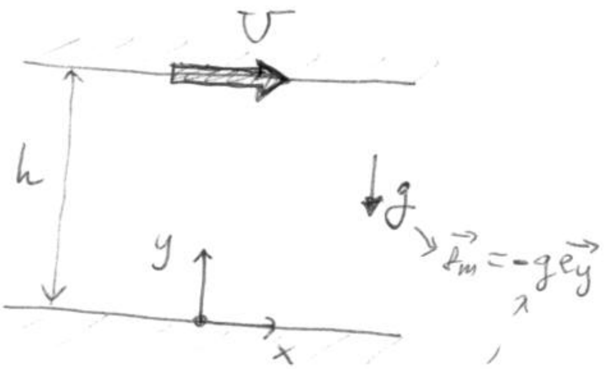
PROBLEMA 1



(1) CONTINUIDAD (ρ = CONSTANTE)

$$\nabla \cdot \vec{v} = \cancel{\frac{\partial v_x}{\partial x}} + \frac{\partial v_y}{\partial y} = 0 \Rightarrow v_y = \text{CONST.}$$

$\hookrightarrow y = 0, h: v_y = 0$

$\Rightarrow \boxed{v_y = 0}$ // $\rightarrow \boxed{\vec{v} = v_x(y)\,\vec{e_x}}$

CANTIDAD DE MOVIMIENTO $\xrightarrow{x}$ $\quad 0 = -\dfrac{\partial p}{\partial x} + \mu \dfrac{\partial^2 v_x}{\partial y^2} + \rho \cancel{\vec{f_m} \cdot \vec{e_x}}$

$\xrightarrow{y}$ $\quad 0 = -\dfrac{\partial p}{\partial y} + \underbrace{\rho \vec{f_m} \cdot \vec{e_y}}_{-\rho g} \rightarrow p + \rho g y = \text{CONSTANTE} = p_0 + \rho g h$

LUEGO $\boxed{p = p_0 + \rho g (h-y)}$ // Y $\boxed{\dfrac{\partial p}{\partial x} = 0} \Rightarrow 0 = \mu \dfrac{\partial^2 v_x}{\partial y^2} \Rightarrow$

$\Rightarrow v_x = Ay + B$ $\begin{cases} y=0: v_x = \boxed{B=0} \\ y=h: v_x = Ah = U \Rightarrow \boxed{A = \dfrac{U}{h}} \end{cases}$ $\boxed{v_x = U\dfrac{y}{h}}$ //

(2) $\boxed{Q = \displaystyle\int_0^h v_x(y)\,dy = \dfrac{Uh}{2}}$ //

(3) TRAYECTORIAS: $\dfrac{dx}{dt} = v_x = \dfrac{Uy}{h} = \dfrac{Uy_i}{h} \Rightarrow \boxed{x = x_i + \dfrac{Uy_i}{h}(t-t_i)}$

$\dfrac{dy}{dt} = v_y = 0 \rightarrow \boxed{y = y_i}$ $\quad \rightarrow$ SI $\begin{rcases} x_i = 0 \\ y_i = \lambda \end{rcases}$ $\boxed{x = \dfrac{Uy}{h}(t-t_i)}$ //

(4) ECUACIÓN DE LA ENERGÍA: $0 = \underbrace{\mu \left(\dfrac{dv_x}{dy}\right)^2}_{\phi_v} + K\dfrac{d^2T}{dy^2}$ $\begin{cases} y=0: T = T_i \\ y=h: T = T_s \end{cases}$ $\quad \phi_v = \mu \dfrac{U^2}{h^2}$

$\Rightarrow T = -\dfrac{\mu}{K}\dfrac{U^2}{h^2}\dfrac{y^2}{2} + Ay + B$ $\quad$ SUSTITUYENDO: $\boxed{T = T_i + (T_s - T_i)\dfrac{y}{h} + \dfrac{\mu U^2}{2K}\dfrac{y}{h}\left(1 - \dfrac{y}{h}\right)}$ //

(5) PARED SUPERIOR $\quad \vec{n} = -\vec{e_y}$

$\vec{f_n} = \bar{\bar{\tau}} \cdot \vec{n} = p_0 \vec{e_y} - \bar{\bar{\tau}}' \cdot \vec{e_y}$; $\bar{\bar{\tau}}' \cdot \vec{e_y} = \tau'_{xy}\vec{e_x} + \cancel{\tau'_{yy}}\vec{e_y}$

$\tau'_{xy} = \mu \dfrac{\partial v_x}{\partial y} = \mu \dfrac{U}{h}$ $\quad \tau'_{yy} = 2\mu \dfrac{\partial v_y}{\partial y} = 0$

