



Escola d'Enginyeria de Telecomunicació  
i Aeroespacial de Castelldefels

UNIVERSITAT POLITÈCNICA DE CATALUNYA

Aerodynamics (AER)

## LESSON 2: INVISCID FLOW

### *Part 1: 2D Potential Flow (Infinite Wings)*

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- Sharp trailing edge – Streamlining
- Hypothesis of Kutta
- Generation of circulation
- Pressure, lift, drag & pitching moment coef.

## 2D POTENTIAL FLOW

### INTRODUCTION

- **Objective** → to calculate the pressure distribution (forces  $l$  &  $d$ ) on 2D airfoils moving horizontally in steady atmosphere
- **Hypotheses:**
  - long or infinite wing → 2D flow
  - $Re \gg 1$  & attached Boundary Layer (BL) → viscosity effects neglected
  - $Fr \gg 1$  → volume forces neglected
  - Bjerknes-Kelvin Theorem
  - properties far upwind → uniform & stationary } irrotational flow
  - $St \ll 1$  → stationary problem
  - low speed:  $M < 0.30$  → compressibility effects neglected:  $\rho = ct$

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## 2D POTENTIAL FLOW

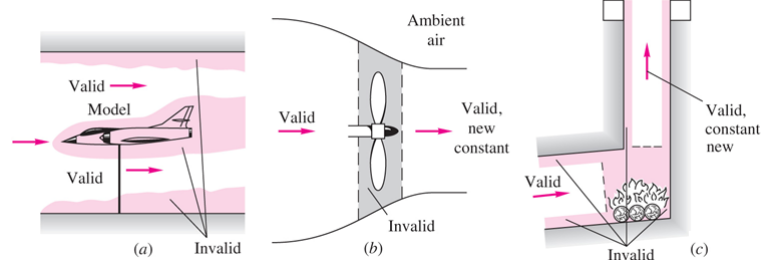
### PROCEDURE TO FULFIL OBJECTIVE

To compute the aerodynamic forces ( $p$  distrib.) on the airfoil:

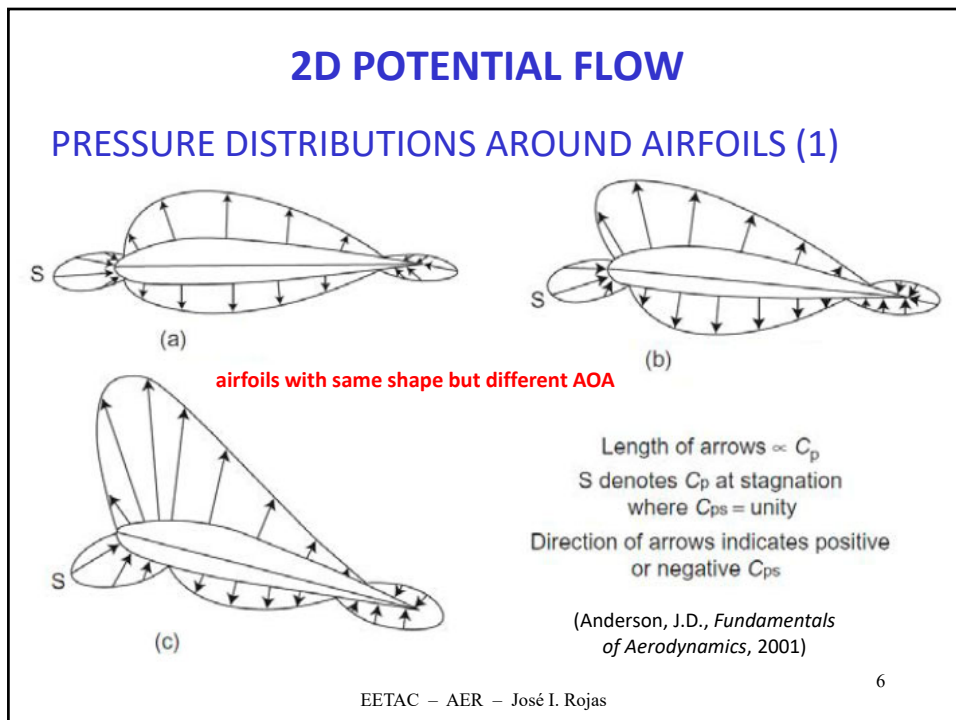
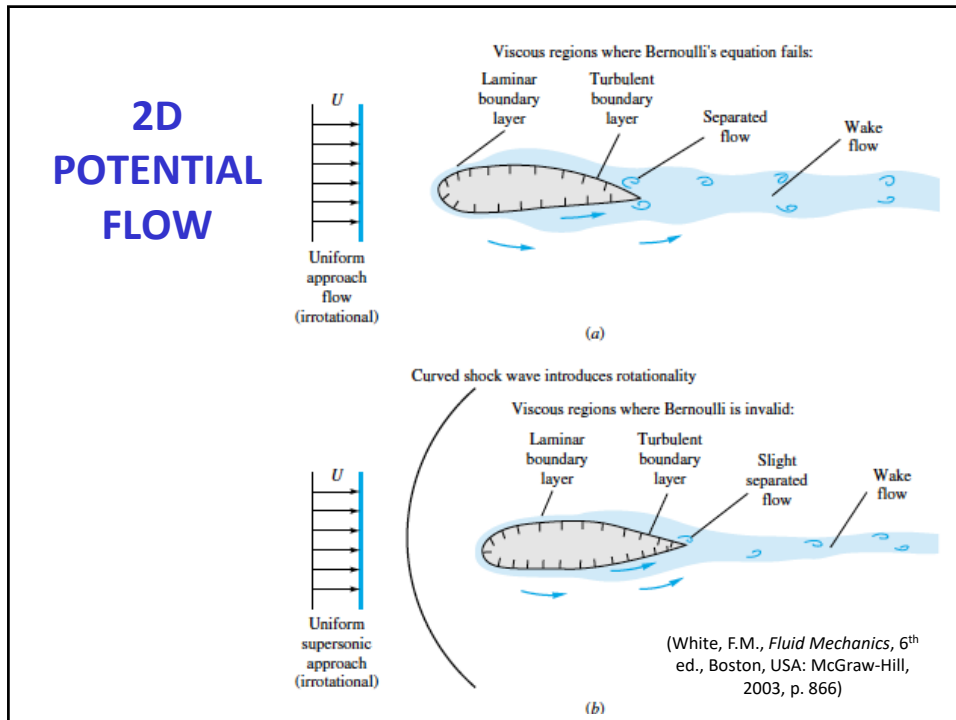
1. compute **velocity potential** from differential Eq. for velocity potential
2. then compute **velocity field** from velocity potential
3. then compute **pressure field** by application of Bernoulli Eq.:

(White, F.M., *Fluid Mechanics*, 6<sup>th</sup> ed., Boston, USA: McGraw-Hill, 2003, p. 866)

$$\frac{1}{2}\rho_{\infty}|\nabla\Phi|^2 + p = \frac{1}{2}\rho_{\infty}U_{\infty}^2 + p_{\infty}$$

