\varepsilon_0$

$\varepsilon_r\ \varepsilon_0$

$$Z_{11} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{120\pi/\sqrt{2.56}}{0.4017} = 586.56 \text{ ohms}$$

e) Ya que el siguiente modo TE es el TE_{21} la anchura de banda del modo único TE_{11} es

$$BW = \frac{3.0542}{1.8412} = 1.6588$$

(Balanis Pag. 483)

Ej. Diseñar una guía de ondas circular rellena con un dieléctrico sin pérdidas de $\varepsilon = 4$. La guía debe de operar en un único modo dominante a lo largo de un ancho de banda de 1 GHz.

a) Encontrar su radio

b) Determinar las frecuencias del ancho de banda.

a) El modo dominante es el TE_{11} cuya frecuencia de corte es:

$$f_{c_{TE_{11}}} = \frac{X'_{11}}{2\pi a\sqrt{\mu\varepsilon}} = \frac{1,8412 \cdot (30\times 10^9)}{2\pi a\sqrt{4}}$$

El siguiente modo es el TM_{01} cuya frecuencia de corte es:

$$f_{c_{TM_{01}}} = \frac{X_{01}}{2\pi a\sqrt{\mu\varepsilon}} = \frac{2.4049(30\cdot 10^9)}{2\pi a\sqrt{4}}$$

Como la diferencia entre ambas frecuencias ha de ser de 1 GHz, se debe cumplir:

$$\frac{(2.4049 - 1.8412)\, 30\cdot 10^9}{2\pi a\sqrt{4}} = 1\times 10^9 \implies a = 1,3457 \text{ cm.}$$

b) Las frecuencias del ancho de banda serán:

$$f_{c_{TE_{11}}} = \frac{1.8412\times(30\times 10^9)}{2\pi\sqrt{4}\times 1,3457} = 3.2664 \text{ GHz}$$

$$f_{c_{TM_{01}}} = \frac{2,4049\times(30\times 10^9)}{2\pi\sqrt{4}\times 1.3457} = 4.2664 \text{ GHz}$$

Guías de ondas Circulares (radio: a)

(Resumen)

Balanis

	$TE_{m,n}$	$TM_{m,n}$
$K_c = \beta_c = \beta_p$	$\dfrac{X'_{m,n}}{a}$ → Ceros de las funciones Derivadas de Bessel.	$\dfrac{X_{m,n}}{a}$ Ceros de las funciones de Bessel
f_c	$\dfrac{X'_{m,n}}{2\pi a\sqrt{\mu\varepsilon}}$	$\dfrac{X_{m,n}}{2\pi a\sqrt{\mu\varepsilon}}$
λ_c	$\dfrac{2\pi a}{X'_{m,n}}$	$\dfrac{2\pi a}{X_{m,n}}$
$\beta_z\ (f \geq f_c)$ constante de fase	$\beta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$	→ igual
$\lambda_g\ (f \geq f_c)$ long. de onda de la guía.	$\dfrac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$	→ igual
$u_p\ (f \geq f_c)$	$\dfrac{u}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$	→ igual.
$Z\ (f \geq f_c)$ Impedancia de la onda	$\dfrac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$	$\eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$