LUEGO $\boxed{\vec{f_n}(y=h) = p_0 \vec{e_y} - \mu \dfrac{U}{h}\vec{e_x}}$ //

PARED INFERIOR $\quad \vec{n}$ $\quad \boxed{\vec{f_n}\Big|_{y=0} = \bar{\bar{\tau}} \cdot \vec{e_y} = -(p_0 + \rho g h)\vec{e_y} + \mu \dfrac{U}{h}\vec{e_x}}$ //

(6) PARED SUPERIOR $\quad \vec{n} = \vec{e_y}$ $\quad \vec{q} \cdot \vec{n} = -K\dfrac{dT}{dy}\Big|_{y=h} = -\dfrac{K(T_s - T_i)}{h} + \dfrac{\mu U^2}{2h}$

PARED INFERIOR $\quad \vec{n} = -\vec{e_y}$ $\quad \vec{q} \cdot \vec{n} = K\dfrac{dT}{dy}\Big|_{y=0} = K\dfrac{(T_s - T_i)}{h} + \dfrac{\mu U^2}{2h}$

(7) $\vec{n}$, V_0

ENERGÍA CINÉTICA $\quad 0 = \underbrace{\displaystyle\int_{\Sigma_0} \vec{v} \cdot \bar{\bar{\tau}}' \cdot \vec{n}\, d\sigma}_{\mu \frac{U^2}{h}} - \underbrace{\displaystyle\int_{V_0} \phi_v\, dV}_{-\mu \frac{U^2}{h}}$

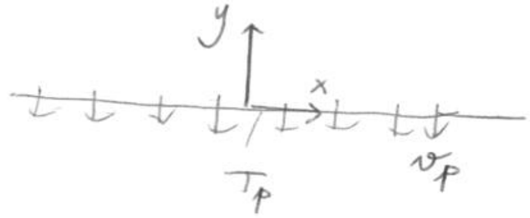
ENERGÍA INTERNA $\quad 0 = -\underbrace{\displaystyle\int_{\Sigma_0} \vec{q} \cdot \vec{n}\, d\sigma}_{+\mu\frac{U^2}{2h} + \mu\frac{U^2}{2h}} + \underbrace{\displaystyle\int \phi_v\, dV}_{-\mu\frac{U^2}{h}}$

PROBLEMA 2

$? \downarrow \longrightarrow U_\infty,\ P_\infty,\ T_\infty$

HIPÓTESIS:
$$\begin{cases} \vec{v} = v_x(y)\,\vec{e_x} + v_y(y)\,\vec{e_y} \\ P = P(x,y) \qquad [P = p + \rho U] \\ T = T(y) \qquad [\vec{f_m} = -\nabla \vec{U}] \end{cases}$$

① $\nabla \cdot \vec{v} = 0 \Rightarrow \dfrac{dv_y}{dy} = 0 \Rightarrow \boxed{v_y = -v_p \ \forall x,y}$ //



CANTIDAD DE MOVIMIENTO

↳ DIRECCIÓN y: $0 = -\dfrac{\partial P}{\partial y} \Rightarrow P = P(x)$ PERO COMO $P = P_\infty\ \forall x$ EN $y \to \infty$,

SE DEDUCE QUE $\boxed{P = P_\infty\ \forall x,y}$ //

DIRECCIÓN x: $\rho v_y \dfrac{\partial v_x}{\partial y} = -\cancel{\dfrac{\partial P}{\partial x}} + \mu \dfrac{\partial^2 v_x}{\partial y^2} \Rightarrow$
$$\boxed{\begin{aligned} & v_x'' + \frac{v_p}{\nu} v_x' = 0 \\ & y = 0:\ v_x = 0 \\ & y \to \infty:\ v_x \to U_\infty \end{aligned}}$$
DONDE PRIMA INDICA DERIVADA RESPECTO A y

CUYA SOLUCIÓN ES $\boxed{v_x = U_\infty \left(1 - e^{-\frac{v_p y}{\nu}}\right)}$ //

② ECUACIÓN DE LA ENERGÍA INTERNA CON $e = cT$: $\rho c v_y \dfrac{\partial T}{\partial y} = -p\cancel{\nabla \cdot \vec{v}} + \phi_v + K \dfrac{\partial^2 T}{\partial y^2}$

SIENDO $\phi_v = \mu \left(\dfrac{dv_x}{dy}\right)^2 = \dfrac{\mu U_\infty^2 v_p^2}{\nu^2} e^{-\frac{2 v_p y}{\nu}} \Rightarrow$
$$\boxed{\begin{aligned} & T'' + \frac{v_p}{\alpha} T' = -\frac{\mu U_\infty^2 v_p^2}{K \nu^2} e^{-\frac{2 v_p y}{\nu}} \\ & y = 0:\ T = T_p \\ & y \to \infty:\ T \to T_\infty \end{aligned}}$$
DONDE $\alpha = \dfrac{K}{\rho c}$ (DIFUSIVIDAD TÉRMICA)