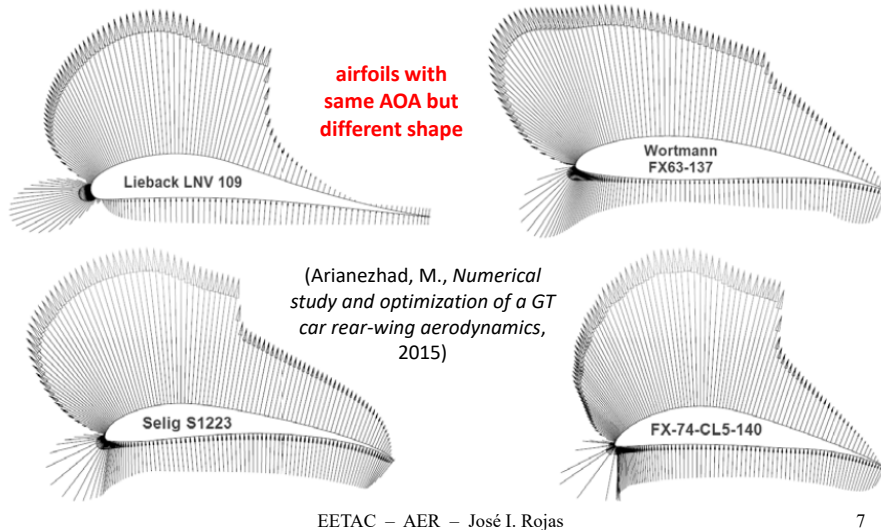


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## 2D POTENTIAL FLOW

### PRESSURE DISTRIBUTIONS AROUND AIRFOILS (2)



## 2D POTENTIAL FLOW

### FORCES ON AIRFOIL IN 2D POTENTIAL FLOW

- relate circulation  $\Gamma$  with Newton's 2<sup>nd</sup> Principle to calculate *lift*
- control vol. containing airfoil & with boundaries not too far from it
- Aerodynamic forces on an airfoil in stationary 2D potential flow:
  - **x-axis:** D'Alembert's paradox → aerodynamic drag on 2D obstacle in a stationary 2D potential flow is **NULL** →
  - **z-axis:** Kutta-Yukovski theorem →

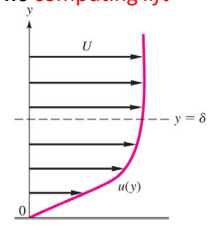
$$d = 0$$

$$l = \rho U_\infty \Gamma$$
- **Conclusions:**
  - **potential flow:** aerodyn. force **perpendicular** to incident flow (only *lift*)
  - airfoils generate *lift* only if circulation is **NOT null**

## 2D POTENTIAL FLOW

### VISCOSITY EFFECTS

- **Contradiction?**
  - **topic 1** → Bjerknes-Kelvin + flying in steady atmosphere → potential flow → **circulation NULL!**
  - **previous slide** → airfoils generate *lift* only if circulation is **NOT NULL!**
- **Explanation** → viscosity effects:
  - if BL thin & not detached,  $p$  distribution on airfoil due to **inviscid potential flow** =  $p$  distrib. due to **real flow** (in BL models, transversal  $p$  gradient is NULL)
  - regardless **viscosity NEGLECTED**, potential flow model allows **computing lift**
- **Yet viscosity plays key role in:**
  - flow motion **around** airfoil
  - generation of **circulation** on airfoil



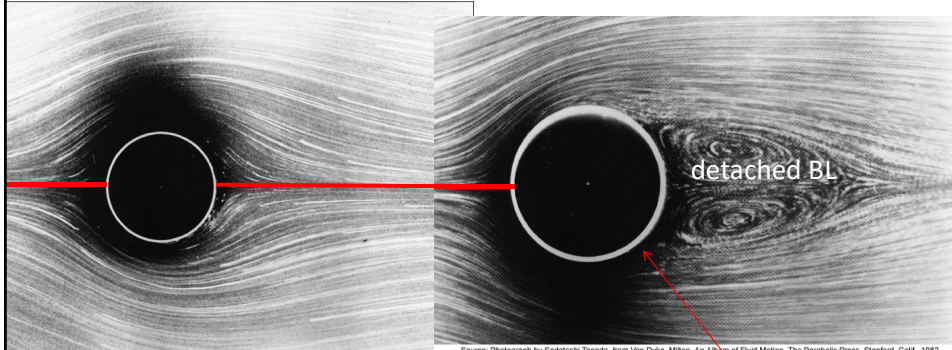
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## 2D POTENTIAL FLOW

### SHARP TRAILING EDGE (1) – STREAMLINING

**potential flow**  
(volunteer for drawing  $p$  distribution)

**real flow**  
(non-streamlined/blunt/bluff body)

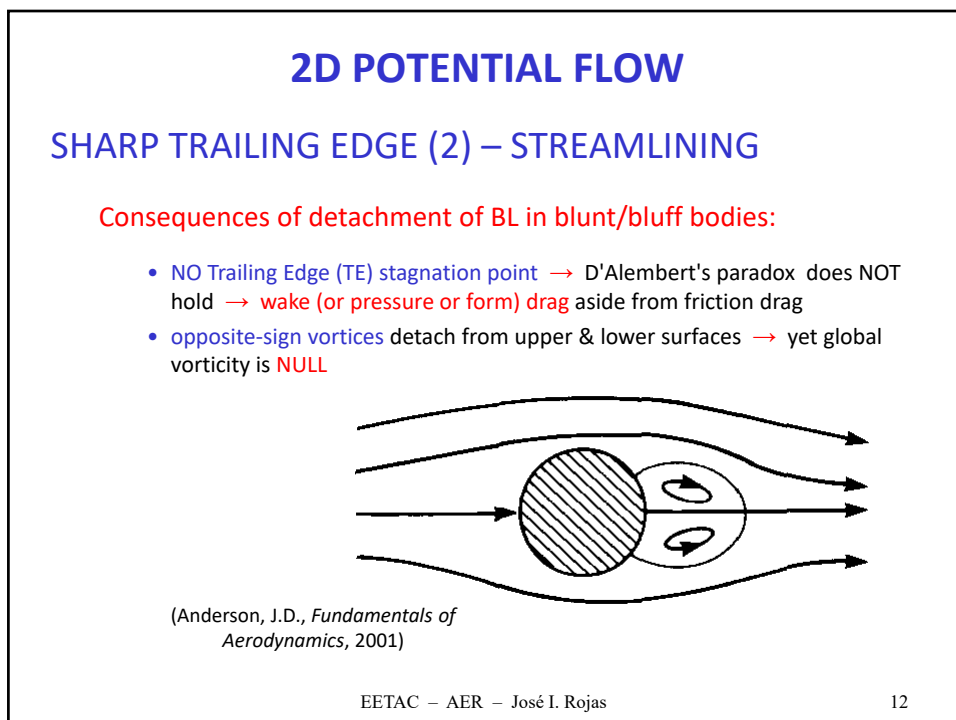
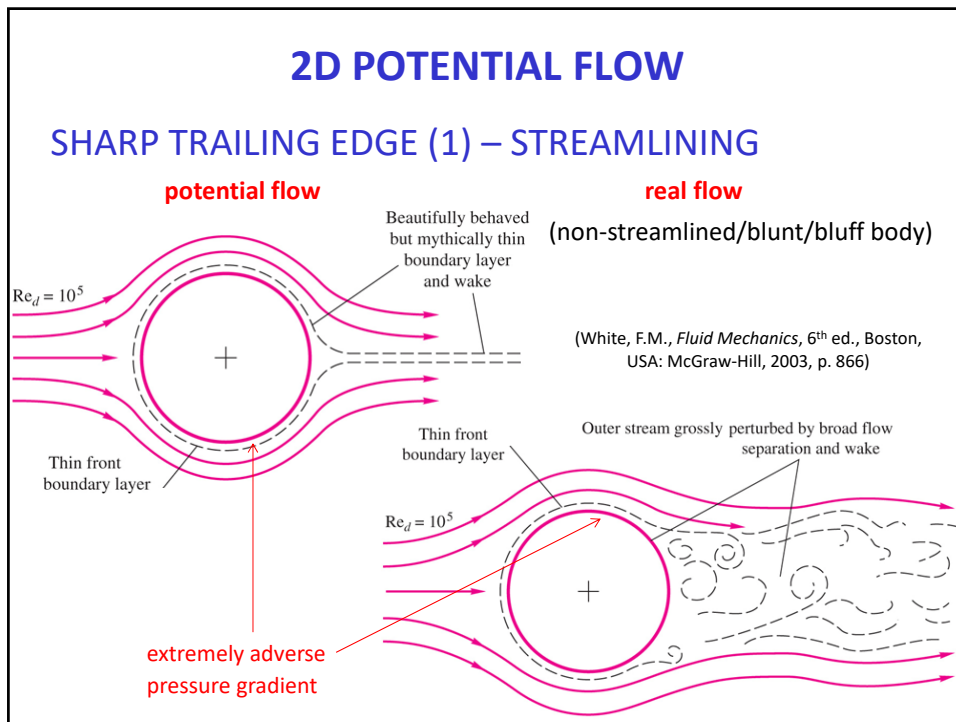


