2º BG. TECNOLÓGICO. EXA 4. Temas 4, 5 y 6.

1º. $f(x)=\frac{(x-1)^2}{x-2}$. $D f$: $\mathbb{R}-\{2\}$. $f'(x)=\frac{(x-1)(x-3)}{(x-2)^2}=\frac{x^2-4x+3}{(x-2)^2}$.

$f''(x)=\frac{2}{(x-2)^2}$.

asíntotas:

. $\lim_{x\to 2^+}\frac{(x-1)^2}{x-2}=+\infty$; $\lim_{x\to 2^-}\frac{(x-1)^2}{x-2}=-\infty$; Asínt. vertical; $x=2$

. $\lim_{x\to\infty}\frac{(x-1)^2}{x-2}=\infty$. No hay asíntota horizontal

. $m=\lim_{x\to\infty}\frac{(x-1)^2}{x^2-2x}=\lim_{x\to\infty}\frac{x^2}{x^2}=1$; $n=\lim_{x\to\infty}\frac{(x-1)^2}{x-2}-x=\lim_{x\to\infty}\frac{1}{x-2}=0$.

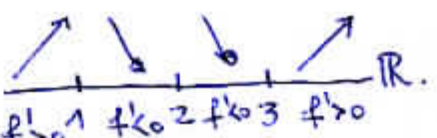
asíntota oblicua $y=x$ tanto si $x\to+\infty$ como si $x\to-\infty$

Extremos:

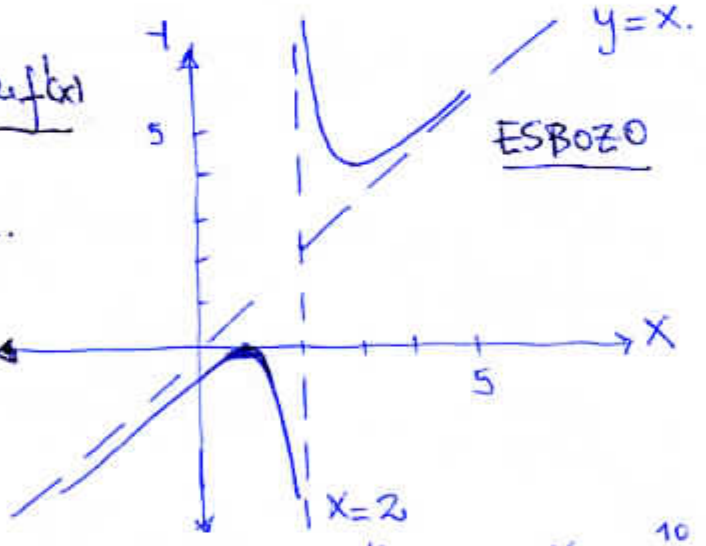
$f'(x)=0$. $x^2-4x+3=0$; $x=\frac{4\pm\sqrt{16-12}}{2}=\frac{4\pm 2}{2}=\begin{cases}3\\1\end{cases}$

$f''(3)=\frac{2}{(3-2)^2}=2>0$. En $x=3$ hay mín. relativo y vale $f(3)=4$.
$f''(1)=\frac{2}{(1-2)^2}=-2<0$ En $x=1$ hay máx. relativo y vale $f(1)=0$

Estudio signo de $f'(x)$



Creciente: $(-\infty,1)\cup(3,+\infty)$
Decreciente: $(1,3)$



2º. $\int (x-10)^9\cdot(x-9)\,dx=\frac{(x-10)^{10}}{10}\cdot(x-9)-\int\frac{(x-10)^{10}}{10}dx=\frac{(x-10)^{10}}{10}(x-9)-\frac{1}{10}\frac{(x-10)^{11}}{11}+$ cte.

Integración por partes: $\int f'(x)g(x)\,dx=f(x)g(x)-\int f(x)g'(x)\,dx$.
$f'(x)=(x-10)^9$; $f(x)=\frac{(x-10)^{10}}{10}$.
$g(x)=x-9$; $g'(x)=1$.