CUYA SOLUCIÓN ES: $\boxed{\dfrac{T - T_\infty}{T_p - T_\infty} = (1 - \Lambda) e^{-\frac{v_p y}{\alpha}} + \Lambda e^{-\frac{2 v_p y}{\nu}}}$ //

SIENDO $\Lambda = \dfrac{U_\infty^2}{c(T_p - T_\infty)} \dfrac{Pr}{2(Pr - 2)}$ DONDE $Pr = \dfrac{\nu}{\alpha}$ ES EL NÚMERO DE PRANDTL

③ LÍNEAS DE CORRIENTE: $\dfrac{dx}{v_x} = \dfrac{dy}{v_y} \Rightarrow dx = -\dfrac{U_\infty}{v_p}\left(1 - e^{-v_p y/\nu}\right) dy \Rightarrow$

$\int_{x_0}^{x}$, $\int_{y_0}^{y}$
$$\boxed{x - x_0 = -\frac{U_\infty}{v_p}\left[y - y_0 + \frac{\nu}{v_p}\left(e^{-\frac{v_p y}{\nu}} - e^{-\frac{v_p y_0}{\nu}}\right)\right]}$$ //

④ ESFUERZO DE FRICCIÓN SOBRE LA PARED: $\vec{\tau_p} = \left(\bar{\bar{\tau}}' \cdot \vec{n}\right)\Big|_{y=0}$ con $\vec{n} = \vec{e_y}$

LUEGO COMO $\bar{\bar{\tau}}' = \begin{pmatrix} \tau'_{xx} & \tau'_{xy} \\ \tau'_{xy} & \tau'_{yy} \end{pmatrix}$ Y $\tau'_{xy} = \mu \dfrac{dv_x}{dy}$ SE TIENE $\boxed{\vec{\tau_p} = \rho U_\infty v_p \vec{e_x}}$ //

⑤ FLUJO DE CALOR FLUIDO → PARED: $q_p = \vec{q} \cdot \vec{n}$ con $\vec{n} = -\vec{e_y}$; $\vec{q} = -K\nabla T$ $\Rightarrow q_p = K \dfrac{dT}{dy}\Big|_{y=0}$

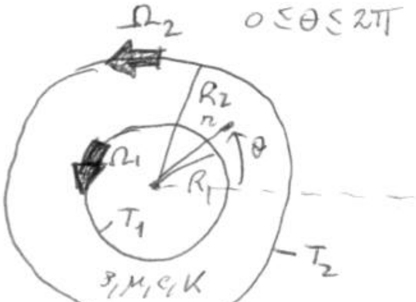
LUEGO $\boxed{q_p = \rho c v_p \left(T_\infty - T_p + \dfrac{U_\infty^2}{2c}\right)}$ //

PROBLEMA 3

HIPÓTESIS $\vec{v} = u_\theta \vec{e_\theta}$

$\frac{\partial}{\partial z} = 0$

$R_1 \le r \le R_2$
$0 \le \theta \le 2\pi$



CONTINUIDAD

① $\nabla \cdot \vec{v} = 0 \Rightarrow \cancel{\frac{\partial u_z}{\partial z}}^{u_z=0} + \frac{1}{r}\cancel{\frac{\partial (r u_r)}{\partial r}}^{u_r=0} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} = 0 \Rightarrow$

$\Rightarrow \frac{\partial u_\theta}{\partial \theta} = 0 \Rightarrow \boxed{u_\theta = u_\theta(r)}$

CANTIDAD DE MOVIMIENTO : → DIRECCIÓN θ : $0 = -\frac{1}{r}\cancel{\frac{\partial p}{\partial \theta}} + \mu \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r u_\theta)}{\partial r}\right) \Rightarrow \boxed{u_\theta = A r + \frac{B}{r}}$ //

DONDE A y B SALEN DE IMPONER $\left.\begin{array}{l} r = R_1 : u_\theta = \Omega_1 R_1 \\ r = R_2 : u_\theta = \Omega_2 R_2 \end{array}\right\}$ $A = \frac{\Omega_2 R_2^2 - \Omega_1 R_1^2}{R_2^2 - R_1^2}$

$B = \frac{(\Omega_1 - \Omega_2) R_1^2 R_2^2}{R_2^2 - R_1^2}$

DIRECCIÓN r : $-\rho \frac{u_\theta^2}{r} = -\frac{\partial p}{\partial r} \Rightarrow \frac{\partial p}{\partial r} = \rho\left(A^2 r + \frac{B^2}{r^3} + \frac{2AB}{r}\right) \Rightarrow$

$\xRightarrow{\int_{R_1}^{r}(\,)dr}$ $\boxed{p - p(R_1) = \rho\left[A^2\frac{(r^2 - R_1^2)}{2} + \frac{B^2}{2}\left(\frac{1}{R_1^2} - \frac{1}{r^2}\right) + 2AB \ln\left(\frac{r}{R_1}\right)\right]}$ //