Source: Photograph by Sadatoshi Tanieda, from Van Dyke, Milton, An Album of Fluid Motion, The Parabolic Press, Stanford, Calif., 1982  
(Anderson, J.D., *Fundamentals of Aerodynamics*, 2001)

**extremely adverse pressure gradient**

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## 2D POTENTIAL FLOW

### SHARP TRAILING EDGE (3) – STREAMLINING

Flow behavior around streamlined bodies: sharp trailing edge (TE):

- POTENTIAL flow → rear stagnation point in upper surf. → NOT realistic
- REAL flow → larger vorticity detached from lower surface  
→ circulation appears on the airfoil:
  - opposite sign respect to detached net vorticity
  - approx. equal in absolute value
  - pushes rear stagnation point towards trailing edge

(Eleni, D.C., *J Mech Eng Res* 4(3) (2012) 100-111)

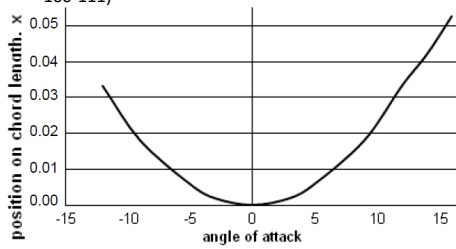
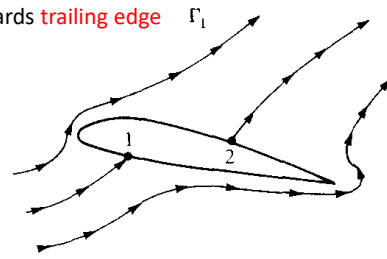


Figure 13. Stagnation point for various angles of attack.

(Anderson, J.D., *Fundamentals of Aerodynamics*, 2001)



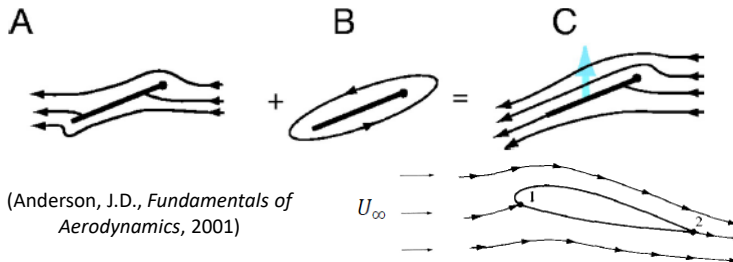
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## 2D POTENTIAL FLOW

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(Anderson, J.D., *Fundamentals of Aerodynamics*, 2001)

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## 2D POTENTIAL FLOW

### HYPOTHESIS OF KUTTA (1)

- **Hypothesis of Kutta:** "Circulation around airfoil is such that the rear stagnation point is located in the trailing edge (or disappears)"
- Upper & lower flow **meet in TE** with **same pressure** →  
→ **MUST** have **same velocity** (Bernoulli Eq.)
- **2 possibilities:**
  - **angular TE:** 2 tangents → velocities can only be equal if **NULL** → **stagnation point** in TE
  - **tangential TE:** unique tangent → velocities do **NOT** need to be null → **NO stagnation point** in TE

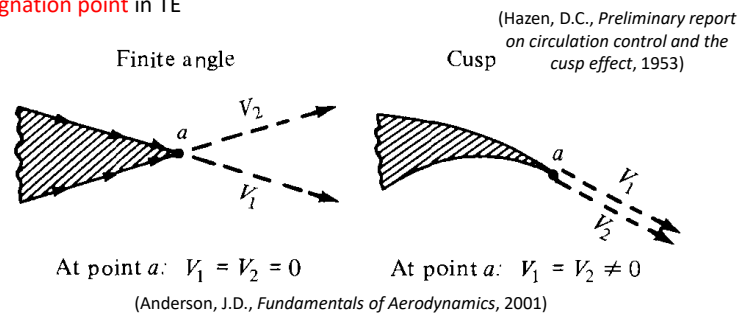
**Question:** Is it correct to state that the Kutta hypothesis can only be fulfilled if the velocity of the upper and lower flow in the TE is null?

## 2D POTENTIAL FLOW

### HYPOTHESIS OF KUTTA (2)

**2 possibilities:**

- **angular TE:** 2 tangents → velocities can only be equal if **NULL** → **stagnation point** in TE
- **tangential TE:** unique tangent → velocities do **NOT** need to be null → **NO stagnation point** in TE

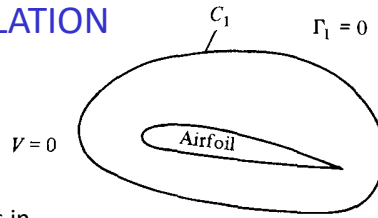




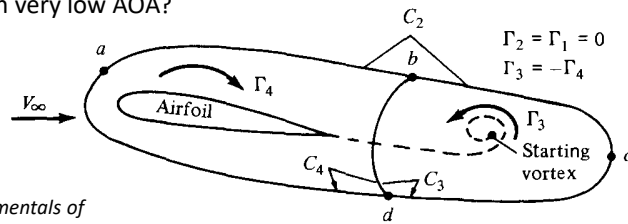
## 2D POTENTIAL FLOW

### GENERATION OF CIRCULATION

Show **video** of starting vortex



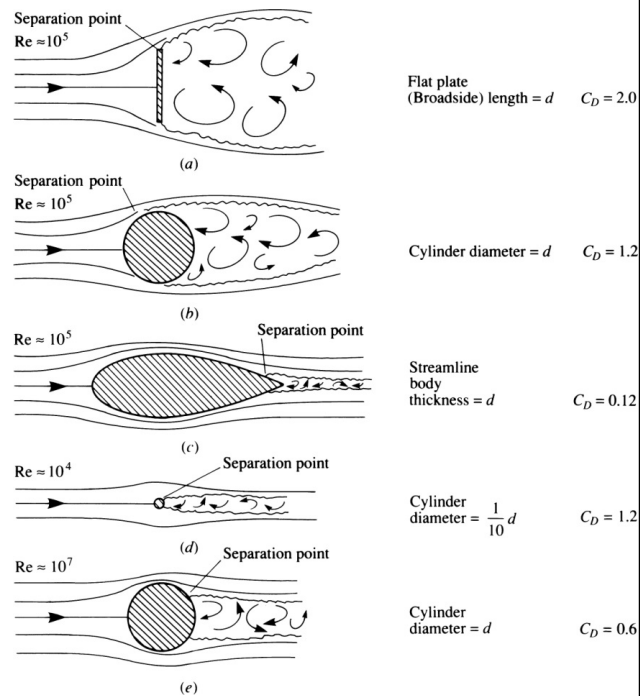
**Question:** What are the differences in behavior of air flow around a cylinder and around an airfoil with very low AOA?



