Escola d'Enginyeria de Telecomunicació i Aeroespacial de Castelldefels
UNIVERSITAT POLITÈCNICA DE CATALUNYA

Aerodynamics & Flight Mechanics (AMV)

LESSON 5: STABILITY AND CONTROL

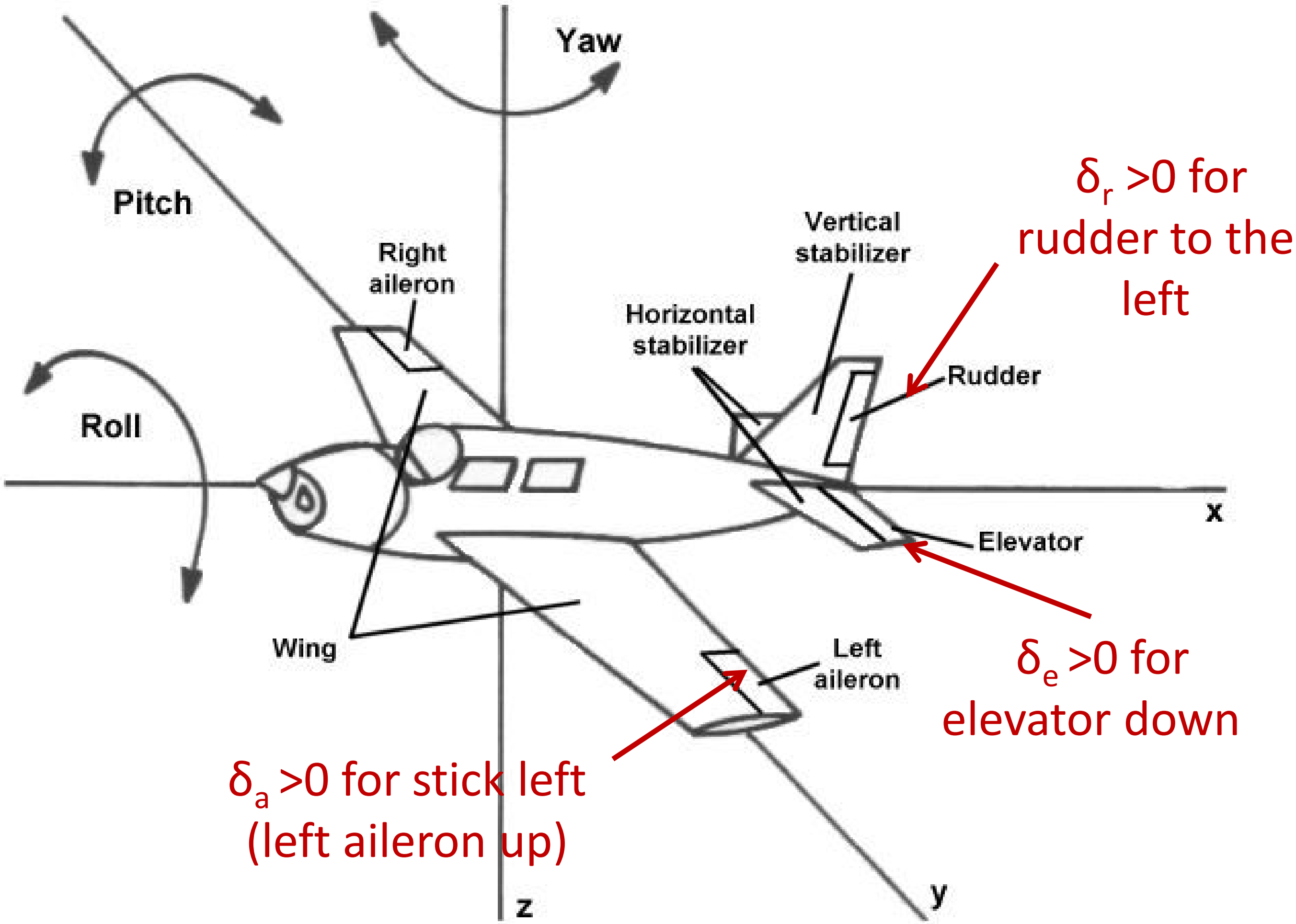
Adeline de Villardi de Montlaur

Santiago Arias

CONTENTS

LESSON 5: STABILITY AND CONTROL

INTRODUCTION



INTRODUCTION

Static stability:

If the forces and moments on the body caused by a disturbance tend initially to return the body toward its equilibrium position, the body is statically stable. The body has positive static stability, Figure 1

If the forces and moments are such that the body continues to move away from its equilibrium position after being disturbed, the body is statically unstable. The body has negative static stability, Figure 2

