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$$\textcircled{1} \quad x' = \begin{pmatrix} 242 & 324 \\ -183 & -245 \end{pmatrix}; \quad x(t_0) = \xi, \quad A = \begin{pmatrix} 242 & 324 \\ -183 & -245 \end{pmatrix}$$

$|\lambda - 25| = 0 \Rightarrow \lambda = -1, \lambda = -2$ autovalores. los correspondientes autovectores son: si $\lambda = -1$ $u = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$, si $\lambda = -2$ $v = \begin{pmatrix} 81 \\ -61 \end{pmatrix}$

$$\text{Si } P = \begin{pmatrix} 4 & 81 \\ -3 & -61 \end{pmatrix} \text{ y } J = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \quad A = PJP^{-1}. \text{ Si } y = P^{-1}x$$

$$\begin{cases} x' = Ax \\ x(t_0) = \xi_0 \end{cases} \Leftrightarrow \begin{cases} y' = Jy \\ y(t_0) = P^{-1}\xi_0 = \eta_0 \end{cases}; \quad \begin{aligned} y_1' &= -y_1 & y_1(t_0) &= \eta_{0,1} \\ y_2' &= -2y_2 & y_2(t_0) &= \eta_{0,2} \end{aligned}$$

El método de Euler aplicado a $y' = Jy$ da

$$\begin{aligned} y_{1,n} &= (1-h)^n \eta_{0,1} \\ y_{2,n} &= (1-2h)^n \eta_{0,2} \end{aligned} \quad y_n = \begin{pmatrix} y_{1,n} \\ y_{2,n} \end{pmatrix} \rightarrow 0 \text{ si } \begin{cases} |1-h| < 1 \\ |1-2h| < 1 \end{cases} \text{ lo}$$

que equivale a $0 < h < 1$.

$$\text{Como } x_n = Py_n \quad x_n \rightarrow 0 \Leftrightarrow y_n \rightarrow 0$$

$$\textcircled{2} \quad x_{n+2} + 2x_{n+1} - (1+\lambda)x_n = h\left(-\frac{1}{2}f_n + \mu f_{n+1}\right)$$

$$p(z) = -(1+\lambda) + 2z + z^2. \text{ Raíces de } p: \quad \begin{aligned} z &= 1 \\ z &= -(1+\lambda) \end{aligned}$$

$$-1 \leq -(1+\lambda) \leq 1 \Leftrightarrow -2 \leq \lambda \leq 0$$

Si $\lambda = 0$: $\begin{cases} z=1 \\ z=-1 \end{cases}$ débilmente estable. Si $\lambda = -2$, $z=1$ doble, inestable

$$\text{Es } 0\text{-estable si } \boxed{-2 < \lambda \leq 0}$$

$$\begin{aligned} \pi(z) &= -\frac{1}{2} + \mu z; & c_0 &= 0 \\ c_1 &= 2 + 2 - \left(-\frac{1}{2} + \mu\right) = 2 - \mu + \frac{5}{2} = 0 & \Rightarrow \mu &= -\frac{1}{2} \\ c_2 &= \frac{1}{2}(2 + 4) - \mu = 0 & \lambda &= -3 \end{aligned}$$

$$\text{Si } \mu = -\frac{1}{2}, \lambda = -3 \quad c_3 = \frac{13}{12} \neq 0. \text{ Orden máximo } 2, \text{ cuando } \lambda = -3, \mu = -\frac{1}{2}$$

Convergente si $\mu = 2 + 5/2$

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① $\begin{cases} x' = -2x \\ x(0) = 1 \end{cases}$

$$x_{n+1} = x_n - 2h x_n = (1-2h)x_n$$

$$x_n = (1-2h)^n x_0 = (1-2h)^n$$

$$x_1 = 1 - 2 \cdot \frac{3}{2} = -2$$

$$x_2 = (-2)^2, \quad x_3 = (-2)^3$$

La solución exacta es $x(t) = e^{-2t} \xrightarrow{t \rightarrow \infty} 0$

Para que $x_n \rightarrow 0$ debe cumplirse que $|1-2h| < 1 \Leftrightarrow 0 < h < 1$