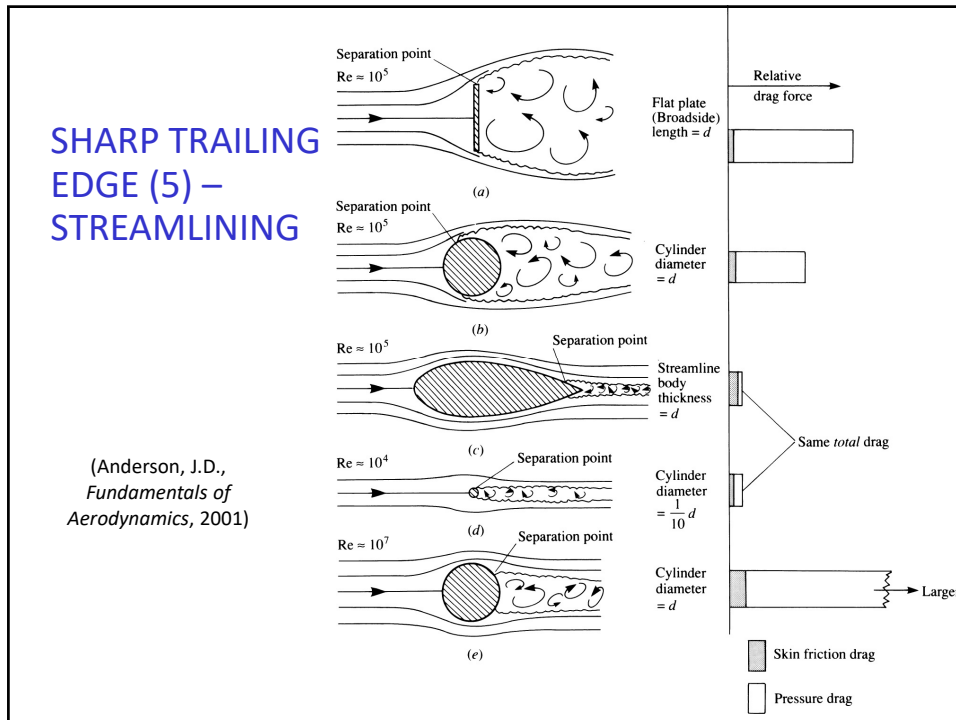
(Anderson, J.D., *Fundamentals of Aerodynamics*, 2001)

(b) Picture some moments after the start of the flow

### SHARP TRAILING EDGE (4) – STREAMLINING



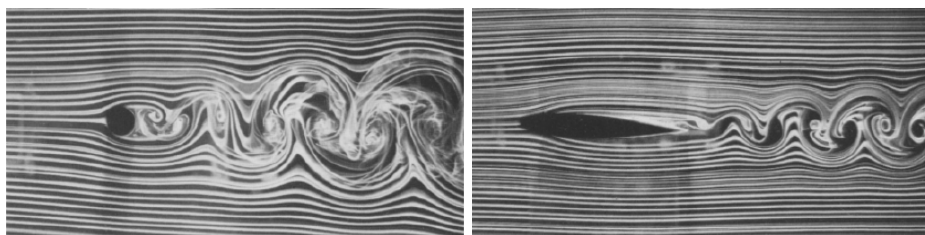
(Anderson, J.D., *Fundamentals of Aerodynamics*, 2001)



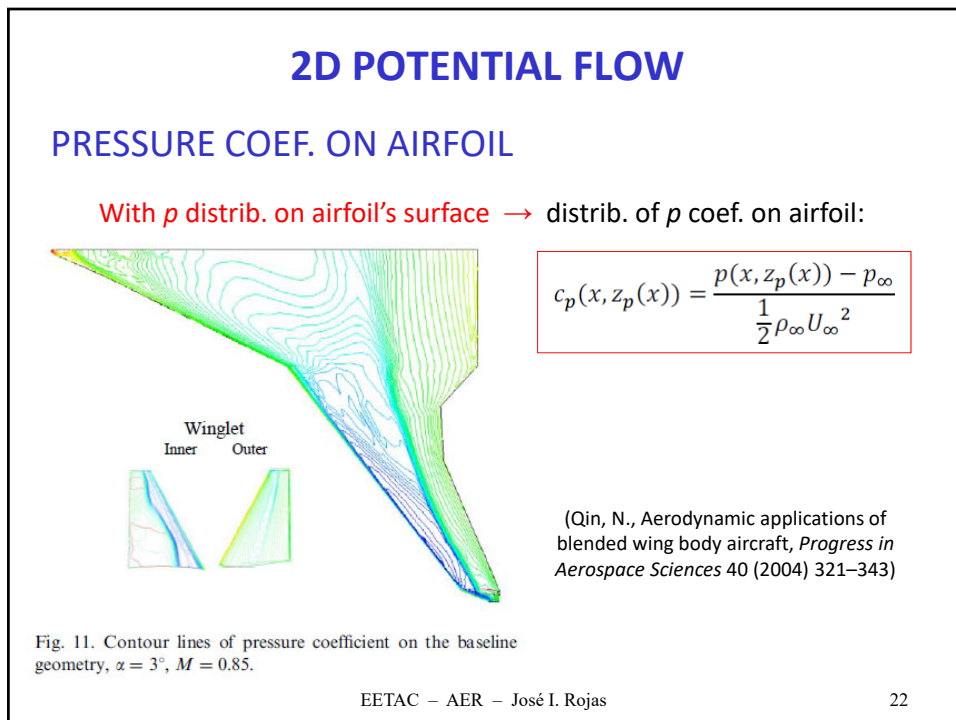
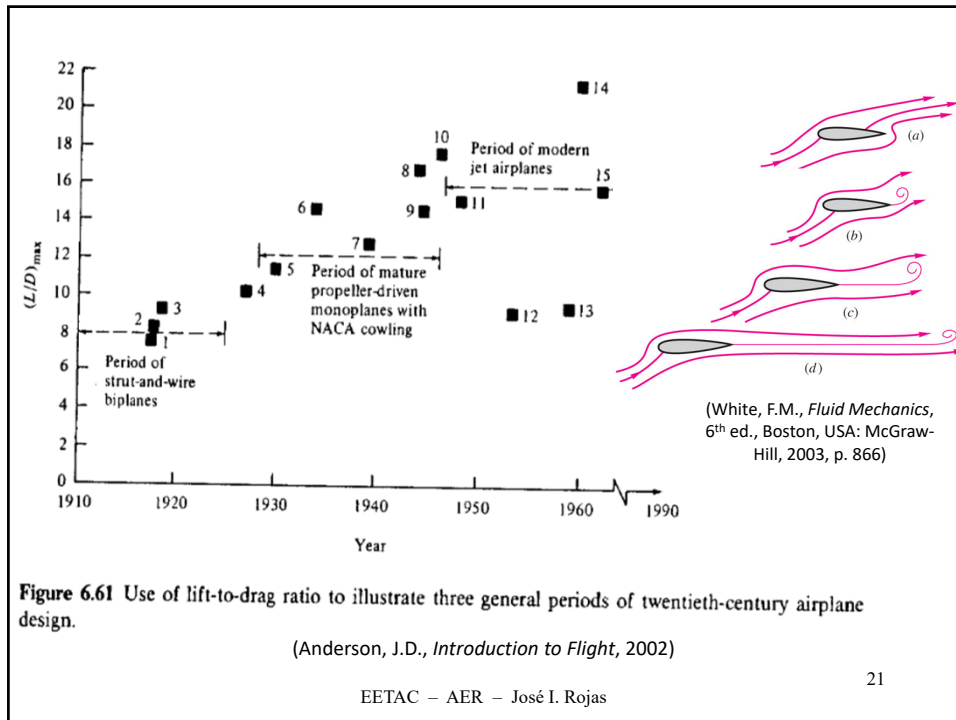
## 2D POTENTIAL FLOW

### SUMMARY

- Potential flow model allows computing  $p$  distribution if BL attached  
 → allows computing lift      limitation: drag is 0! not realistic!
- even if BL attached → drag not null (viscosity in BL causes friction)
- streamlining very efficient → lift  $\gg$  drag (e.g. lift/drag  $\approx$  30)



