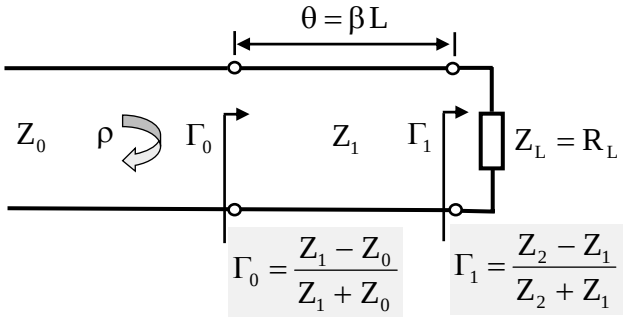
LÍNEAS DE TRANSMISIÓN

ADAPTACIÓN DE IMPEDANCIAS EN BANDA ANCHA

Banda ancha

Teoría de pequeñas reflexiones



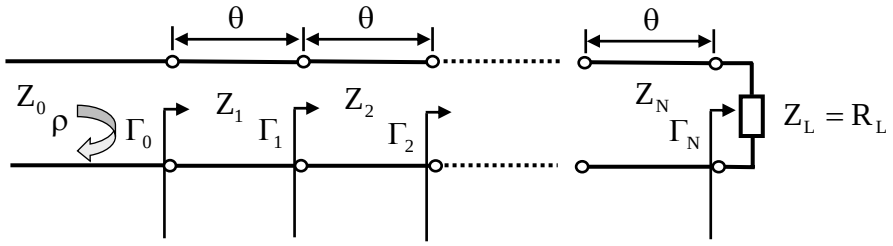
$$\Gamma_0 = \frac{Z_1 - Z_0}{Z_1 + Z_0} \qquad \Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$\rho = \frac{\Gamma_0 + \Gamma_1 e^{-j2\theta}}{1 + \Gamma_0 \Gamma_1 e^{-j2\theta}}$$

Si $|\Gamma_0|, |\Gamma_1| << 1 \Longrightarrow \rho \approx \Gamma_0 + \Gamma_1 e^{-j2\theta}$

Banda ancha

Teoría de pequeñas reflexiones



$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

$$\rho(\theta) = \Gamma_0 + \Gamma_1 e^{-j2\theta} + \Gamma_2 e^{-j4\theta} + ... + \Gamma_N e^{-j2N\theta}$$

Banda ancha

Si la red es simétrica: $\Longrightarrow$ $\Gamma_0 = \Gamma_N,\ \Gamma_1 = \Gamma_{N-1}, ...$

$$\rho(\theta) = \begin{cases} e^{-jN\theta}\left[\Gamma_0\left(e^{jN\theta} + e^{-jN\theta}\right) + \Gamma_1\left(e^{j(N-2)\theta} + e^{-j(N-2)\theta}\right) + ... + \Gamma_{(N-1)/2}\left(e^{j\theta} + e^{-j\theta}\right)\right] & \text{N impar} \\ e^{-jN\theta}\left[\Gamma_0\left(e^{jN\theta} + e^{-jN\theta}\right) + \Gamma_1\left(e^{j(N-2)\theta} + e^{-j(N-2)\theta}\right) + ... + \Gamma_{N/2}\right] & \text{N par} \end{cases}$$

$$\rho(\theta) = \begin{cases} 2e^{-jN\theta}\left[\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + ... + \Gamma_n \cos(N-2n)\theta + ... + \frac{1}{2}\Gamma_{(N-1)/2} \cos\theta\right] & \text{N impar} \\ 2e^{-jN\theta}\left[\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + ... + \Gamma_n \cos(N-2n)\theta + ... + \frac{1}{2}\Gamma_{N/2}\right] & \text{N par} \end{cases}$$

Dado $\rho(\theta) \Rightarrow Z_1, Z_2, ... Z_n$

Banda ancha

Transformador binomial

$$\rho(\theta) = A\left(1 + e^{-j2\theta}\right)^N = A\sum_{n=0}^{N} C_n^N\, e^{-j2n\theta} = \Gamma_0 + \Gamma_1 e^{-j2\theta} + \Gamma_2 e^{-j4\theta} + ... + \Gamma_N e^{-j2N\theta}$$

$$\Gamma_n = A\, C_n^N$$

$$\rho(\theta) = A\, e^{-jN\theta}\left(e^{j\theta} + e^{-j\theta}\right)^N = 2^N A\, e^{-jN\theta}\cos^N\theta$$