Figure 1

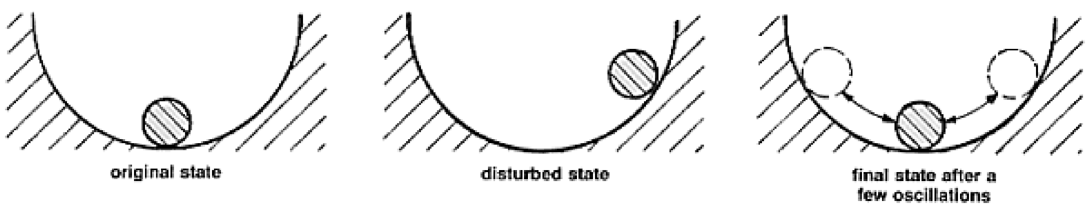
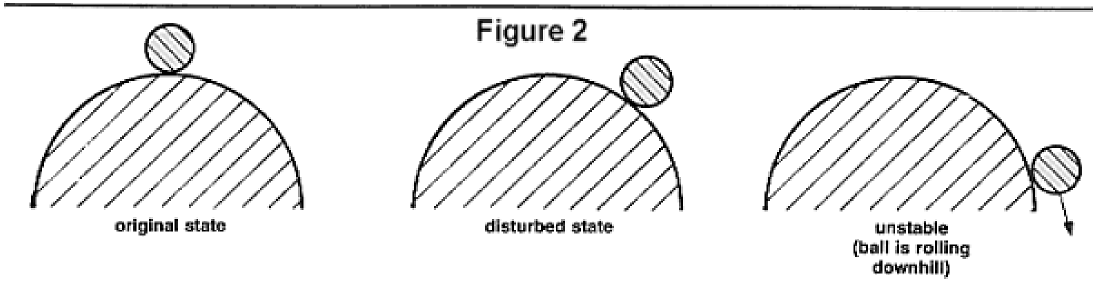


Figure 2



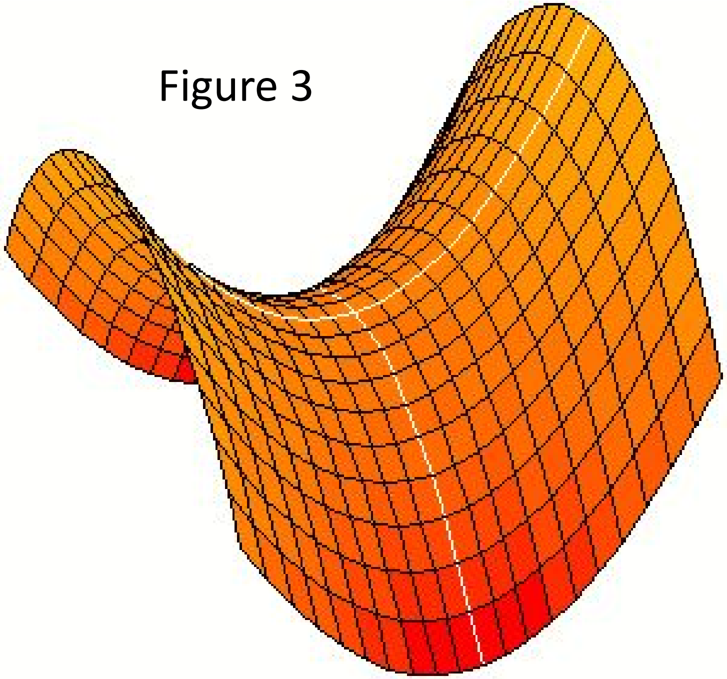
INTRODUCTION

Static stability:

Note that a 3D body can be stable with respect to one of its axis and unstable with respect to another, see for example a saddle point, Figure 3