②

$\boxed{\begin{array}{l}\vec{n_1} = \vec{e_r} \\ \vec{n_2} = -\vec{e_r}\end{array}}$

P.U. DE LONGITUD

$\vec{M_1} = \int \vec{x_1} \wedge (\bar{\bar{\tau}} \cdot \vec{n_1})\, d\sigma_1$ DONDE $\vec{x_1} = R_1 \vec{e_r}$, $d\sigma_1 = R_1 d\theta$

$\vec{M_2} = \int \vec{x_2} \wedge (\bar{\bar{\tau}} \cdot \vec{n_2})\, d\sigma_2$ $\vec{x_2} = R_2 \vec{e_r}$, $d\sigma_2 = R_2 d\theta$

$\vec{x_1} \wedge (\bar{\bar{\tau}} \cdot \vec{n_1}) = R_1 \vec{e_r} \wedge \left[(-p\vec{e_r}) + \bar{\bar{\tau}}' \cdot \vec{e_r}\right] = R_1 \vec{e_r} \wedge (\bar{\bar{\tau}}' \cdot \vec{e_r})$

PERO $\bar{\bar{\tau}}' \cdot \vec{e_r} = \begin{pmatrix} \cancel{\tau'_{rr}} & \tau'_{r\theta} \\ \tau'_{r\theta} & \cancel{\tau'_{\theta\theta}} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \tau'_{r\theta}\, \vec{e_\theta} \Rightarrow \vec{x_1} \wedge (\bar{\bar{\tau}} \cdot \vec{n_1}) = R_1 \tau'_{r\theta}(r = R_1)\underbrace{\vec{e_r} \wedge \vec{e_\theta}}_{\vec{e_z}}$

DONDE $\boxed{\tau'_{r\theta} = \mu r \frac{d}{dr}\left(\frac{u_\theta}{r}\right) = -2\mu\frac{B}{r^2}}$ LUEGO $\boxed{\vec{M_1} = -4\pi\mu B \vec{e_z}}$ //

Y DEL MISMO MODO, $\boxed{\vec{M_2} = 4\pi\mu B \vec{e_z}}$ //

③ ENERGÍA INTERNA $e = cT \Rightarrow \rho c \cancel{\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}} = \phi_v + \frac{K}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$ DONDE

$\boxed{\phi_v = \mu r^2\left[\frac{d}{dr}\left(\frac{v_\theta}{r}\right)\right]^2 = 4\mu B^2 r^{-4}} \Rightarrow \left\{\begin{array}{l} \frac{d}{dr}\left(r\frac{dT}{dr}\right) = \frac{-4\mu B^2}{K}\frac{1}{r^3} \\ r = R_1 : T = T_1 \\ r = R_2 : T = T_2 \end{array}\right.$

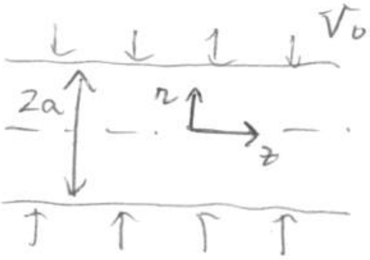
CUYA SOLUCIÓN SE PUEDE ESCRIBIR COMO:

$$\frac{T - T_1}{T_2 - T_1} = \Lambda\left(1 - \frac{1}{\eta^2}\right) - \frac{\ln \eta}{\ln(R_2/R_1)}\left[1 - \Lambda\left(\frac{R_1^2}{R_2^2} - 1\right)\right]$$

DONDE $\Lambda = \frac{\mu B^2}{K(T_2 - T_1)R_1^2}$ Y $\eta = \frac{r}{R_1}$

PROBLEMA 4

$$\begin{cases} \vec{v} = v_r(r)\,\vec{e}_r \\ P = P(r) \\ T = T(r) \end{cases}$$



[1] $\nabla\cdot\vec{v} = \frac{1}{r}\frac{\partial(r v_r)}{\partial r} = 0 \Rightarrow r v_r = -a V_0 \Rightarrow$

$\Rightarrow \boxed{v_r = -\frac{a V_0}{r}}$ //

EL CAMPO DE PRESIÓN REDUCIDA $P(r)$ VIENE DADO POR LA ECUACIÓN DE CANTIDAD DE MOVIMIENTO RADIAL:

$$\rho v_r \frac{\partial v_r}{\partial r} + \frac{\partial P}{\partial r} = 0 \Rightarrow \frac{\partial}{\partial r}\left(\frac{P}{\rho} + \frac{v_r^2}{2}\right) = 0 \Rightarrow \boxed{\frac{P - P_\infty}{\rho} = -\frac{V_0^2}{2}\left(\frac{a}{r}\right)^2}$$ //