Máxima respuesta plana:

$$\left.\frac{d^n\left|\rho(\theta)\right|}{d\theta^n}\right|_{\theta=\frac{\pi}{2},\, l=\frac{\lambda}{4}} = 0; \quad n = 1, 2, ..., N-1$$

$$\rho(0) = 2^N A = \frac{Z_L - Z_0}{Z_L + Z_0} \quad \Longrightarrow \quad A = 2^{-N}\frac{Z_L - Z_0}{Z_L + Z_0}$$

Banda ancha

Transformador binomial

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} \approx \frac{1}{2}\ln\frac{Z_{n+1}}{Z_n}$$

$$\ln\frac{Z_{n+1}}{Z_n} \approx 2\Gamma_n = 2A\, C_n^N = 2\cdot 2^{-N}\frac{Z_L - Z_0}{Z_L + Z_0} C_n^N \approx 2^{-N} C_n^N \ln\frac{Z_L}{Z_0}$$

Ancho de banda:

$$\left|\rho_m\right| = 2^N\left|A\right|\cos^N\theta_m \qquad \theta_m = \arccos\left[\frac{1}{2}\left(\frac{\left|\rho_m\right|}{\left|A\right|}\right)^{1/N}\right]$$

$$\frac{\Delta f}{f_0} = \frac{2\left(f_0 - f_m\right)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4}{\pi}\arccos\left[\frac{1}{2}\left(\frac{\left|\rho_m\right|}{\left|A\right|}\right)^{1/N}\right]$$

$N \uparrow, \Delta f \uparrow$

Banda ancha

Transformador binomial Ejemplo:

$Z_L = 50\,\Omega,\ Z_0 = 100\,\Omega,\ N = 3,\ \rho_m = 0{,}05$

$$A = 2^{-N}\frac{Z_L - Z_0}{Z_L + Z_0} \approx \frac{1}{2^{N+1}}\ln\frac{Z_L}{Z_0} = -0{,}0433$$

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\arccos\left[\frac{1}{2}\left(\frac{|\rho_m|}{|A|}\right)^{1/N}\right] = 0{,}70 \rightarrow 70\,\%$$

$$C_0^3 = \binom{3}{0} = \frac{3!}{3!0!} = 1 \qquad C_1^3 = \binom{3}{1} = \frac{3!}{2!1!} = 3 \qquad C_2^3 = \binom{3}{2} = \frac{3!}{1!2!} = 3$$

Banda ancha

Transformador binomial Ejemplo:

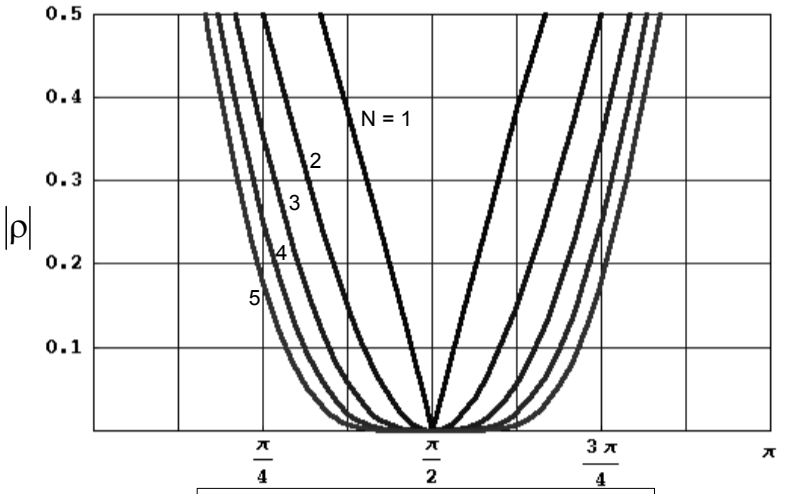
$Z_L = 50\,\Omega,\ Z_0 = 100\,\Omega,\ N = 3,\ \rho_m = 0{,}05$