(Bienkiewicz, B., A flow visualization technique for low-speed wind-tunnel studies, 1987)



## 2D POTENTIAL FLOW

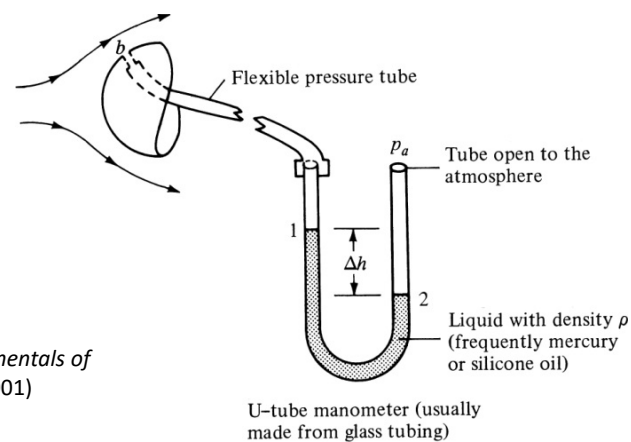
### PRESSURE COEF. ALONG CHORD (1)

- instead of  $p$  coef. on airfoil, work with  **$p$  coef. along chord**:  $c_p(x)$
- $c_p(x) \rightarrow$  projection along  $z$  axis of force per unit length on each point of airfoil
- it can be demonstrated that  $c_p(x) = c_p(x, z_p(x)) = \frac{p(x, z_p(x)) - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2}$
- **Graphically**, we usually represent  $-c_p(x)$
- When the flow velocity increases, so does  $-c_p(x)$

## 2D POTENTIAL FLOW

### PRESSURE COEF. ALONG CHORD (2)

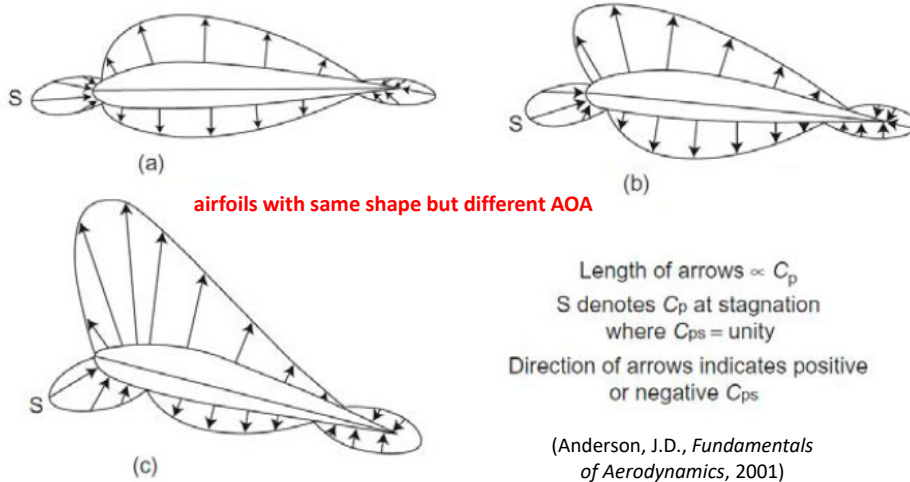
**Graphically**, we usually represent  $-c_p(x)$



(Anderson, J.D., *Fundamentals of Aerodynamics*, 2001)

## 2D POTENTIAL FLOW

### PRESSURE DISTRIBUTIONS AROUND AIRFOILS (1)



## 2D POTENTIAL FLOW

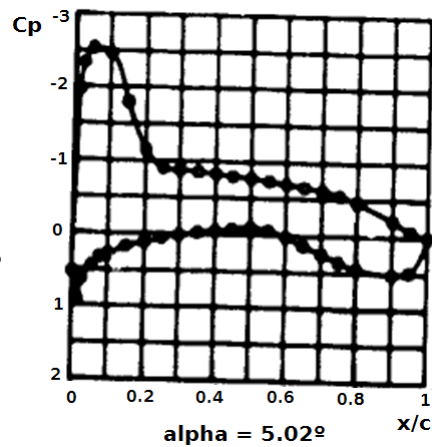
### PRESSURE COEF. ALONG CHORD (3)

Graphically, we usually represent  $-c_p(x)$

**TEST**      **187**  
**RUN**        **9**  
**MACH**      **.000**  
*Re*           **$10.0 \times 10^6$**

**Question:**  
 How much is the global airfoil's lift coefficient?

(Jenkins, R.V., *Aerodynamic performance & pressure distributions for NASA SC(2)-0714 airfoil*, 1988)



## 2D POTENTIAL FLOW

### LIFT COEF. ALONG CHORD & GLOBAL LIFT COEF. (1)

Distribution of lift coef. along chord:

$$c_l(x) = c_{p,lower}(x) - c_{p,upper}(x)$$

Global airfoil's lift coef.:

$$c_l = \frac{1}{c} \int_{x_{LE}}^{x_{TE}} c_l(x) dx$$

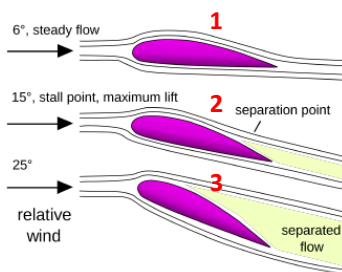
- for  $|\alpha| \ll 1 \rightarrow c = x_{TE} - x_{LE}$
- for  $|\alpha| \ll 1 \rightarrow$  lift coef. linear with  $\alpha$ :  $c_l \propto \alpha$  (slope is constant)
- theory fails if  $|\alpha|$  large:
  - as  $\alpha$  grows/decreases  $\rightarrow$  BL eventually detaches, slope starts to decrease & reaches 0  $\rightarrow$  max./min. lift coef.
  - if  $\alpha$  continues to grow/decrease  $\rightarrow$  slope becomes increasingly negative  
» we say that the airfoil is in **STALL**
- typically,  $c_{l,max} \approx 1.5$  for airfoils with **no hyper-lift devices**

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## 2D POTENTIAL FLOW

### GLOBAL LIFT COEF. vs. AOA



$c_l [-]$

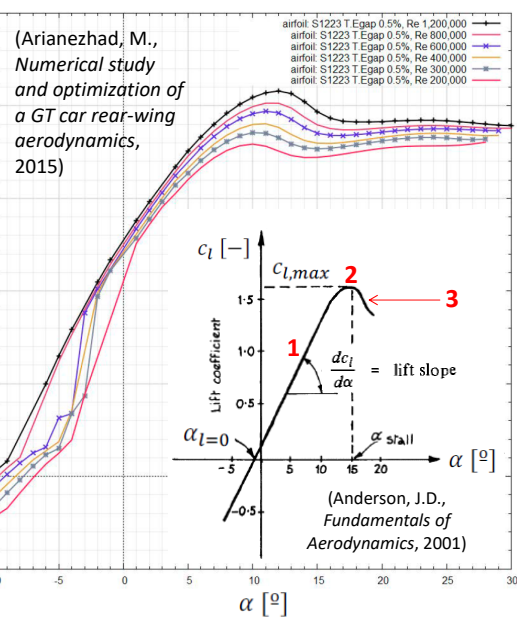


Figure 2.7: Performance comparison in different Reynolds numbers computed by XFOIL

## 2D POTENTIAL FLOW

### LIFT COEF. ALONG CHORD & GLOBAL LIFT COEF. (2)

Detachment of BL:

(Anderson, J.D., *Fundamentals of Aerodynamics*, 2001)

(White, F.M., *Fluid Mechanics*, 6<sup>th</sup> ed., Boston, USA: McGraw-Hill, 2003, p. 866)

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## 2D POTENTIAL FLOW

### LIFT COEF. ALONG CHORD & GLOBAL LIFT COEF. (3)

Detachment of BL:

(Rojas, J.I., 2006)

(Deng, J.-J., On exploring application of MEMS in aerodynamic flow control, *Proceedings of CANEUS Conference*, 2006, Toulouse, France)

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## 2D POTENTIAL FLOW

### LIFT COEF. ALONG CHORD & GLOBAL LIFT COEF. (4)

Global airfoil's *lift* coef.:

$$c_l = \frac{1}{c} \int_{x_{LE}}^{x_{TE}} c_l(x) dx$$

- for  $|\alpha| \ll 1 \rightarrow$  lift coef. linear with  $\alpha$ :  $c_l \propto \alpha$

– E.g.: Yukovsky transformation  $\longrightarrow c_l = 2\pi \frac{\delta + (1 + \gamma)\alpha}{1 + \frac{\gamma}{1 + 2\gamma}}$

– E.g.: thin airfoil  $\longrightarrow c_l = 2\pi(\delta + (1 + \gamma)\alpha)$

– E.g.: flat plate  $\longrightarrow c_l = 2\pi\alpha$

**Question:** Compute the global lift coefficient for a flat plate with angle of attack (AOA)  $10^\circ$ . Compute the global lift coefficient for a symmetric thin airfoil.