Para que $x_n > 0 \quad \forall n, \quad 1-2h > 0 \Rightarrow \boxed{h < 1/2}$

② $x_{n+1} = x_n + h f(t_n + \frac{h}{2}, \frac{x_n + x_{n+1}}{2})$. Aplicado a $x' = 2x$,

$$x_{n+1} = x_n + 2h \frac{x_n + x_{n+1}}{2} \Leftrightarrow x_{n+1} = \left(\frac{1 + \bar{h}/2}{1 - \bar{h}/2} \right) x_n \quad \bar{h} = 2h$$

región de estabilidad $|1 + \bar{h}/2| < |1 - \bar{h}/2| \Leftrightarrow |2 + \bar{h}| < |2 - \bar{h}| \Leftrightarrow$

$\Leftrightarrow \operatorname{Re}(\bar{h}) < 0$. Es A-estable.

③ $\begin{cases} -x'' + q(x)x = r \\ \mu x(t_0) - x'(t_0) = \alpha \\ x(T) = \beta \end{cases}$

①

Consideramos un nodo extra $t_{-1} = t_0 - h$ y un valor aproximado x_{-1}

Para $0 \leq n \leq N$ aproximamos a t_n mediante

$$= \frac{x_{n+1} - 2x_n + x_{n-1}}{h^2} + q_n x_n = r_n \quad \begin{matrix} N+1 \text{ ecuaciones} \\ N+2 \text{ incógnitas} \end{matrix}$$

$$-\alpha + \mu x(t_0) = x'(t_0) = \frac{x(t_0+h) - x(t_0-h)}{2h} - \frac{h^2}{6} x'''(t_0) \quad (3)$$

Nuevas ecuaciones: $-\alpha + \mu x_0 = \frac{x_1 - x_{-1}}{2h} \Leftrightarrow -x_{-1} - 2\mu h x_0 + x_1 = -2\alpha h$
 $\Leftrightarrow x_{-1} + 2\mu h x_0 - x_1 = 2\alpha h$

$$\begin{pmatrix} 1 & 2\mu h & -1 & 0 & \dots & 0 \\ -1 & 2 + q_0 h^2 & -1 & 0 & \dots & 0 \\ 0 & -1 & 2 + q_1 h^2 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \end{pmatrix} \begin{pmatrix} x_{-1} \\ x_0 \\ x_1 \\ \vdots \end{pmatrix} = \begin{pmatrix} 2\alpha h \\ h^2 r_0 \\ h^2 r_1 \\ \vdots \end{pmatrix} \quad \begin{matrix} N+2 \text{ ecuaciones} \\ N+2 \text{ incógnitas} \end{matrix}$$

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③ El sistema no es tridiagonal, pero podemos eliminar la incógnita x_1 sumando las dos primeras ecuaciones

$$\begin{pmatrix} 2(1+\mu h) + q_0 h^2 & -2 & 0 & \dots & 0 \\ -1 & 2 + q_1 h^2 & -1 & \dots & 0 \\ 0 & -1 & 2 + q_2 h^2 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ \vdots \end{pmatrix} = \begin{pmatrix} h^2 r_0 + 2\alpha h \\ h^2 r_1 \\ \vdots \end{pmatrix}$$

Ahora es triangular y diagonalmente fuertemente dominante

El sistema tiene solución única

El error local de truncamiento τ_0 es

$$\tau_0 = \left[\frac{2(1+\mu h)}{h^2} + q_0 \right] x(t_0) - \frac{2}{h^2} x(t_0 + h) - r(t_0) - \frac{2\alpha}{h} =$$

$$= -\frac{h^2}{12} x^{(4)}(\xi) - \frac{h}{3} x^{(3)}(\xi) \quad \text{de manera análoga al problema 7}$$

de la hoja 6

Nuevamente se obtiene que $\max |E_n| \leq h^2 \left[\frac{(T-t_0)^2}{24} \eta_4 + \frac{(T-t_0)}{6} \eta_3 \right]$

