

## 7 Problem Set 5.i

### Installments

#### Problem 1

Suppose that a financial institution, X, offers you an investment whereby you do not receive anything during the first two years. Thereafter and during five years you receive 1.000€ per year (postpayable). Suppose that the interest rates are expected to be constant and equal to 10% per year

1. How much should the investor deposit so that the institution does not lose money?
  - (a) How would your answer change if the conditions are as described but the payments are prepayable, rather than postpayable?

#### Problem 2

Consider a sequence of payments of 5.000€ per year over five years with a 10% annual interest rate which doesn't start until 2019 (today is 2016).

1. What would the final value of these payments be if they are postpayable?
  - (a) Compute the initial value (at date zero) of the previous installment payments both if it is postpayable and if it is prepayable.

#### Problem 3

Mr Fernandez has just bought a house but it hasn't been built yet. Construction will take two years to complete, but when it is done he will be able to let it out for sure, as it is right next to the university. His calculations indicate that in two year's time he will be able to receive 4500 euros a year in rent. As his son grows, he will be able to receive rent for another ten years, when his son will go to university (in 12 years' time).

1. Determine the present value of the rent money he will receive using a 5% TAE (and postpayable payments)
  - (a) Determine how much money he will have at his disposal to pay for his son's university education if the rent money is saved at 5% TAE

#### Problem 4

Suppose you are the manager of a company and that you have been recently fired by the board of directors. You have the right to receive unemployment benefits for the next 6 years, of 50.000€ per year. The first payment is due at the end of the current year, and the rest at the end of the following years (in the same way). You expect the term structure of interest to be as follows: constant and equal to 5% for the first 3 years, and then 6% for the following years. Determine

1. What is the present value of the unemployment benefits?
  - (a) What would be the present value of the unemployment benefits if in addition to the above, your payment on the third year was doubled (to 100.000€) for that year? What the present value be different if the payment that doubled was not that of the third, but that of the fourth year? Which of the two is preferable?
  - (b) Suppose the term structure of interest rates is flat, that is all interest rates are the same and equal to 5%. What would the present value of the unemployment benefits in the first question be? Use the annuity formula
  - (c) What is the final value of the payments (in the last question)?
  - (d) If the payments would have been prepayable, rather than postpayable, what would have been the present value of the payments? And the final value?
  - (e) Suppose that the present value should be 300.000 euros. What would the annual amount received have to be? How about if the final value should be 400.000

**Problem 5**

Determine the present value of the following annual payments that last for 8 years: the first payment is 150€. Then, the payments grow at 9% per year until the fifth year/payment. After that, it grows at 8% per year. Payments are post-payable. The return is 6% per year with annual compounding.

**Problem 6**

One of your family members has just arrived from talking to his financial advisor about his retirement. His expected income and expenditures are such that when he retires (at 65) his monthly income will be less than his monthly expenditures to cover his regular expenses. This monthly deficit forces him to consider a savings plan. His current financial status allows him to save 10,000€ once every two years (four payments, first in two years). He decides to put this money in an 8 year savings plan with a 5% effective APR. Once he reaches his retirement age (in 8 years), it will not be convenient to receive everything as a lump sum payment, and can leave it at the bank for a 3% nominal APR with monthly compounding. This he plans to use up at a rate of 656€ per month (extracted at the end of each month), and the plan is that this should last for 5 years (60 monthly payments).

1. Compute how much money will be in the bank 8 years from now.
  - (a) Compute the balance saved 5 years from today.
  - (b) Suppose that once he retires, and after 3 months, he needs all the money—how much will he be able to withdraw?

**Past Exam Questions****Problem 7 (EX 2013)**

Determine the present value of the following sequence of payments: 3000€ every year during 5 years (the first is in 3 years time). The interest rate is 4% effective APR.

**Problem 8 (EX 2013)**

Determine how much you will have saved after 20 years if you deposit 2000€ at the end of each year in an account that offers you 3% nominal APR with monthly capitalization.