## 2D POTENTIAL FLOW

### FORCES ON AIRFOIL

In 2D, we compute forces per unit span [N/m]:

• lift  $\rightarrow$   $l = \frac{1}{2} \rho U_\infty^2 c c_l$

• drag  $\rightarrow$   $d = \frac{1}{2} \rho U_\infty^2 c c_d$

**Question:** Compute lift for previous plate at SL in ISA at 60 km/h, if chord is 30 cm.

Global airfoil's *aerodynamic drag* coef.:  $c_d$

- typically,  $c_{d,min} \approx 0.004$  for a laminar airfoil



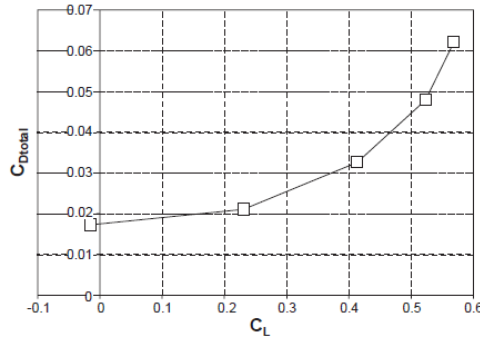
## 2D POTENTIAL FLOW

### AERDYNAMIC DRAG COEF. – POLAR CURVE

- **Polar curve:** relationship between *lift & drag* coef. →  $c_d = f(c_l)$
- **E.g.:** laminar airfoil → laminar BL in a small range of  $\alpha$ /lift coef.
- catalogues listing airfoil data:

e.g.: UIUC:

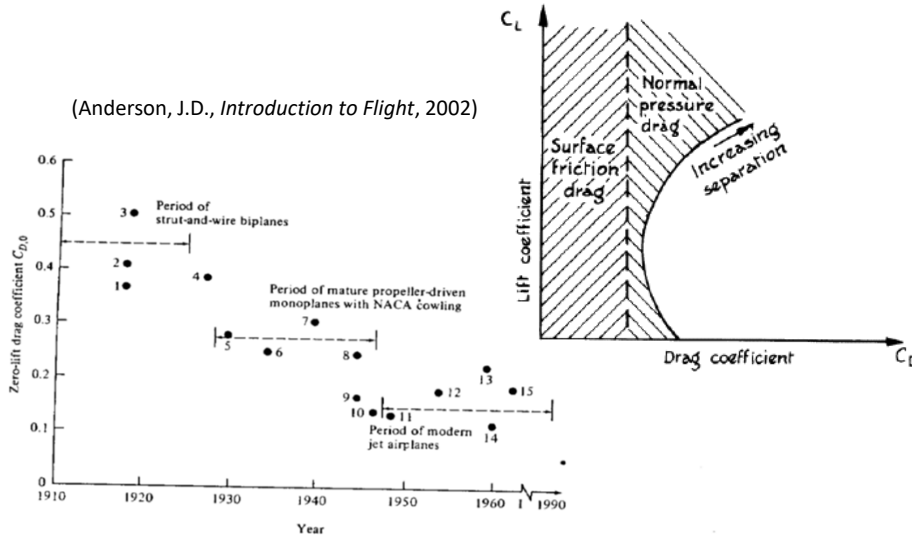
<http://www.ae.uiuc.edu/m-selig/ads.html>



(Qin, N., Aerodynamic applications of blended wing body aircraft, *Progress in Aerospace Sciences* 40 (2004) 321–343)

Fig. 8. Lift-drag polar for baseline geometry  $M = 0.85$ .

(Anderson, J.D., *Introduction to Flight*, 2002)



**Figure 6.60** Use of zero-lift drag coefficient to illustrate three general periods of twentieth-century airplane design. The numbered data points correspond to the following aircraft: (1) SPAD XIII, (2) Fokker D-VII, (3) Curtiss JN-4H Jenny, (4) Ryan NYP (*Spirit of St. Louis*), (5) Lockheed Vega, (6) Douglas DC-3, (7) Boeing B-17, (8) Boeing B-29, (9) North American P-51, (10) Lockheed P-80, (11) North American F-86, (12) Lockheed F-104, (13) McDonnell F-4E, (14) Boeing B-52, (15) General Dynamics F-111D.

## 2D POTENTIAL FLOW

### PITCHING MOMENT & PITCHING MOMENT COEF. (1)

- aerodynamic loading **NOT fully defined** by simply stating *lift & drag*
- it is necessary to provide also:
  - point of application of *lift & drag*; OR
  - moment produced by *lift & drag* in a reference point
- typical reference points:
  - pressure center (*cp*): point of application of *lift & drag* (moment is NULL)
  - aerodynamic center (*ca*): pitching moment coef. independent of  $\alpha$ , for  $|\alpha| \ll 1$ 
    - subsonic regime: *ca* located approx. in 25% of chord
    - supersonic regime: *ca* located approx. in 50% of chord

## 2D POTENTIAL FLOW

### PITCHING MOMENT & PITCHING MOMENT COEF. (2)

Pitching moment coef. respect to a generic point:

$$c_{m,point} = \frac{-1}{c^2} \int_{x_{LE}}^{x_{TE}} c_l(x)(x - x_{point})dx$$

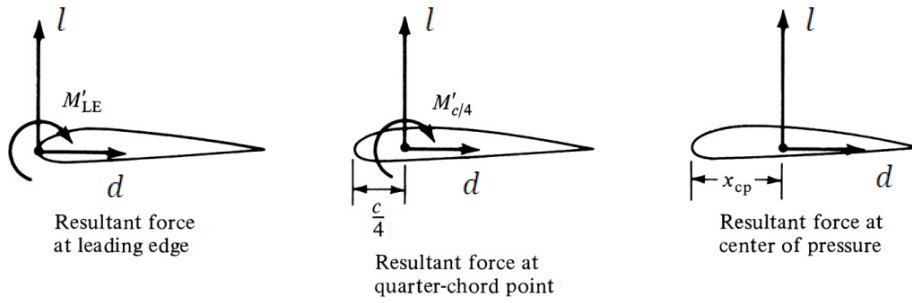
$$c_{m,2} = c_{m,1} - c_l \left( \frac{x_1}{c} - \frac{x_2}{c} \right)$$