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$$T(z) = 0 \cdot 0 \cdot 0 \cdot 1$$

$$C_0 = \alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 = 0$$

①

$$P(z) = \alpha_0 \alpha_1 \alpha_2 \alpha_3$$

$$C_1 = \alpha_1 + 2\alpha_2 + 3\alpha_3 - 1 = 0$$

$$C_2 = \frac{1}{2}(\alpha_1 + 4\alpha_2 + 9\alpha_3) - 3 = 0$$

$$C_3 = \frac{1}{6}(\alpha_1 + 8\alpha_2 + 27\alpha_3) - \frac{1}{2}9 = 0$$

$$C_4 = \frac{1}{24}(\alpha_1 + 16\alpha_2 + 81\alpha_3) - \frac{1}{6}27 = 0$$

En forma matricial y resolviendo por el método de Gauss

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 1 \\ 0 & 1 & 4 & 9 & 6 \\ 0 & 1 & 8 & 27 & 27 \\ 0 & 1 & 16 & 81 & 108 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 1 \\ 0 & 0 & 2 & 6 & 5 \\ 0 & 0 & 4 & 18 & 21 \\ 0 & 0 & 8 & 54 & 81 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 1 \\ 0 & 0 & 2 & 6 & 5 \\ 0 & 0 & 0 & 6 & 11 \\ 0 & 0 & 0 & 18 & 39 \end{array} \right)$$

Las dos últimas ecuaciones son incompatibles. Solo podemos asegurar $C_0 = C_1 = C_2 = C_3$ es decir orden 3.

La solución es $\alpha_3 = \frac{11}{6}$, $\alpha_2 = -3$, $\alpha_1 = \frac{3}{2}$, $\alpha_0 = -\frac{1}{3}$

$$P(z) = -\frac{1}{3} + \frac{3}{2}z - 3z^2 + \frac{11}{6}z^3 = (z-1) \left(\frac{11}{6}z^2 - \frac{7}{6}z + \frac{1}{3} \right)$$

Los roots de $P(z)$ son $z=1$, $z = \frac{7 \pm \sqrt{39}i}{22}$

$$|z|^2 = \frac{49 + 39}{22^2} = \frac{2}{11} \Rightarrow |z| < 1 \quad \text{Es 0-estable.} \quad \left. \begin{array}{l} \text{Es de orden 3} \\ \end{array} \right\} \Rightarrow \text{Es convergente}$$

De hecho es el método BDF3

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(2) Sea y_n el valor de la solución $x(t)$ en $t=t_n$: $y_n = x(t_n)$

Sea

~~$$y_{n+2} = y_{n+1} + \frac{h}{4} \left[f(t_{n+1}, y_{n+1}) + 3f\left(t_{n+1} + \frac{2h}{3}, y_{n+1} + \frac{2h}{3} f(t_{n+1}, y_{n+1})\right) \right]$$~~

~~$$z_{n+2} = y_{n+1} + \frac{h}{4} \left[f(t_{n+1}, y_{n+1}) + 3f\left(t_{n+1} + \frac{2h}{3}, y_{n+1} + \frac{2h}{3} f(t_{n+1}, y_{n+1})\right) \right]$$~~

$$y_{n+2} = y_n + \frac{h}{3} \left[f(t_{n+2}, z_{n+2}) + 4f(t_{n+1}, y_{n+1}) + f(t_n, y_n) \right] + h\sigma(t) \tau_n^{\text{PEC}}$$

$$\left(\sigma(t) \text{ corresponde a Simpson } \begin{matrix} p: -1, 0, 1 \\ \sigma: \frac{1}{3}, \frac{4}{3}, \frac{1}{3} \end{matrix} \right)$$