**Problem 9 (EX 2014)**

Your multiple investments offer you a monthly income of 2000€. Currently, you are living with your family and have no plans to move out for at least 6 years, so you don't need to use this money. Hence, you decide to use it to guarantee a loan and buy a house. What loan amount can you borrow using the income from those 6 years as guarantee if the effective APR is 5%?

**Problem 10 (EX 2013)**

A friend of your mother's is receiving a net yearly salary of 40000€ and saves 25% of that income. This money is deposited at the end of the year in a financial institution that capitalizes at a 4% effective APR. Determine the amount that will be available after 4 years if you estimate that the salary will increase at a cumulative rate of 2% per year, after the first year.

**Problem 11 (EX 2014)**

Imagine that a large corporation wants to setup a permanent endowment fund to pay for education scholarships. This fund can only use the interest in order to keep the endowment unaltered. This fund is created today with one million euros at a 2% effective APR. Determine

1. The amount of money that can be used for scholarships each year
  - (a) The amount for the first and second years if the scholarship amounts have to grow at a 1% per year to cover inflation

**Problem 12 (EX 2014)**

In exactly 3 years, you will need 12000€ to pay for the registration fee of your postgraduate studies. How much money do you have to save at the end of every month from today (April 1, 2017) until the time of registration (September 1, 2021) if your bank offers you a 4% nominal APR with monthly payments?

**Problem 13 (EX 2014)**

Mr Zima owns office space which he rents out for 2500€ per month (prepayable). The contract is for 5 years, but you would like to propose an amendment on the contract so that rent is paid every three months (also prepayable). If the market is offering a 5% effective APR, how much will you offer as payment every three months in order to convince him to let you change the rental payment period?

**Problem 14 (EX 2014)**

A member of your family made contributions to a pension plan every year for 20 years. The pension plan guaranteed a minimum return of 3% effective APR. The first contribution was 5000€ but it has increased every year since at a rate of 1% per year. After 20 years, she retired and started to receive an income of 1000€ per month. Determine how much money is available in the plan 10 years after retirement taking into account that the interest rate after retirement changed to a 1% nominal APR with monthly capitalization.

**Problem 15 (EX 2013)**

You plan to start a motorcycle rental business. To analyze the viability of the business you have to determine the present value of both income and expenses for the next 5 years. The return you want to obtain from the business is a 15% annual rate.

The income and expenses are

1. buying motorcycles: 100000€ each year at the beginning of the first five years. Payment for the motorcycles is made three months after purchase
  - (a) you expect an average daily income from renting the motorcycles of 2000€
  - (b) you have to pay for rental of office space: 10000€ per month
  - (c) you expect to spend 600000€ per year in salaries, and this number increases 5% per year
  - (d) electricity and other utilities: 400€ every two months

**Problem 16 (2014)**

Ms Mónica García signs up to a pension plan on the day of her 43rd birthday. She will contribute 3000€ per year, at the end of each year, with an annual increase of 5%.

1. Determine the accumulated balance on her fund at retirement (67 years) if the insurance company guarantees an 8% effective APR.
  - (a) She can either receive this money at retirement in the form of a single lumpsum payment, or as a monthly salary. Determine how much she will receive each month if the monthly payment is computed for a 15 year duration and valued using a 7% effective APR.

**Problem 17 (EX 2017)**

You work for a financial company that offers to buy houses from the elderly and pay them in the form of a fixed "salary" every month, while they continue to live in their homes (this is called a reverse mortgage). The contract establishes that the current owners will receive the salary for 20 years.

1. Using a 4% nominal APR with monthly capitalization, determine the monthly payment for someone who sells the house for 200.000€
  - (a) You propose an alternative to your boss for those who sell for 200.000€: guaranteed higher early payments, of 2000€ per month, for the first five years, and lower later ones. Determine what will be the constant monthly payments after the initial five years. Again use a 4% nominal APR with monthly capitalization.