Pitching moment respect to a generic point:

$$m_{point} = \frac{1}{2} \rho U_\infty^2 c^2 c_{m,point}$$

## 2D POTENTIAL FLOW

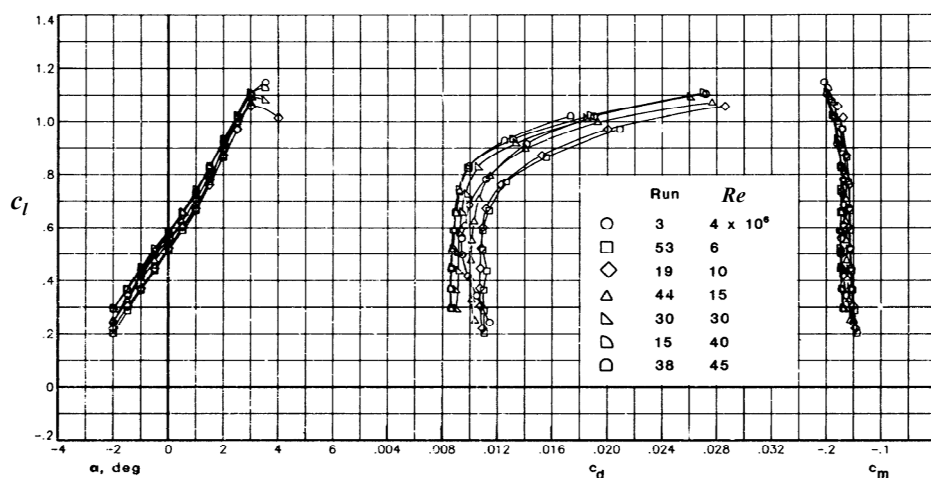
### PITCHING MOMENT & PITCHING MOMENT COEF. (3)



(Anderson, J.D., *Fundamentals of Aerodynamics*, 2001)

## 2D POTENTIAL FLOW

### PITCHING MOMENT & PITCHING MOMENT COEF. (4)

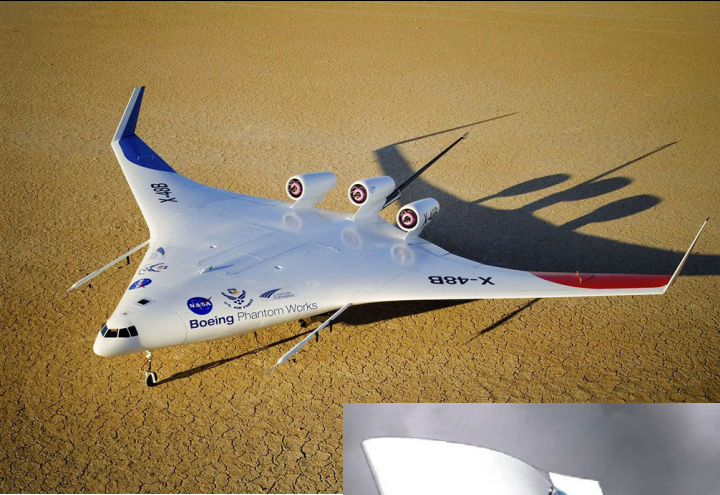

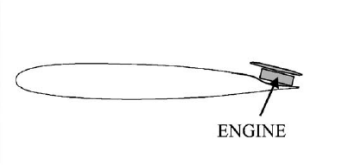
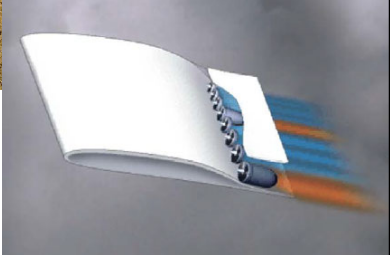


(Jenkins, R.V., *Aerodynamic performance & pressure distributions for NASA SC(2)-0714 airfoil*, 1988)

(e)  $M = 0.72$

ALL WING A

(Sehra, A.K., Propulsion and power for 21st century aviation, *Progress in Aerospace Sciences* 40 (2004) 199–235)

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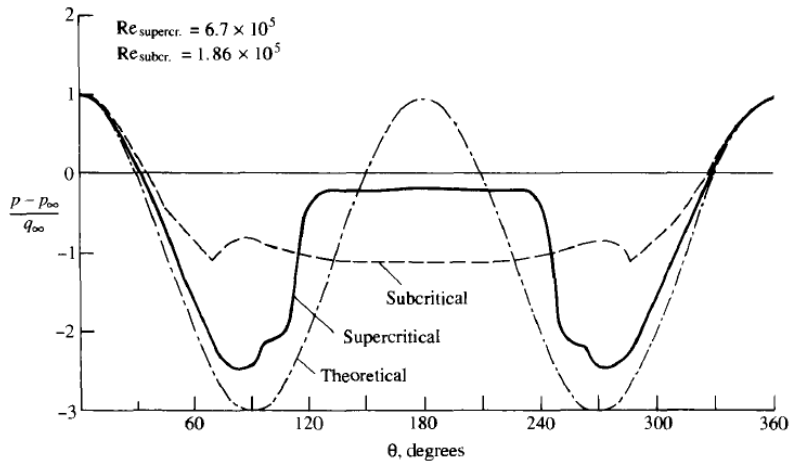
## LESSON 2: INVISCID FLOW

### *Part 1: 2D Potential Flow (Infinite Wings)*

**THANKS FOR YOUR ATTENTION  
ANY QUESTION?**

(Anderson, J.D., *Fundamentals of Aerodynamics*, 2001)

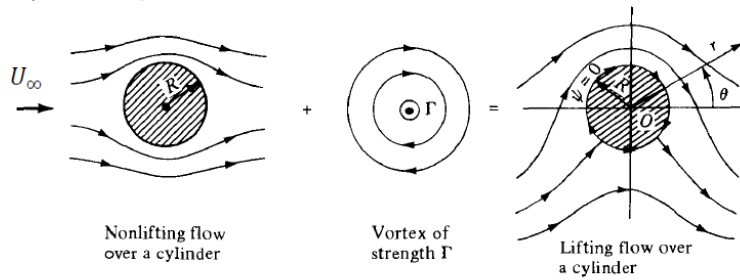
### CYLINDER



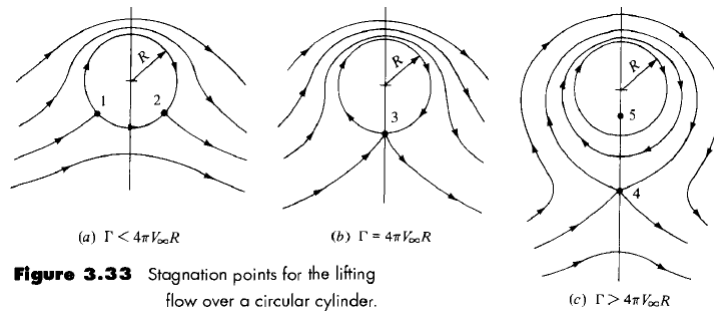
**Figure 3.49** Pressure distribution over a circular cylinder in low-speed flow. Comparison of the theoretical pressure distribution with two experimental pressure distributions—one for a subcritical Re and the other for a supercritical Re.