$$n = 0 \rightarrow \ln Z_1 = \ln Z_0 + 2^{-3} C_0^3 \ln\frac{Z_L}{Z_0} = 4{,}518 \qquad Z_1 = 91{,}7\,\Omega$$

$$n = 1 \rightarrow \ln Z_2 = \ln Z_1 + 2^{-3} C_1^3 \ln\frac{Z_L}{Z_0} = 4{,}26 \qquad Z_2 = 70{,}7\,\Omega$$

$$n = 2 \rightarrow \ln Z_3 = \ln Z_2 + 2^{-3} C_2^3 \ln\frac{Z_L}{Z_0} = 4{,}00 \qquad Z_3 = 54{,}5\,\Omega$$

Transformador Binomial



Banda ancha

Transformador binomial

Z_L/Z_0	$N=2$		$N=3$			$N=4$			
	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1067	1.3554	1.0520	1.2247	1.4259	1.0257	1.1351	1.3215	1.4624
2.0	1.1892	1.6818	1.0907	1.4142	1.8337	1.0444	1.2421	1.6102	1.9150
3.0	1.3161	2.2795	1.1479	1.7321	2.6135	1.0718	1.4105	2.1269	2.7990
4.0	1.4142	2.8285	1.1907	2.0000	3.3594	1.0919	1.5442	2.5903	3.6633
6.0	1.5651	3.8336	1.2544	2.4495	4.7832	1.1215	1.7553	3.4182	5.3500
8.0	1.6818	4.7568	1.3022	2.8284	6.1434	1.1436	1.9232	4.1597	6.9955
10.0	1.7783	5.6233	1.3409	3.1623	7.4577	1.1613	2.0651	4.8424	8.6110

Z_L/Z_0	$N=5$					$N=6$					
	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_6/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0128	1.0790	1.2247	1.3902	1.4810	1.0064	1.0454	1.1496	1.3048	1.4349	1.4905
2.0	1.0220	1.1391	1.4142	1.7558	1.9569	1.0110	1.0790	1.2693	1.5757	1.8536	1.9782
3.0	1.0354	1.2300	1.7321	2.4390	2.8974	1.0176	1.1288	1.4599	2.0549	2.6577	2.9481
4.0	1.0452	1.2995	2.0000	3.0781	3.8270	1.0225	1.1661	1.6129	2.4800	3.4302	3.9120
6.0	1.0596	1.4055	2.4495	4.2689	5.6625	1.0296	1.2219	1.8573	3.2305	4.9104	5.8275
8.0	1.0703	1.4870	2.8284	5.3800	7.4745	1.0349	1.2640	2.0539	3.8950	6.3291	7.7302
10.0	1.0789	1.5541	3.1623	6.4346	9.2687	1.0392	1.2982	2.2215	4.5015	7.7030	9.6228

Banda ancha

Transformador Chebyshev

$$\rho(\theta) = 2e^{-jN\theta}\left[\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + ... + \Gamma_n \cos(N-2n)\theta + ...\right]$$

$$\rho(\theta) = A\, e^{-jN\theta}\, T_N(\sec\theta_m \cos\theta) \quad \Rightarrow \quad \Gamma_n$$

Polinomios de Chebyshev:

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

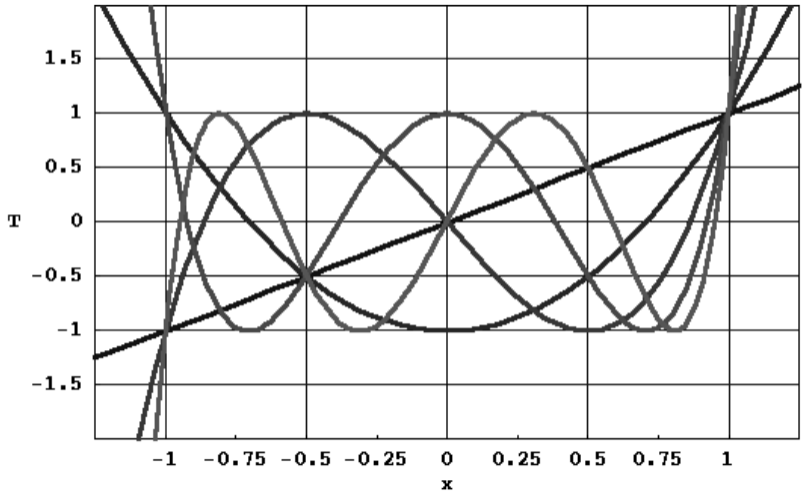
$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