$$y_{n+2} = y_n + \frac{h}{3} \left[f(t_{n+2}, y_{n+2}) + 4 \dots \text{igual} \right] + h\sigma(t) \tau_n^{\text{Simpson}}$$

$$0 = \frac{h}{3} \left[f(t_{n+2}, z_{n+2}) - f(t_{n+2}, y_{n+2}) \right] + h(\tau_n^{\text{PEC}} - \tau_n)$$

$$\Rightarrow \tau_n^{\text{PEC}} = \tau_n + \frac{1}{3} \left[f(t_{n+2}, y_{n+2}) - f(t_{n+2}, z_{n+2}) \right]$$

$$y_{n+2} = y_{n+1} + \frac{h}{4} \left[f(t_{n+1}, y_{n+1}) + 3f\left(t_{n+1} + \frac{2h}{3}, y_{n+1} + \frac{2h}{3} f(t_{n+1}, y_{n+1})\right) \right]$$

$$+ h \tau_n^{\text{Hem}}$$

$$\text{de modo que } y_{n+2} - z_{n+2} = h \tau_n^{\text{H}}$$

$$|\tau_n^{\text{PEC}}| \leq |\tau_n| + \frac{1}{3} L |y_{n+2} - z_{n+2}| \leq |\tau_n| + \frac{1}{3} L h |\tau_n^{\text{H}}|$$

↓
n

2+1

logu orde 3

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$$\textcircled{3} \quad \begin{cases} Lx = r \\ (BP) \quad \begin{cases} x'(2) = 6 \\ x(5) + x'(5) = 4 \quad (B_2 x = 4) \end{cases} \end{cases}$$

$$(PVI1) \quad \begin{cases} Lx = r \\ x'(2) = 6 \\ x(2) = 0 \end{cases}$$

$\downarrow$
 $u(t)$

$$(PVI2) \quad \begin{cases} Lx = 0 \\ x'(2) = 0 \\ x(2) = 1 \end{cases}$$

$\downarrow$
 $v(t)$

$\therefore x(t)$ es solución de BP y $x(2)$ es su valor a $t=2$

$$w = u(t) + x(2)v(t) \quad \text{sabemos} \quad \begin{aligned} Lw &= r \\ w'(2) &= u'(2) = 6 = x'(2) \\ w(2) &= x(2) \end{aligned}$$

por unicidad $x(t) = u(t) + x(2)v(t)$.

Procedamos a la inversa escribamos $x(t) = u(t) + s v(t)$

y calculemos s de modo que

$$x(5) + x'(5) = u(5) + s v(5) + u'(5) + s v'(5) = 4 \quad (\Leftrightarrow)$$

$$\Leftrightarrow s(v(5) + v'(5)) = 4 - (u(5) + u'(5))$$

$$s B_2 v = 4 - B_2 u \Rightarrow \boxed{s = \frac{4 - B_2 u}{B_2 v}} \quad \text{si } B_2 v \neq 0 \text{ siempre.}$$

si $B_2 v = 0$ y $B_2 u = 4$ infinitas soluciones

si $B_2 v = 0$ y $B_2 u \neq 4$ no hay solución.

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$$x' = \begin{pmatrix} 2 & 6 \\ 0 & 4 \end{pmatrix} x$$

$$x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$0 \leq t \leq 1/4 \quad h = \frac{1}{4N} \quad t_i = ih.$$

a) solve exact. $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

b) $x_1 = x_0 + \frac{h}{2} (Ax_0 + Ax_1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{h}{2} \left[\begin{pmatrix} 2 \\ 0 \end{pmatrix} + Ax_1 \right]$