(Anderson, J.D., *Fundamentals of Aerodynamics*, 2001)

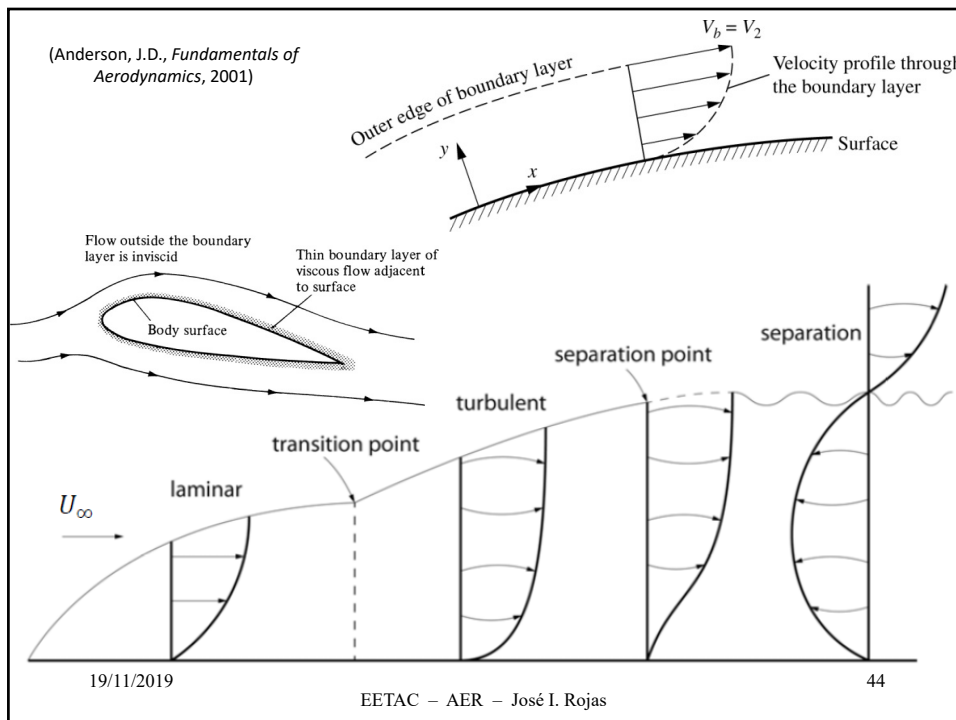
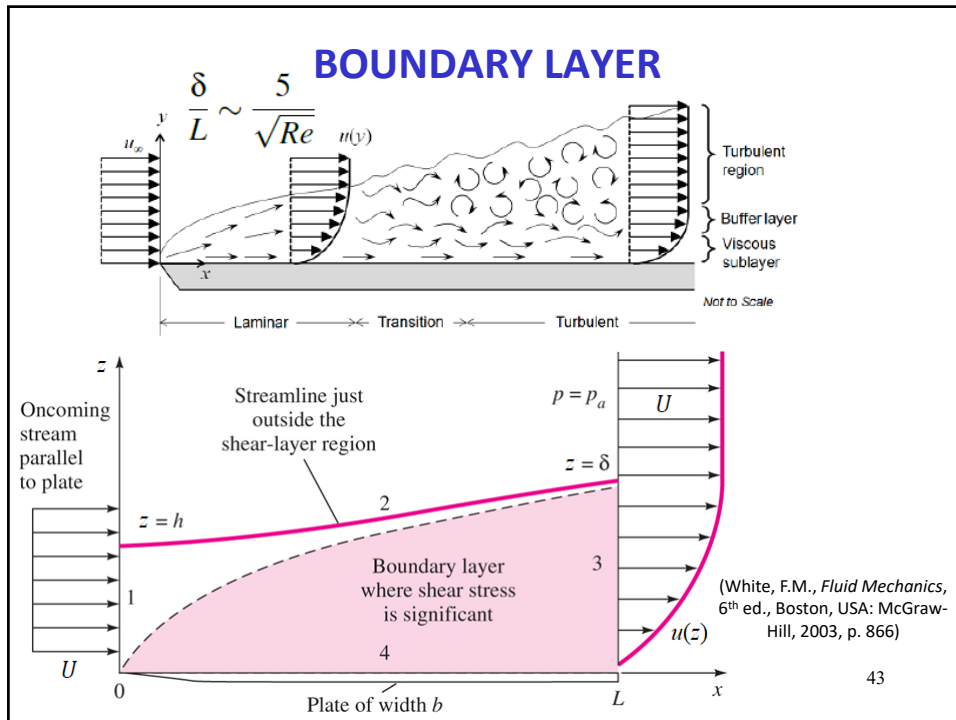
### MAGNUS EFFECT



**Figure 3.32** The synthesis of lifting flow over a circular cylinder.

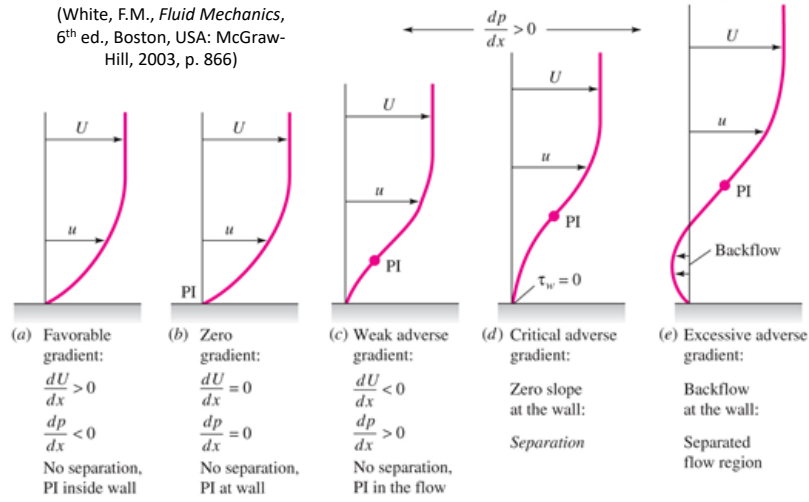


**Figure 3.33** Stagnation points for the lifting flow over a circular cylinder.



## FLOW VELOCITY IN BOUNDARY LAYER

(White, F.M., *Fluid Mechanics*, 6th ed., Boston, USA: McGraw-Hill, 2003, p. 866)



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## BOUNDARY LAYER

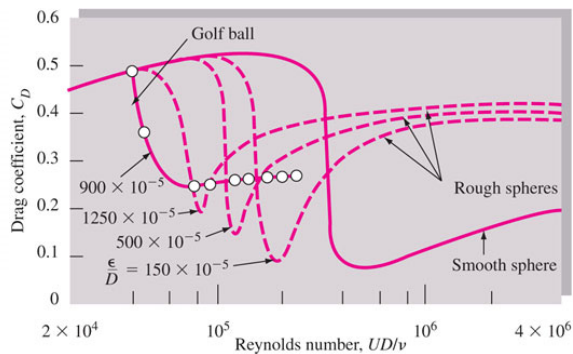
### LAMINAR & TURBULENT BOUNDARY LAYER (1)

#### Surface of golf balls:

- characteristic length (diameter) & velocity not high → **small  $Re$**
- **plain surface** → laminar BL → detaches earlier → **thicker wake** →  $d \uparrow$
- **carved surface** → forces transition to turbulent BL → detaches later → **thinner wake** →  $d \downarrow$



(White, F.M., *Fluid Mechanics*, 6th ed., Boston, USA: McGraw-Hill, 2003, p. 866)

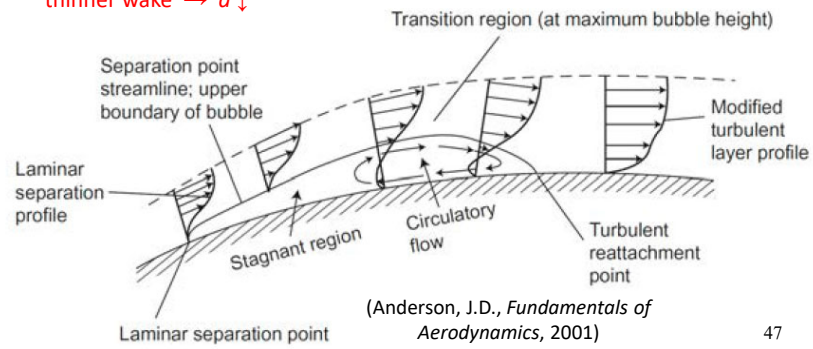


## BOUNDARY LAYER

### LAMINAR & TURBULENT BOUNDARY LAYER (2)

#### Surface of golf balls:

- characteristic length (diameter) & velocity not high → **small  $Re$**
- **plain surface** → laminar BL → detaches earlier → **thicker wake** →  $d \uparrow$
- **carved surface** → forces transition to turbulent BL → detaches later → **thinner wake** →  $d \downarrow$



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