→ this could also be the case for an airplane

Figure 3

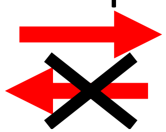


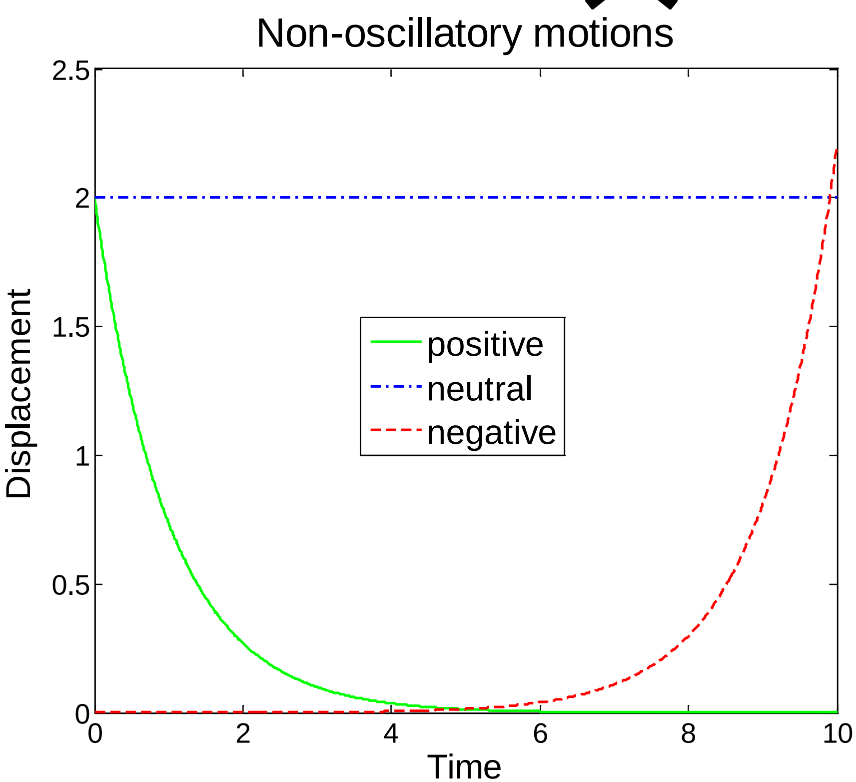
INTRODUCTION

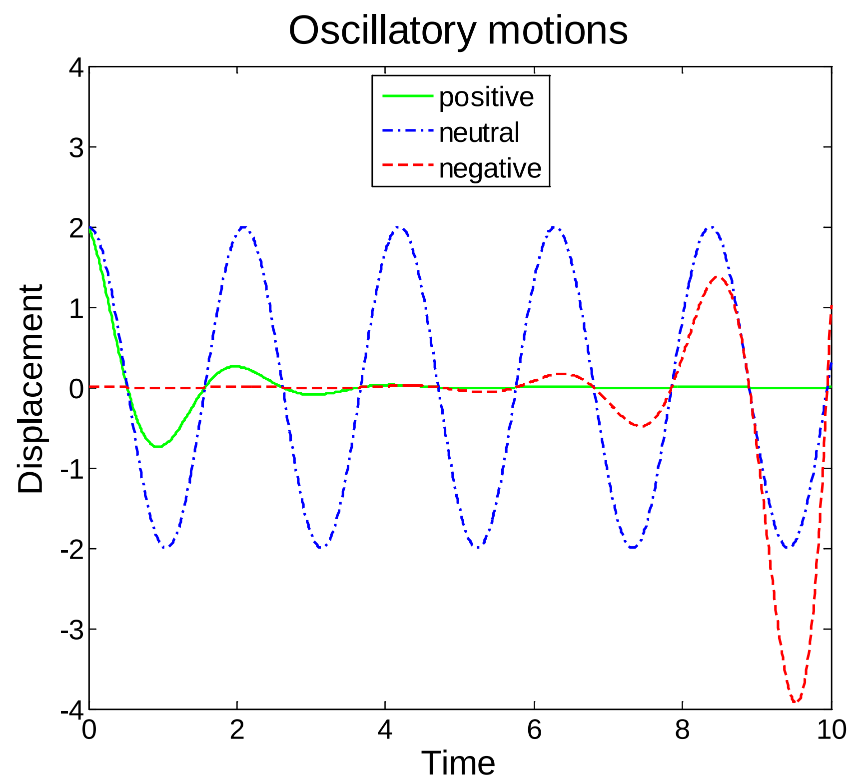
Dynamic stability:

Deals with time history of vehicle's motion after initial response to its static stability

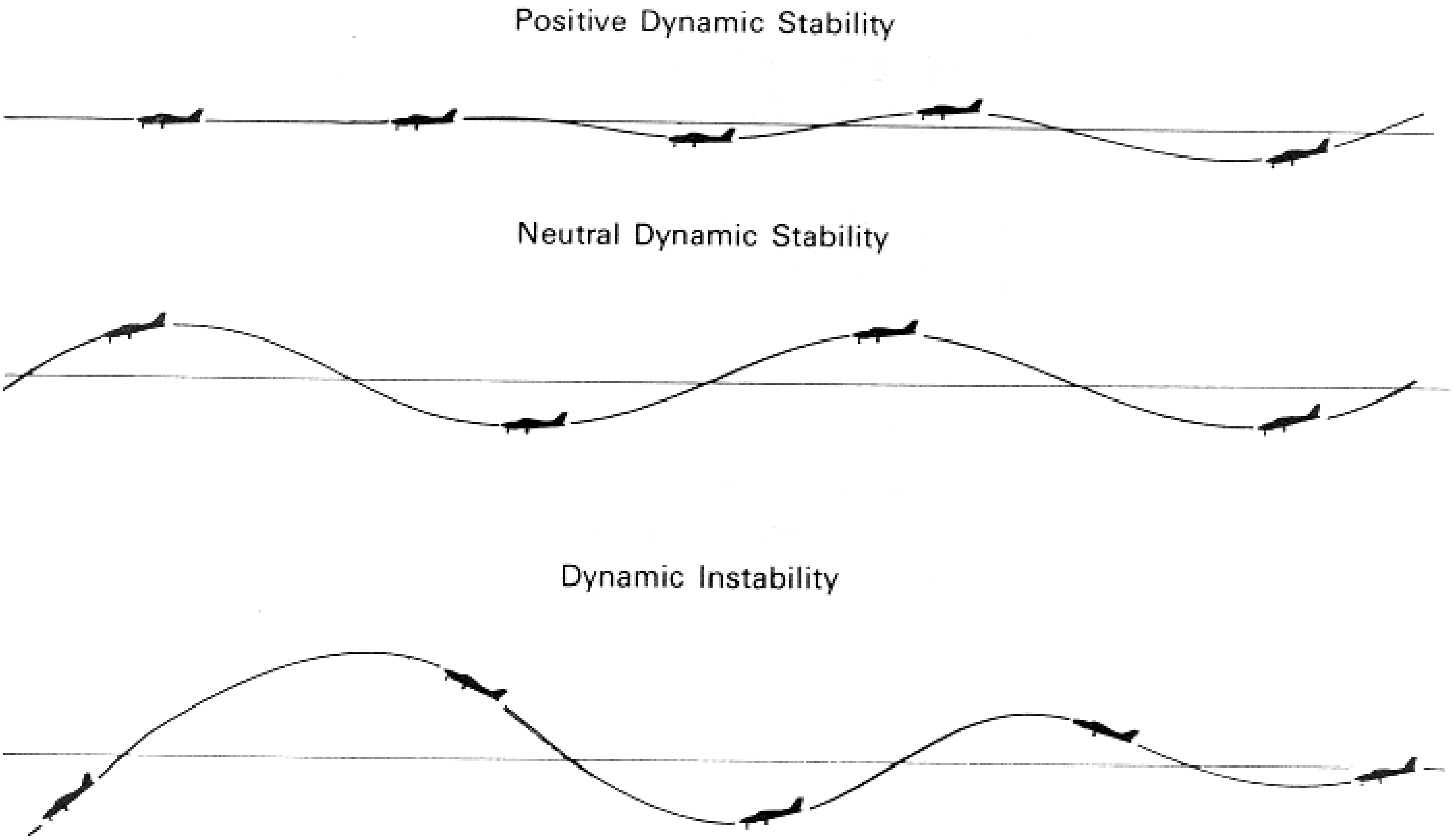
A body is dynamically stable if it eventually returns to, and remains at, its equilibrium position over a period of time