$$x = \frac{\cos\theta}{\cos\theta_m}$$

Polinomios de Chebyshev



Banda ancha

Transformador Chebyshev

$$\rho(0) = A\,T_N(\sec\theta_m) = \frac{Z_L - Z_0}{Z_L + Z_0} \Longrightarrow A = \frac{Z_L - Z_0}{Z_L + Z_0}\frac{1}{T_N(\sec\theta_m)} = \rho_m$$

$$T_N(\sec\theta_m) = \frac{1}{\rho_m}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| \Longrightarrow \theta_m \Longrightarrow \frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi}$$

- Dado ρ_m → máximo Δf
- Dado Δf → mínimo ρ_m

- Mayor ancho de banda que con el transformador binomial
- Rizado en la banda de paso

Banda ancha

Transformador Chebyshev Ejemplo:

$$Z_L = 100\,\Omega,\ Z_0 = 50\,\Omega,\ N = 3,\ \rho_m = 0{,}05$$

$$\begin{aligned}\rho(\theta) &= 2e^{-j3\theta}\left(\Gamma_0\cos 3\theta + \Gamma_1\cos\theta\right) = A\,e^{-j3\theta}\,T_3(\sec\theta_m\cos\theta)\\ &= A\,e^{-j3\theta}\left(4\sec^3\theta_m\cos^3\theta - 3\sec\theta_m\cos\theta\right)\\ &= A\,e^{-j3\theta}\left[\sec^3\theta_m(\cos 3\theta + 3\cos\theta) - 3\sec\theta_m\cos\theta\right]\end{aligned}$$

$$A = \frac{Z_L - Z_0}{Z_L + Z_0}\frac{1}{T_3(\sec\theta_m)} = \rho_m \Longrightarrow \sec\theta_m$$

$$\theta_m = 44{,}7^o \Longrightarrow \frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi} = 101\,\%$$

Banda ancha

Transformador Chebyshev Ejemplo:

$$Z_L = 100\,\Omega,\ Z_0 = 50\,\Omega,\ N = 3,\ \rho_m = 0{,}05$$

$$2\Gamma_0 = A\sec^3\theta_m \quad \rightarrow \quad \Gamma_0 = 0{,}0698 = \Gamma_3$$

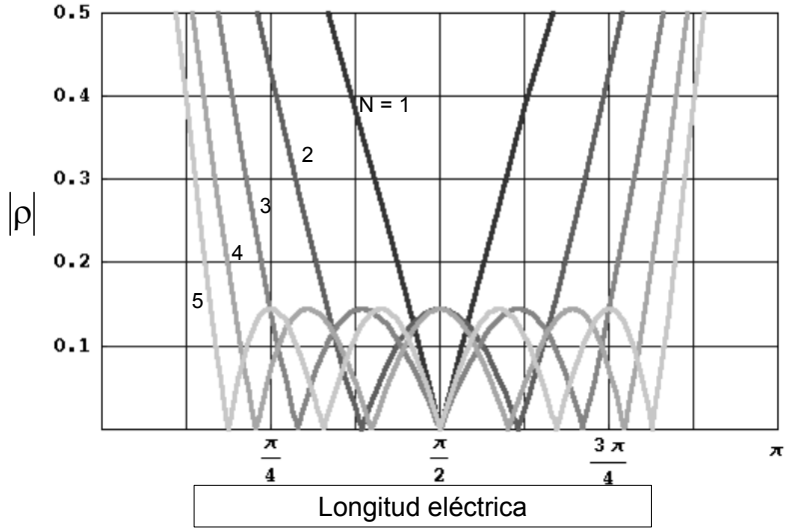
$$2\Gamma_1 = A\left(3\sec^3\theta_m - 3\sec\theta_m\right) \quad \rightarrow \quad \Gamma_1 = 0{,}1037 = \Gamma_2$$