[2] $\left.\begin{array}{l} \frac{dr}{dt} = v_r = \frac{-V_0 a}{r} \\ r(t_i) = r_i > a \end{array}\right\}$ $\int_{r_i}^{r} x\,dx = -aV_0(t - t_i) \Rightarrow \boxed{r^2 = r_i^2 - 2aV_0(t - t_i)}$ //

TIEMPO DE SUCCIÓN t_s TAL QUE $r(t_s) = a \Rightarrow$

$\Rightarrow \boxed{t_s = t_i + \frac{r_i^2 - a^2}{2V_0 a}}$ //

[4] $\begin{cases} \rho c v_r \frac{dT}{dr} = \cancel{\phi_v} + \frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) \\ r = a:\ T = T_0 \\ r \to \infty:\ T \to T_\infty \end{cases}$ CON $\phi_v = 2\mu\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{v_r}{r}\right)^2\right]$

$\rightarrow$ DISIPACIÓN $\phi_v \sim \frac{\mu V_0^2}{a^2}$ DESPRECIABLE FRENTE A CONDUCCIÓN

$\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) \sim \frac{k(T_\infty - T_p)}{a^2}$ SI $V_0^2 \ll P_r\, c(T_\infty - T_0)$

DONDE $P_r = \frac{\mu c}{k}$

LA SOLUCIÓN DESPRECIANDO ϕ_v ES $T = T_\infty + (T_0 - T_\infty)\left(\frac{a}{r}\right)^{Re\,Pr}$

$\rightarrow$ DONDE $Re = \frac{\rho V_0 a}{\mu}$

[3] $\bar{\bar{\tau}}'\cdot\vec{n} = \begin{pmatrix} \tau'_{rr} & \tau'_{rz} \\ \tau'_{rz} & \tau'_{zz} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \tau'_{rr}(r=a)\,\vec{e}_r + \cancel{\tau'_{rz}(r=a)}\,\vec{e}_z$

$\uparrow \vec{n} = \vec{e}_r$

$\tau'_{rz} = 0$

$\tau'_{rr} = 2\mu\frac{dv_r}{dr} = \frac{2\mu a V_0}{r^2} \Rightarrow \tau'_{rr}(r=a) = \frac{2\mu V_0}{a} \Rightarrow \boxed{\bar{\bar{\tau}}'\cdot\vec{n} = \frac{2\mu V_0}{a}\vec{e}_r}$ //

MIENTRAS QUE LA FUERZA POR UNIDAD DE LONGITUD ES CERO POR SIMETRÍA YA QUE $\int_0^{2\pi} \vec{e}_r\,d\theta = 0$

[5] $\boxed{q_p = \vec{q}\cdot\vec{n} = -\vec{e}_r\cdot(-k\nabla T) = k\left.\frac{dT}{dr}\right|_{r=a}}$ Y DESPRECIANDO ϕ_v SE TIENE

q_p
$\downarrow \vec{n} = -\vec{e}_r$

$\boxed{q_p = \rho c V_0 (T_\infty - T_0)}$ Y POR UNIDAD DE LONGITUD PASA UN

CALOR POR UNIDAD DE TIEMPO $2\pi a \rho c V_0 (T_\infty - T_0)$

PROBLEMA 5

1

$\frac{\partial}{\partial\varphi} = \frac{\partial}{\partial\theta} = 0$, $V_\theta = V_\varphi = 0$ $\quad V_r = V$

CONTINUIDAD $\quad \nabla\cdot\bar{v} = 0 \quad \boxed{V = \frac{C(t)}{r^2} = \frac{R^2\dot{R}}{r^2}} \rightarrow \underline{\nabla\wedge\bar{v} = 0}$

2 CANTIDAD DE MOVIMIENTO $\quad \frac{\partial}{\partial r}\left(\frac{P}{\rho} + \frac{v_r^2}{2}\right) = -\frac{\partial v_r}{\partial t} = -\frac{2R\dot{R}^2 + R^2\ddot{R}}{r^2}$

$$\boxed{\frac{P}{\rho} - \frac{P_\infty}{\rho} = \frac{2R\dot{R}^2 + R^2\ddot{R}}{r} - \frac{R^4\dot{R}^2}{2r^4}}$$

ELIJO PARA EL ANALISIS EL V_c

EN GENERAL

$$\frac{d}{dt}\left[\int_{V_c}\rho\frac{|\bar{v}|^2}{2}dV\right] + \int_{\Sigma_c}\rho\frac{|\bar{v}|^2}{2}(\bar{v}-\bar{v}_c)\cdot\bar{n}\,d\sigma = -\int_{\Sigma_c}P\bar{v}\cdot\bar{n}\,d\sigma + \int_{\Sigma_c}\bar{v}\cdot\bar{\bar{\tau}}'\cdot\bar{n}\,d\sigma + \int_{V_c}\rho\bar{f}_m\cdot\bar{v}\,dV - \left[-\int_{V_c}P\bar{v}\cdot\bar{n}\,d\sigma + \int_{V_c}\Phi_v\,dV\right]$$