Dynamically stable  statically stable





INTRODUCTION



INTRODUCTION

Control:

Conventional control surfaces (elevators, ailerons, and rudder) used to

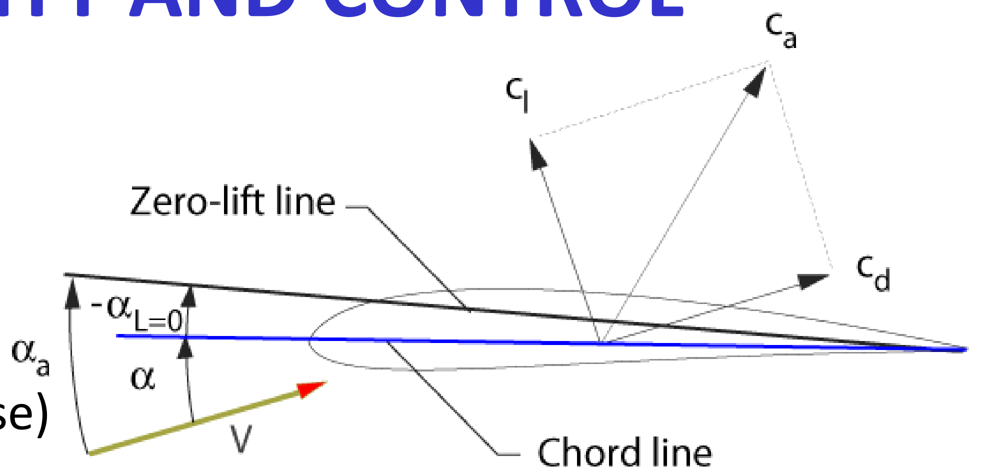
- change the airplane from one equilibrium position to another,
- produce non-equilibrium accelerated motions such as maneuvers

Airplane control: study of

- deflections of the ailerons, elevators and rudder necessary to make the airplane do what we want
- amount of force to be exerted by the pilot to deflect these control surfaces

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability



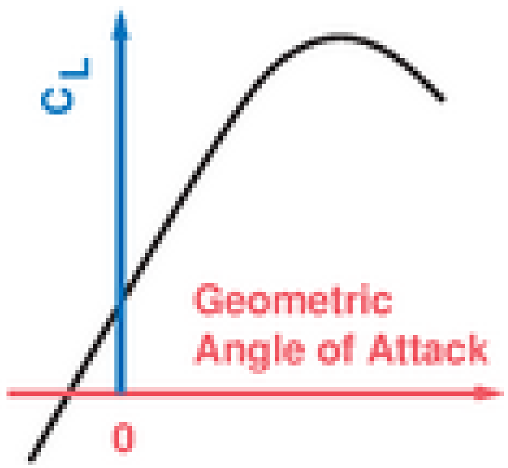
Absolute angle of attack α_a:

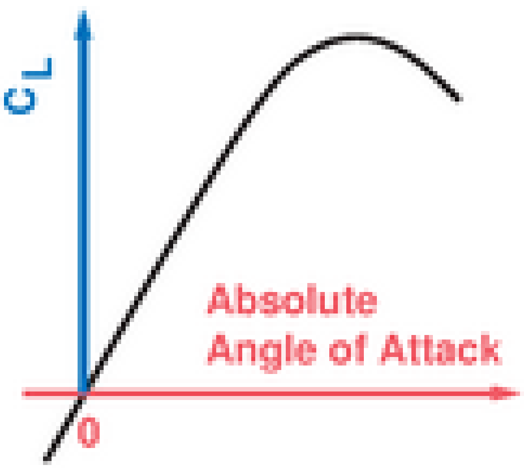
= geometric angle of attack

+ zero-lift angle of attack (absolute sense)

General cambered airfoils: zero-lift angle of attack slightly <0

Use of α_a instead of α is common in studies of stability and control (advantage: when α_a=0 then L=0 no matter the camber of airfoil)





$$\alpha_a = \alpha - \alpha_{L=0}$$

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Moments on the airplane:

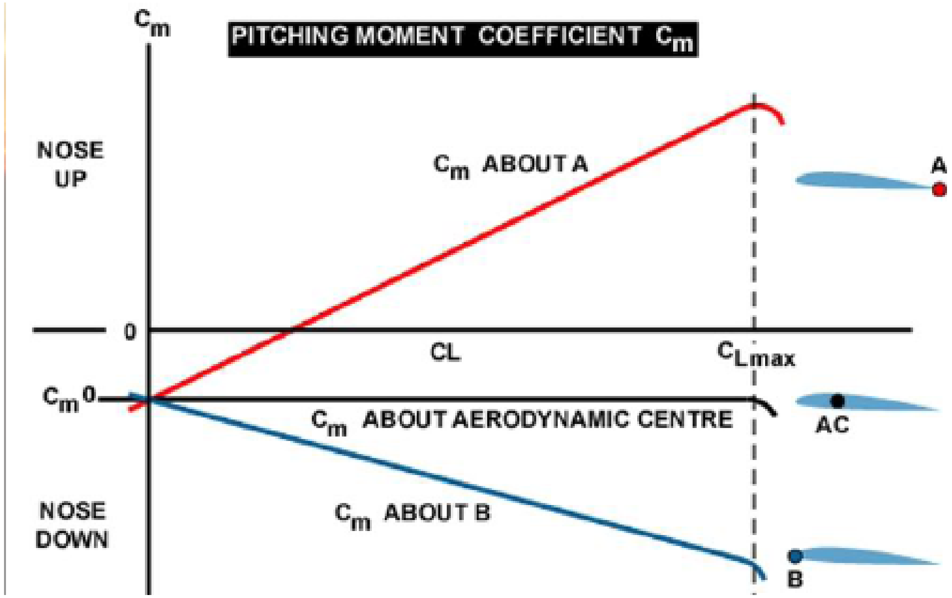
Pressure & shear stress distribution produce pitching moment

Aerodynamic center: point about which the moments are independent of the angle of attack

$$C_{M,ac} \equiv \frac{M_{ac}}{q_\infty S c}$$

$C_{M,ac}$ (constant with α) obtained from value of moment coefficient about any point when wing is at zero-lift angle of attack $\alpha_{L=0}$

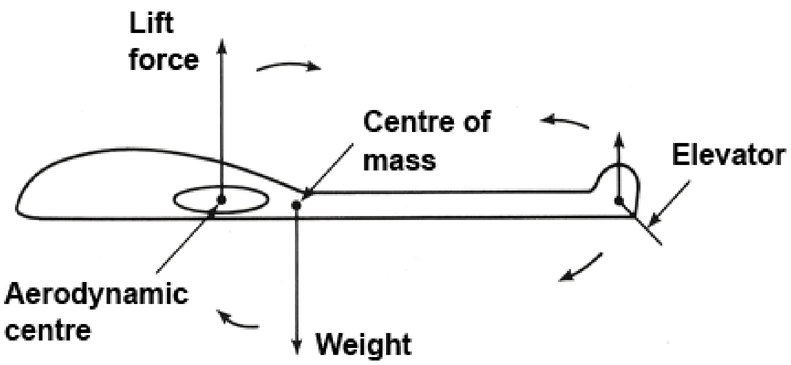
M_{ac}: sometimes called zero-lift moment



1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Moments on the airplane:



M_{cg}: pitching moment about c.g. of airplane

created by: - L, D and M_{ac} of wing

- lift of tail
- thrust
- aerodynamics forces and moments on other parts of airplane, such as fuselage + engine nacelles

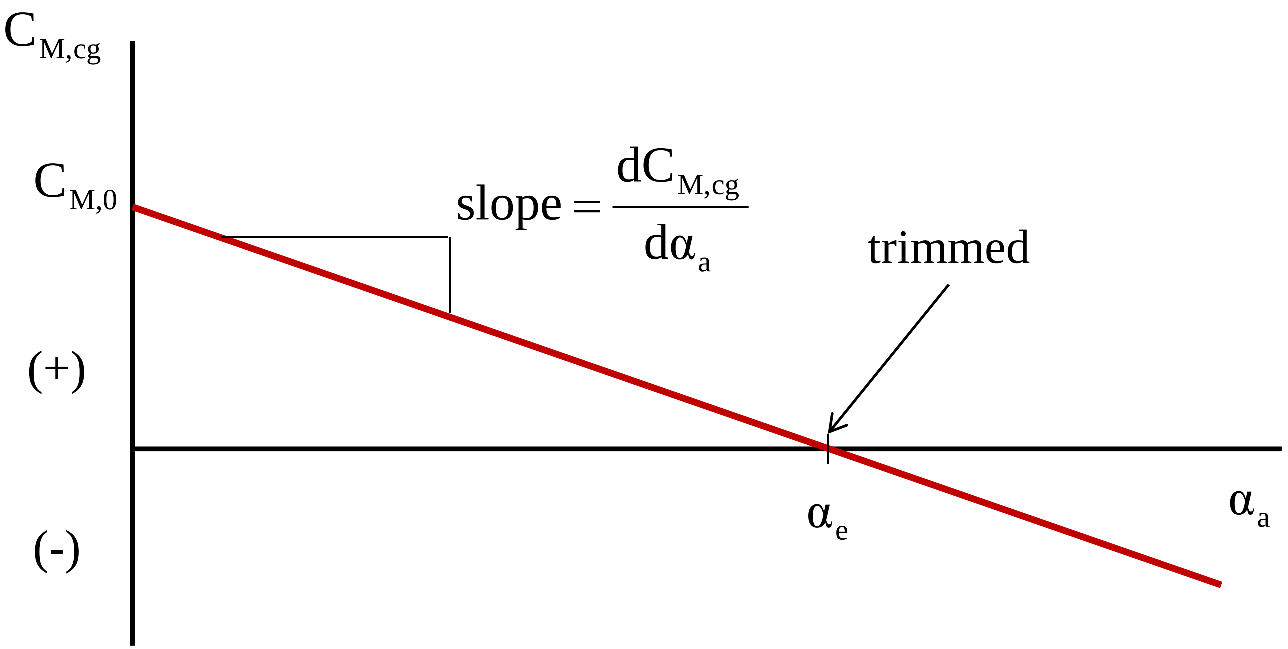
- $$C_{M,cg} \equiv \frac{M_{cg}}{q_\infty S c}$$
- airplane is in equilibrium (in pitch) when moment about CG is zero