$$x_1 = \begin{pmatrix} 1+h \\ 0 \end{pmatrix} + \frac{h}{2} Ax_1 \quad \Leftrightarrow \left(\frac{h}{2} A - I \right) x_1 = \begin{pmatrix} -(1+h) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} h-1 & 3h \\ 0 & 2h-1 \end{pmatrix} x_1 = \begin{pmatrix} -(1+h) \\ 0 \end{pmatrix} \quad x_1 = \begin{pmatrix} \frac{h+1}{1-h} \\ 0 \end{pmatrix}$$

c) $x(t_1) = x(h) = \begin{pmatrix} e^{2h} \\ 0 \end{pmatrix}$

$$x_1 - x(t_1) = \begin{pmatrix} \frac{h+1}{1-h} - e^{2h} \\ 0 \end{pmatrix}$$

$$\frac{h+1}{1-h} - e^{2h} = \begin{pmatrix} -1 + \frac{2}{1-h} - e^{2h} \\ 0 \end{pmatrix}$$

$$-1 + \frac{2}{1-h} = -1 + 2(1+h+h^2+h^3+\dots) = 1+2h+2h^2+2h^3+\dots$$

$$e^{2h} = 1+2h+\frac{1}{2}4h^2+\frac{1}{6}8h^3+\dots$$

$$-1 + \frac{2}{1-h} - e^{2h} = \left(2 - \frac{4}{3}\right)h^3 + \dots$$

$$\|x_1 - x(t_1)\| = O(h^3)$$

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(2)

$$x_{n+2} = x_n + \frac{h}{4} [F_1 + 3F_3]$$

$$F_1 = f(t_n, x_n)$$

$$F_2 = f(t_n + \frac{h}{3}, x_n + \frac{h}{3} F_1)$$

$$F_3 = f(t_n + \frac{2h}{3}, x_n + \frac{2h}{3} F_2)$$

$$\begin{array}{c|ccc} & 0 & 1/3 & 2/3 \\ \hline 0 & 0 & 0 & 0 \\ 1/3 & 1/3 & 0 & 0 \\ 2/3 & 0 & 2/3 & 0 \\ \hline & 1/4 & 0 & 3/4 \end{array}$$

$$c_1 + c_2 + c_3 = 1$$

$$b_1 c_1 + 2b_2 c_2 + b_3 c_3 = \frac{2}{3} \cdot \frac{3}{4} = \frac{1}{2}$$

$$a_1 c_1 + a_2 c_2 + a_3 c_3 = 0 + \frac{3}{4} \cdot \frac{2}{3} = \frac{1}{2}$$

$$c_3 a_{32} h_2 = 1/6$$

$$\begin{aligned} x_{n+1} &= x_n + \frac{h}{4} \left[2x_n + 3 \left(x_n + \frac{2h}{3} f \left(x_n + \frac{h}{3} x_n \right) \right) \right] = \\ &= x_n + \frac{h}{4} \left[2x_n + 3 \left(x_n + \frac{2h}{3} \left(x_n + \frac{h}{3} x_n \right) \right) \right] = \\ &= x_n + \frac{h}{4} \left[x_n + 3x_n + 2h \left(x_n + \frac{h}{3} x_n \right) \right] = \\ &= x_n \left(1 + \bar{h} + \frac{\bar{h}^2}{2} + \frac{\bar{h}^3}{6} \right) \end{aligned}$$

$$(3) \begin{cases} -x'' + f(t, x) = 0 \\ x(0) = x(1) = 0 \end{cases}$$

$$- \frac{x(t_{n-1}) - 2x(t_n) + x(t_{n+1}))}{h^2} + f(t_n, x(t_n)) =$$

$$= \tau_n = - \frac{h^2}{12} x^{(4)}(\theta_n)$$

$$- \frac{\varepsilon_{n-1} - 2\varepsilon_n + \varepsilon_{n+1}}{h^2} + \underbrace{f(t_n, x(t_n)) - f(t_n, x_n)}_{=0} = \tau_n$$

$$\frac{\partial f}{\partial x}(t_n, x_n) \varepsilon_n$$