3 DISIPACION VISCOSA

$$\Phi_v = 2\mu\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + 2\left(\frac{v_r}{r}\right)^2\right] = 12\mu\frac{R^4\dot{R}^2}{r^6}$$

ENERGIA TOTAL DISIPADA POR VISCOSIDAD POR UNIDAD DE TIEMPO

$$\boxed{\int_{V_c}\Phi_v\,dV = 12\mu R^4\dot{R}^2\int_R^\infty\frac{4\pi}{r^4}dr = 16\mu\pi R\dot{R}^2}$$

4 TRABAJO COMUNICADO POR EL GLOBO AL LIQUIDO $\quad \bar{n} = (-1,0,0)$

$$\boxed{-\int P\bar{v}\cdot\bar{n}\,d\sigma + \int\bar{v}\cdot\bar{\bar{\tau}}'\cdot\bar{n}\,d\sigma = P(R)\,4\pi R^2\dot{R} + 16\pi\mu R\dot{R}^2 = 4\pi R^2\dot{R}\rho\left[\frac{3}{2}\dot{R}^2 + R\ddot{R}\right] + 16\pi\mu R\dot{R}^2 + P_\infty 4\pi R^2\dot{R}}$$

¿DONDE VA ESE TRABAJO?

$$\frac{d}{dt}\left[\int_{V_c}\rho\frac{|\bar{v}|^2}{2}dV\right] = \frac{d}{dt}\left[\frac{\rho}{2}(R^2\dot{R})^2\int_R^{R_\infty}\frac{4\pi r^2}{r^4}dr\right] = 4\pi\rho R^2\dot{R}\left[\frac{3}{2}\dot{R}^2 + R\ddot{R}\right]$$

ADEMAS, EN EL INFINITO $\quad -\int P\bar{v}\cdot\bar{n}\,d\sigma = -P_\infty R^2\dot{R}4\pi$

DEL TRABAJO COMUNICADO POR EL GLOBO AL FLUIDO:

$16\pi\mu R\dot{R}^2 \rightarrow$ SE DISIPA POR VISCOSIDAD Y VA A AUMENTAR LA ENERGIA INTERNA DEL FLUIDO + LA ENERGIA QUE SE VA POR CONVECCION POR EL INFINITO

- $4\pi R^2\dot{R}\left[P(R) - P_\infty\right] \rightarrow$ VA A AUMENTAR LA ENERGIA CINETICA DEL FLUIDO

$4\pi R^2\dot{R}\,P_\infty \rightarrow$ SE PIERDE POR EL INFINITO DEBIDO A QUE $P(r\to\infty) = P_\infty \neq 0$.

PROBLEMA 6

1 $\frac{\partial}{\partial r}(r^2 v_r) = 0 \Rightarrow \boxed{v_r = -V_0 \left(\frac{a}{r}\right)^2}$

2 $\frac{dr}{dt} = -V_0\left(\frac{a}{r}\right)^2, \quad t = t_i, \; r = r_i \Rightarrow \boxed{r^3 - r_i^3 = -3V_0 a^2 (t - t_i)}$

$r = a, \quad \boxed{t = t_i + \frac{r_i^3 - a^3}{3V_0 a^2}}$

3 $\rho v_r \frac{\partial v_r}{\partial r} = -\frac{\partial p}{\partial r} \Rightarrow p + \frac{1}{2}\rho v_r^2 = \text{CTE} \Rightarrow \boxed{p - p_\infty = -\frac{1}{2}\rho V_0^2 \left(\frac{a}{r}\right)^4}$

4 $v_r \frac{\partial T}{\partial r} = \frac{\phi_v}{\rho c} + \alpha \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right), \quad \phi_v = \mu\left\{2\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + 2\left(\frac{v_r}{r}\right)^2\right]\right\} = 12\mu \frac{V_0^2 a^4}{r^6}$