(when M_{cg}= $C_{M,cg}$=0) → airplane is said to be *trimmed*

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Criteria for longitudinal static stability:



1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Criteria for longitudinal balance:

α_e: equilibrium or trim angle of attack (value of α_a where M_{cg}=0)

airplanes moves through a range of angle of attack as it flies through its velocity range from V_{stall} (largest α_a) to V_{max} (smallest α_a)

→ value of α_e must fall within this flight range of angle of attack

Necessary criteria for static stability and longitudinal balance :

- $C_{M,0}$ must be >0
- $\frac{dC_{M,cg}}{d\alpha_a}$ must be <0
- α_e must fall within flight range of angle of attack

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

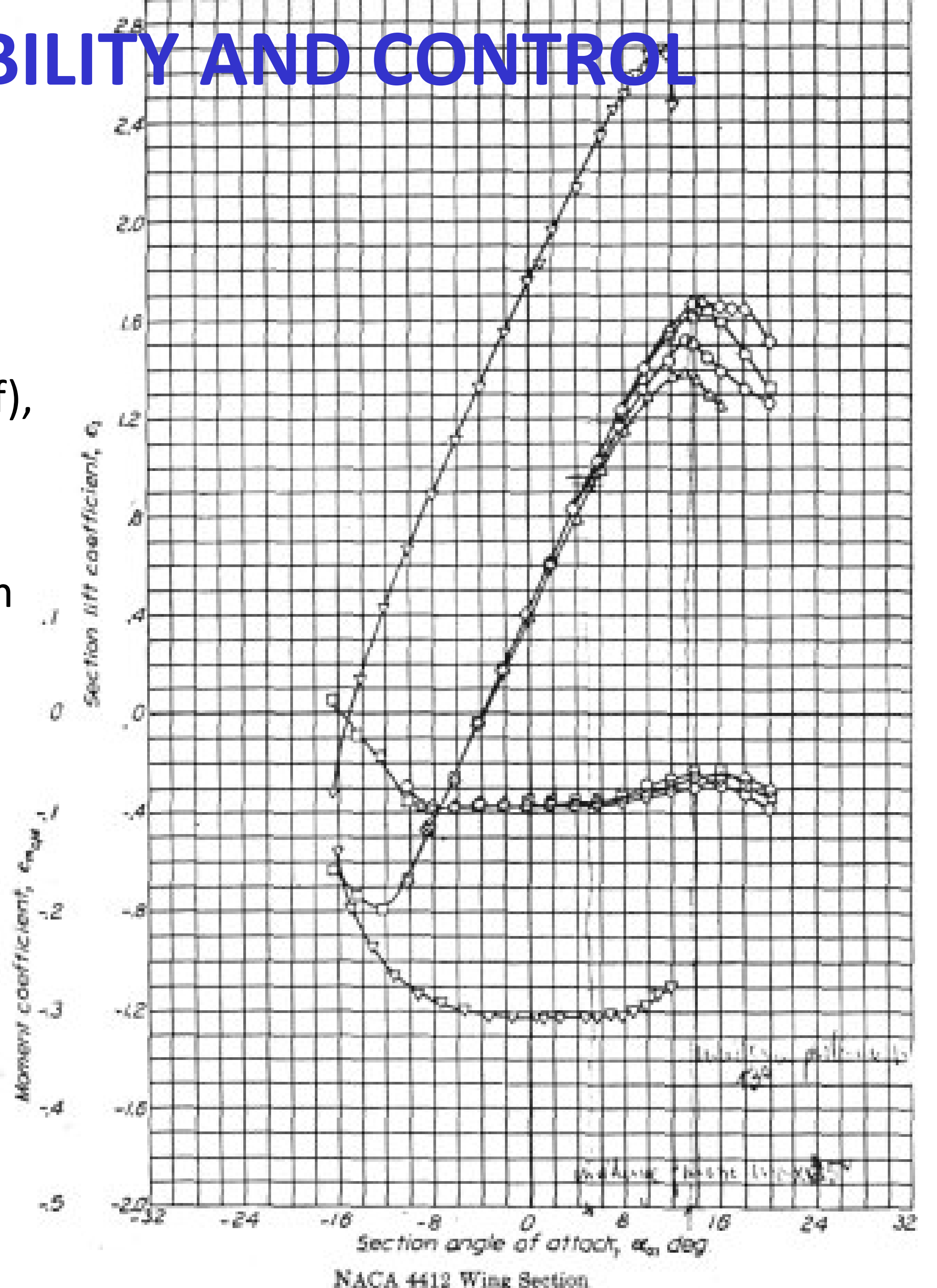
Wing-tail combination:

Consider an ordinary wing (by itself), with a conventional airfoil.