$$\Gamma_0 = \frac{Z_1 - Z_0}{Z_1 + Z_0} \Longrightarrow Z_1 = 57{,}5\,\Omega$$

$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1} \Longrightarrow Z_2 = 70{,}7\,\Omega$$

$$\Gamma_2 = \frac{Z_3 - Z_2}{Z_3 + Z_2} \Longrightarrow Z_3 = 87\,\Omega$$

Transformador Chebyshev



Banda ancha

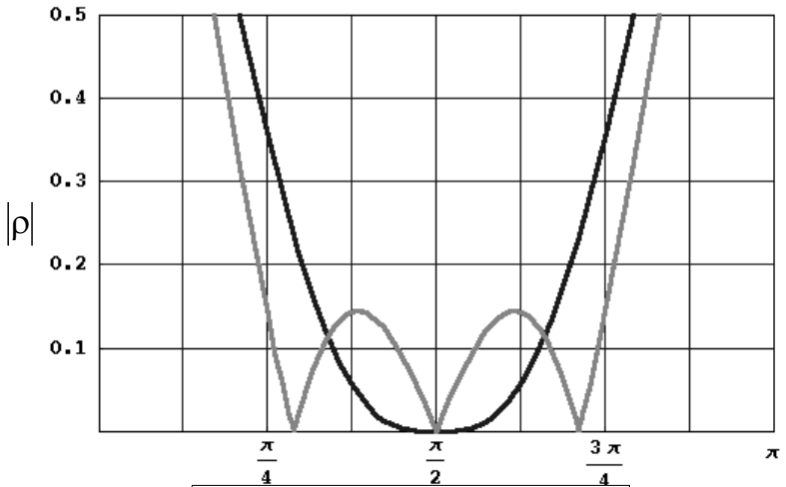
Transformador Chebyshev

	$N=2$				$N=3$					
	$\Gamma_m = 0.05$		$\Gamma_m = 0.20$		$\Gamma_m = 0.05$			$\Gamma_m = 0.20$		
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1347	1.3219	1.2247	1.2247	1.1029	1.2247	1.3601	1.2247	1.2247	1.2247
2.0	1.2193	1.6402	1.3161	1.5197	1.1475	1.4142	1.7429	1.2855	1.4142	1.5558
3.0	1.3494	2.2232	1.4565	2.0598	1.2171	1.7321	2.4649	1.3743	1.7321	2.1829
4.0	1.4500	2.7585	1.5651	2.5558	1.2662	2.0000	3.1591	1.4333	2.0000	2.7908
6.0	1.6047	3.7389	1.7321	3.4641	1.3383	2.4495	4.4833	1.5193	2.4495	3.9492
8.0	1.7244	4.6393	1.8612	4.2983	1.3944	2.8284	5.7372	1.5766	2.8284	5.0742
10.0	1.8233	5.4845	1.9680	5.0813	1.4385	3.1623	6.9517	1.6415	3.1623	6.0920

	$N=4$							
	$\Gamma_m = 0.05$				$\Gamma_m = 0.20$			
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0892	1.1742	1.2775	1.3772	1.2247	1.2247	1.2247	1.2247
2.0	1.1201	1.2979	1.5409	1.7855	1.2727	1.3634	1.4669	1.5715
3.0	1.1586	1.4876	2.0167	2.5893	1.4879	1.5819	1.8965	2.0163
4.0	1.1906	1.6414	2.4369	3.3597	1.3692	1.7490	2.2870	2.9214
6.0	1.2290	1.8773	3.1961	4.8820	1.4415	2.0231	2.9657	4.1623
8.0	1.2583	2.0657	3.8728	6.3578	1.4914	2.2428	3.5670	5.3641
10.0	1.2832	2.2268	4.4907	7.7930	1.5163	2.4210	4.1305	6.5950

Comparación Binomial-Chebyshev

Transformador de tres secciones



Longitud eléctrica