↳ DESPRECIABLE SI $\frac{V_0^2}{c} \ll T_\infty - T_p$

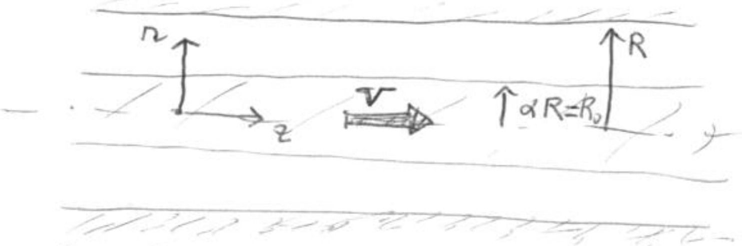
$\Downarrow$

$T = A\, e^{\frac{V_0 a}{\alpha}\frac{a}{r}} + B, \quad r = a, \; T = T_0; \quad r \to \infty, \; T = T_\infty \Rightarrow \boxed{T = \frac{T_0\left(e^{\frac{V_0 a}{\alpha}\frac{a}{r}} - 1\right)}{e^{\frac{V_0 a}{\alpha}} - 1} + \frac{T_\infty\left(e^{\frac{V_0 a}{\alpha}} - e^{\frac{V_0 a}{\alpha}\frac{a}{r}}\right)}{e^{\frac{V_0 a}{\alpha}} - 1}}$

5 $\boxed{Q = k \left.\frac{\partial T}{\partial r}\right|_a 4\pi a^2 = 4\pi a^2 k = \frac{4\pi a^2 \rho c V_0 (T_\infty - T_0)}{e^{\frac{V_0 a}{\alpha}} - 1}} \rightarrow \boxed{Q = 4\pi a k (T_\infty - T_0)}$

si $\frac{V_0 a}{\alpha} \ll 1$

PROBLEMA 7 $\begin{cases} \frac{\partial}{\partial z} = 0 \\ \frac{\partial}{\partial \theta} = 0 \end{cases}$



1 $\frac{\cancel{\partial v_z}}{\cancel{\partial z}} + \frac{1}{r}\frac{\partial (r v_r)}{\partial r} = 0$

$\hookrightarrow r v_r = C$ Y COMO $v_r = 0$ EN $r = R_0$ SE TIENE $\boxed{v_r = 0}$

LUEGO $\boxed{\vec{v} = v_z(r)\,\vec{e}_z}$

2 CANTIDAD DE MOVIMIENTO EN DIRECCIÓN z: $\left\{ 0 = -\cancel{\frac{\partial p}{\partial z}} + \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) \Rightarrow \right.$

$\Rightarrow v_z(r) = A \ln r + B$ E IMPONIENDO LAS CONDICIONES DE CONTORNO

$r = \underset{\alpha R}{R_0}: v_z = V$

$r = R: v_z = 0$

$$\left.\begin{aligned} r = \alpha R:&\quad A\ln(\alpha R) + B = V \\ r = R:&\quad A\ln(R) + B = 0 \end{aligned}\right\} \quad A = \frac{V}{\ln\alpha} \quad B = -V\frac{\ln R}{\ln \alpha} \Rightarrow \boxed{v_z(r) = V\frac{\ln(r/R)}{\ln(\alpha)}}$$

3 $Q = \int_{R_0}^{R} v_z(r)\, 2\pi r\, dr \Rightarrow \boxed{Q = \pi R^2 V\left(\frac{1}{2}\frac{\alpha^2 - 1}{\ln\alpha} - \alpha^2\right)}$ //

4 $\bar{\bar{\tau}}' = \begin{pmatrix} \cancel{\tau'_{rr}}^{\,0} & \tau'_{rz} \\ \tau'_{rz} & \cancel{\tau'_{zz}}_{\,0} \end{pmatrix} = \mu \begin{pmatrix} 0 & \frac{dv_z}{dr} \\ \frac{dv_z}{dr} & 0 \end{pmatrix}$ CON $\frac{dv_z}{dr} = \frac{V}{\ln\alpha}\frac{1}{r}$

$\bar{\bar{\tau}}'\cdot\vec{n} = \frac{\mu V}{\ln(\alpha)\,\alpha r}\vec{e}_z$

$\vec{n} = \vec{e}_r$

FUERZA POR UNIDAD DE LONGITUD: $\vec{F}_v = 2\pi \cancel{\alpha R}\frac{\mu V}{\cancel{\alpha R}\ln(\alpha)}\vec{e}_z \Rightarrow \boxed{\vec{F}_v = \frac{2\pi\mu V}{\ln\alpha}\vec{e}_z}$

(NÓTESE QUE $\ln\alpha < 0$ AL SER $\alpha < 1$)

5 $$\left.\begin{aligned} &0 = \mu\left(\frac{dv_z}{dr}\right)^2 + \frac{K}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) \\ &r = \alpha R: T = T_V \\ &r = R: T = T_0 \end{aligned}\right\}$$

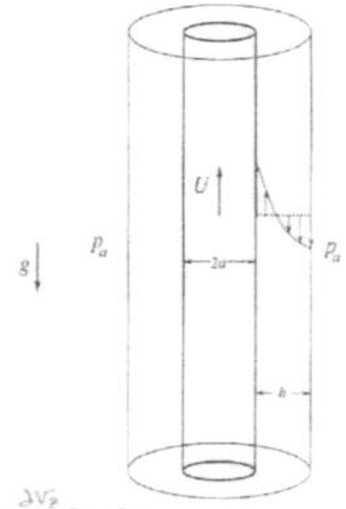
$\vec{n} = \vec{e}_r$

$$\boxed{T(r) = T_0 + (T_V - T_0)\frac{\ln(r/R)}{\ln\alpha} + \frac{\mu V^2}{2K}\frac{\ln(r/R)}{\ln\alpha}\left(1 - \frac{\ln(r/R)}{\ln\alpha}\right)}$$