For a positive camber, $C_{M,ac}<0$ (from NACA data) and for zero lift $C_{M,ac}=C_{M,cg}=C_{M,0}$

Hence $C_{M,0}<0$ and such a wing by itself is unbalanced

→ horizontal tail must be added to the airplane



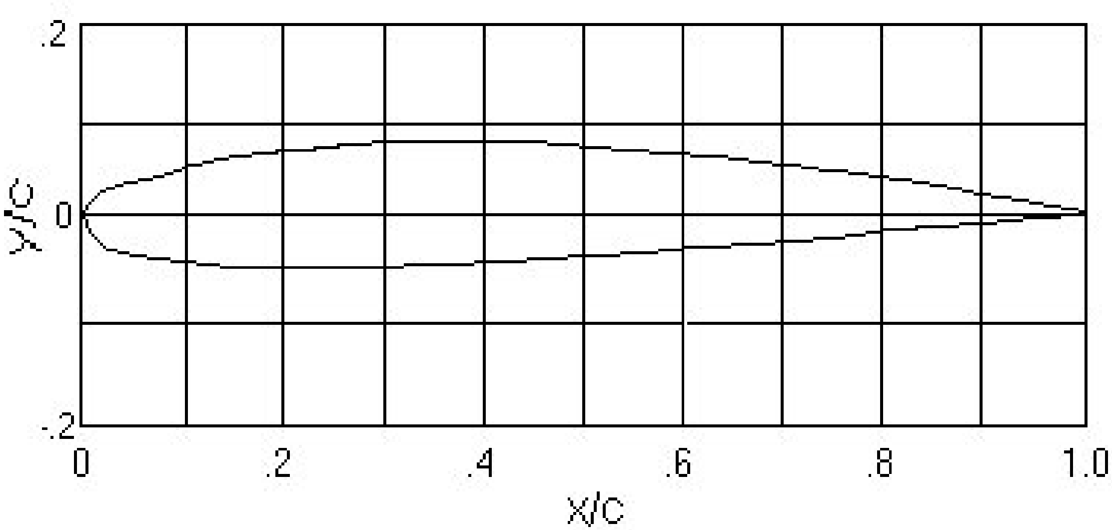
1. STATIC STABILITY AND CONTROL

NACA airfoils: a large bulk of experimental airfoil data was compiled over the years by the National Advisory Committee for Aeronautics (NACA: later absorbed in the creation of National Aeronautics and Space Administration-NASA)

lift, drag and moment coefficient were systematically measured for many airfoil shapes in low-speed subsonic wind tunnels

NACA four-digit wing sections define the profile by

- One digit describing maximum camber as percentage of the chord.
- One digit describing the distance of maximum camber from the airfoil leading edge in tens of percents of the chord.
- Two digits describing maximum thickness of airfoil as % of chord.

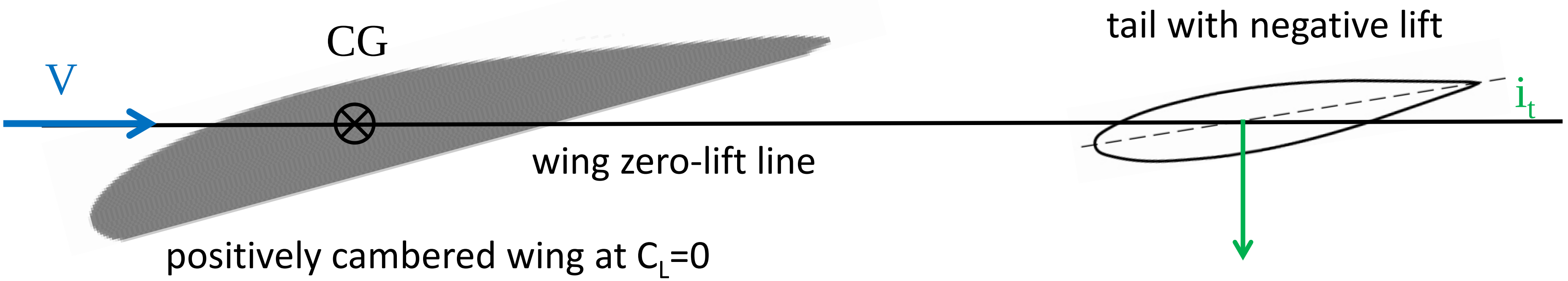


NACA 2412 airfoil has a maximum camber of 2% located 40% (0.4 chords) from the leading edge with a maximum thickness of 12% of the chord

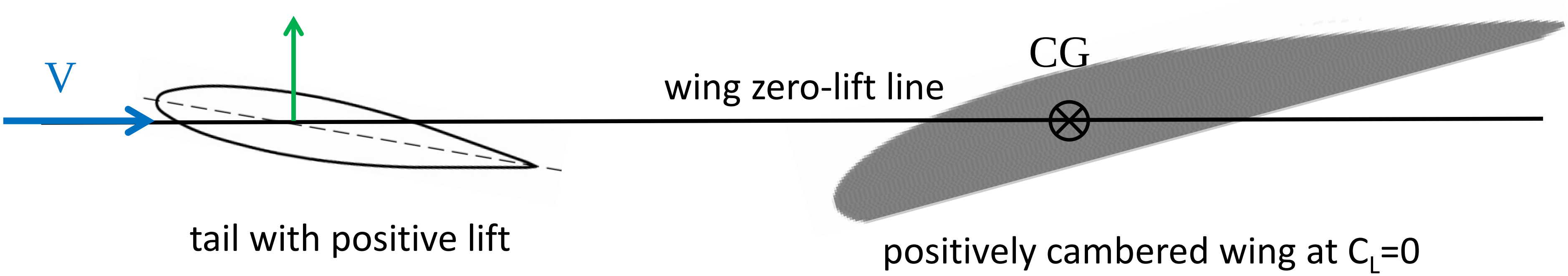
1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Conventional wing-tail combination