3° $\int \frac{x^2}{\sqrt[3]{1+2x}}\,dx = *$ Cambio de variable: $1+2x=t^3$;
$x=\frac{t^3-1}{2}$; $dx=\frac{3t^2}{2}dt$.

$$* = \int \frac{\left[\frac{t^3-1}{2}\right]^2}{t}\cdot\frac{3t^2}{2}\,dt = \frac{3}{8}\int (t^3-1)^2\cdot t\,dt = \frac{3}{8}\int (t^7-2t^4+t)\,dt$$

$$= \frac{3}{8}\left[\frac{t^8}{8}-\frac{2t^5}{5}+\frac{t^2}{2}\right]+cte = \frac{3}{8}\left[\frac{\sqrt[3]{(1+2x)^8}}{8}-\frac{2\sqrt[3]{(1+2x)^5}}{5}+\frac{\sqrt[3]{(1+2x)^2}}{2}\right]+cte$$

4° $$\int \frac{3x-7}{x^2+x+5}\,dx = \frac{3}{2}\int\frac{\frac{2}{3}(x-7)}{x^2+x+5}\,dx = \frac{3}{2}\int\frac{2x-\frac{14}{3}+1-1}{x^2+x+5}\,dx =$$

$$= \frac{3}{2}\left[\int\frac{2x+1}{x^2+x+5}\,dx + \int\frac{-14/3-1}{x^2+x+5}\,dx\right] = \frac{3}{2}\ln|x^2+x+5| +$$

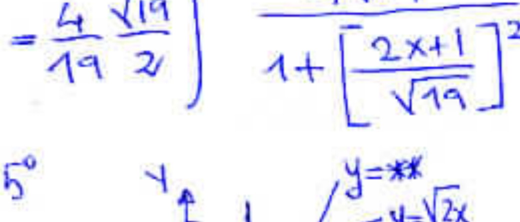
$$+\frac{3}{2}\cdot\left(-\frac{17}{3}\right)\int\frac{1}{x^2+x+5}\,dx \underset{*}{=} \frac{3}{2}\ln|x^2+x+5| - \frac{17}{\cancel{2}}\frac{\cancel{2}\sqrt{19}}{19}\,\mathrm{arctg}\left[\frac{2x+1}{\sqrt{19}}\right]+cte$$

$$\int\frac{1}{x^2+x+5}\,dx = \int\frac{1}{(x+\frac{1}{2})^2+5-\frac{1}{4}}\,dx = \int\frac{dx}{\frac{19}{4}+(x+\frac{1}{2})^2} =$$

$$= \int\frac{dx}{\frac{19}{4}\left[1+\frac{(x+1/2)^2}{19/4}\right]} = \frac{4}{19}\int\frac{dx}{1+\left[\frac{2(x+1/2)}{\sqrt{19}}\right]^2} = \frac{4}{19}\int\frac{dx}{1+\left[\frac{2x+1}{\sqrt{19}}\right]^2} =$$

$$= \frac{4}{19}\frac{\sqrt{19}}{2}\int\frac{2/\sqrt{19}}{1+\left[\frac{2x+1}{\sqrt{19}}\right]^2}\,dx = \frac{2\sqrt{19}}{19}\,\mathrm{arctg}\left[\frac{2x+1}{\sqrt{19}}\right]+cte.$$

5°



$$A = \int_0^2 \overbrace{\sqrt{2x}-(-\sqrt{2x})}^{2\sqrt{2}x^{1/2}}\,dx + \int_2^4\left[\sqrt{2x}-(1+\sqrt{2})x+4+2\sqrt{2}\right]dx$$

$$= 2\sqrt{2}\cdot\frac{2}{3}x^{3/2}\Big|_0^2 + \sqrt{2}\,\frac{2}{3}x^{3/2}\Big|_2^4 - (1+\sqrt{2})\frac{x^2}{2}\Big|_{+2}^4 +$$

$$+(4+2\sqrt{2})x\Big|_2^4 = \frac{4\sqrt{2}}{3}2^{3/2} + \frac{16\sqrt{2}}{3} - \frac{8}{3} -$$

$$-(1+\sqrt{2})8 + (1+\sqrt{2})2 + 8+4\sqrt{2} =$$

$$= \frac{16}{3}+\frac{16\sqrt{2}}{3}-\frac{8}{3}-6-6\sqrt{2}+8+4\sqrt{2} = \dots = \boxed{\frac{14+10\sqrt{2}}{3}u^2}$$

** $\left[\begin{array}{l}(2,-2)\text{ y }(4,2\sqrt{2}).\\ y=(1+\sqrt{2})x-4-2\sqrt{2}.\end{array}\right.$