T_0

FLUJO DE CALOR: $q_0 = -K\left.\frac{dT}{dr}\right|_{r=R} \Rightarrow \boxed{q_0 = -\frac{2\pi}{\ln\alpha}\left[K(T_V - T_0) + \frac{\mu V^2}{2}\right]}$ //

PROBLEMA 8



① CONT: $\frac{\partial V_z}{\partial z} = 0 \Rightarrow \boxed{V_z = V_z(r)}$; $\frac{\partial V_z}{\partial \theta} = 0$ POR SIMETRIA

CM r: $0 = -\frac{\partial p}{\partial r} \Rightarrow \boxed{P = p_a}$

CM z: $0 = -\rho g + \frac{\mu}{r}\frac{d}{dr}\left(r\frac{dV_z}{dr}\right)$ $\begin{cases} r = a: & V_z = U \\ r = a+h: & \frac{dV_z}{dr} = 0 \end{cases}$

INTEGRANDO 1 VEZ $\rightarrow r\frac{dV_z}{dr} = \frac{g}{2\nu}r^2 + C_1 = \frac{g}{2\nu}\left[r^2 - (a+h)^2\right]$; $r = a+h$, $\frac{dV_z}{dr} = 0$

INTEGRANDO OTRA VEZ CON $V_z = U$ EN $r = a \Rightarrow \boxed{V_z = U + \frac{g}{2\nu}\left[\frac{r^2 - a^2}{2} - (a+h)^2 \ln\left(\frac{r}{a}\right)\right]}$

② $\boxed{\bar{F} = +\int \bar{\bar{\tau}}' \cdot \bar{n}\, d\sigma = -2\pi\rho g a h\left[1 + \frac{h}{2a}\right]\bar{e}_z}$

$\begin{pmatrix} \tau_{rr} & \tau_{rz} \\ \tau_{zr} & \tau_{zz} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \mu \left.\frac{dV_z}{dr}\right|_{r=a}\bar{e}_z = -\rho g h\left[1 + \frac{h}{2a}\right]$

③ $\boxed{Q} = \int \bar{v} \cdot \bar{n}\, d\sigma = \int_a^{a+h} V_z\, 2\pi r\, dr = \pi U\left[(a+h)^2 - a^2\right] + \frac{g\pi}{\nu}\left\{\frac{(a+h)^4}{8} + \frac{h^2(2a+h)^2}{4} - \frac{a^4}{8} - \frac{(a+h)^4}{2}\ln\frac{a+h}{a}\right\}$

④ $0 = \phi_v + \frac{K}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)$ $\begin{cases} r = a: & T = T_p \\ r = a+h: & T = T_a \end{cases}$

$\phi_v = \mu\left(\frac{dV_z}{dr}\right)^2 = \mu\left(\frac{g}{2\nu}\right)^2\frac{1}{r^2}\left[r^2 - (a+h)^2\right]^2$

$\Rightarrow \boxed{T = C_2 + C_1 \ln r - \underbrace{\frac{\mu}{K}\left(\frac{g}{2\nu}\right)^2\left[\frac{r^4}{16} - \frac{(a+h)^2}{2}r^2 + \frac{(a+h)^4}{2}(\ln r)^2\right]}_{f(r)}} =$

$= T_p + \left\{T_a - T_p + f(a+h) - f(a)\right\}\frac{\ln(r/a)}{\ln\left(\frac{a+h}{a}\right)} + f(a) - f(r)$

⑤ $\bar{q} \cdot \bar{n} = K\left.\frac{dT}{dr}\right|_{r=a} = K\frac{T_a - T_p + f(a+h) - f(a)}{a\ln\left(\frac{a+h}{a}\right)} - \mu\left(\frac{g}{2\nu}\right)^2\left[\frac{a^3}{4} - (a+h)^2 a + (a+h)^4 \frac{\ln a}{a}\right] \simeq K\frac{T_a - T_p}{a\ln((a+h)/a)}$

$\bar{q} = -K\frac{dT}{dr}$, $\bar{n} = -\bar{e}_r$

$\left(\text{si } \frac{\mu}{K(T_p - T_a)}\left(\frac{g}{\nu}\right)^2 a^4 \ll 1\right)$

$\frac{T - T_p}{T_a - T_p} = \frac{\ln(r/a)}{\ln\frac{a+h}{a}}$