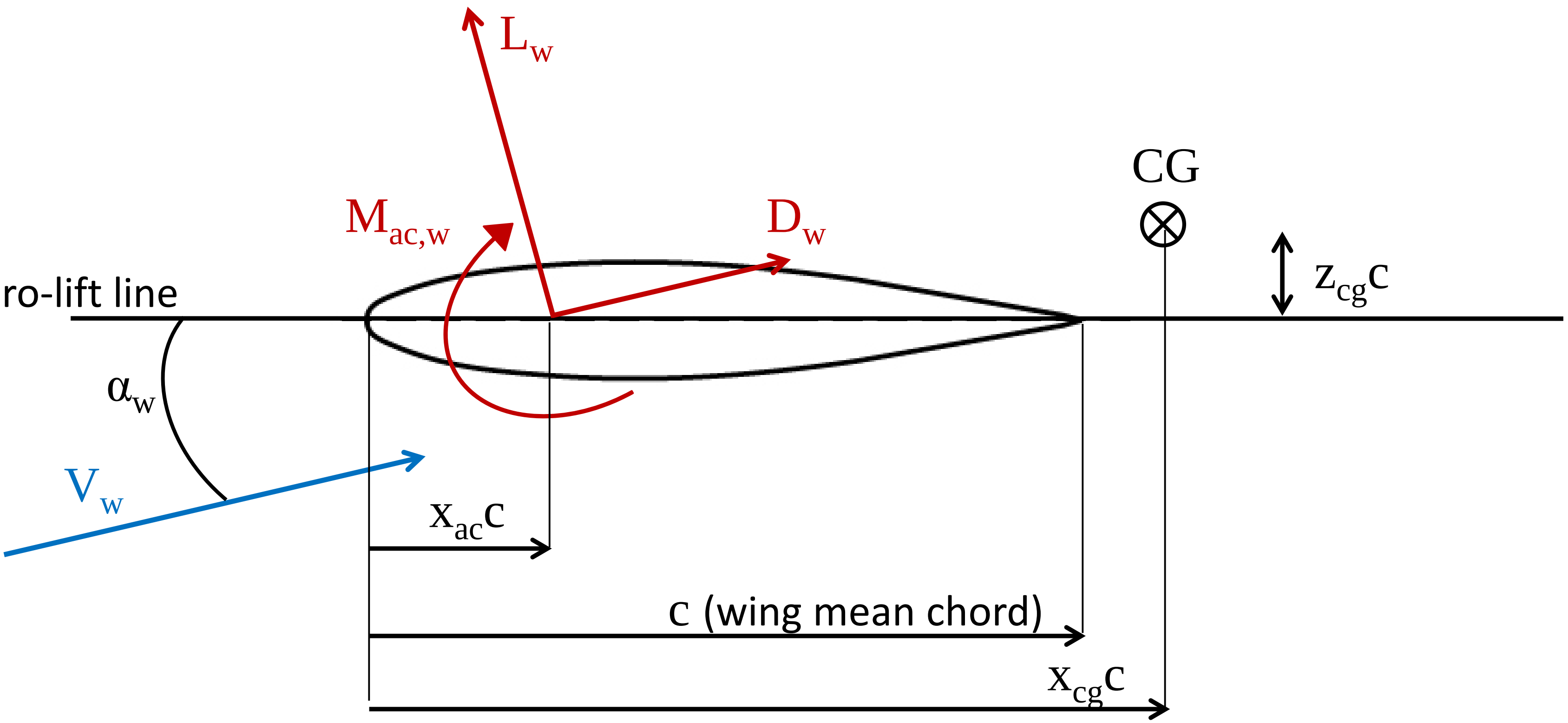
Canard wing-tail combination



1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Contribution of the wing to M_{cg}:



after simplification: $$C_{M,cg_w} = C_{M,ac_w} + C_{L_w}\left(x_{cg} - x_{ac_w}\right)$$

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Contribution of the wing+body to M_{cg}:

Consider individually contribution of wing + fuselage + tail moments about center of gravity of airplane → obtain total M_{cg}

1. Obtain results for wing only

$$C_{M,cg_{wb}} = C_{M,ac_{wb}} + C_{L_{wb}}\left(x_{cg} - x_{ac_{wb}}\right)$$

2. Results are slightly modified if fuselage is added to wing

Interference effects: when flow over wing affects fuselage flow and vice versa are extremely difficult to predict

→ lift, drag and moments of a wing-body combination usually obtained from wind tunnel measurements

Generally adding a fuselage to a wing shifts AC forward, increases lift curve slope and contributes a negative increment to the moment about AC

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Contribution of the tail to M_{cg}:

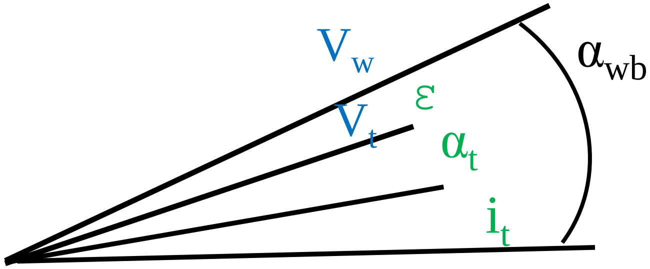
2 interference effects influence tail aerodynamics

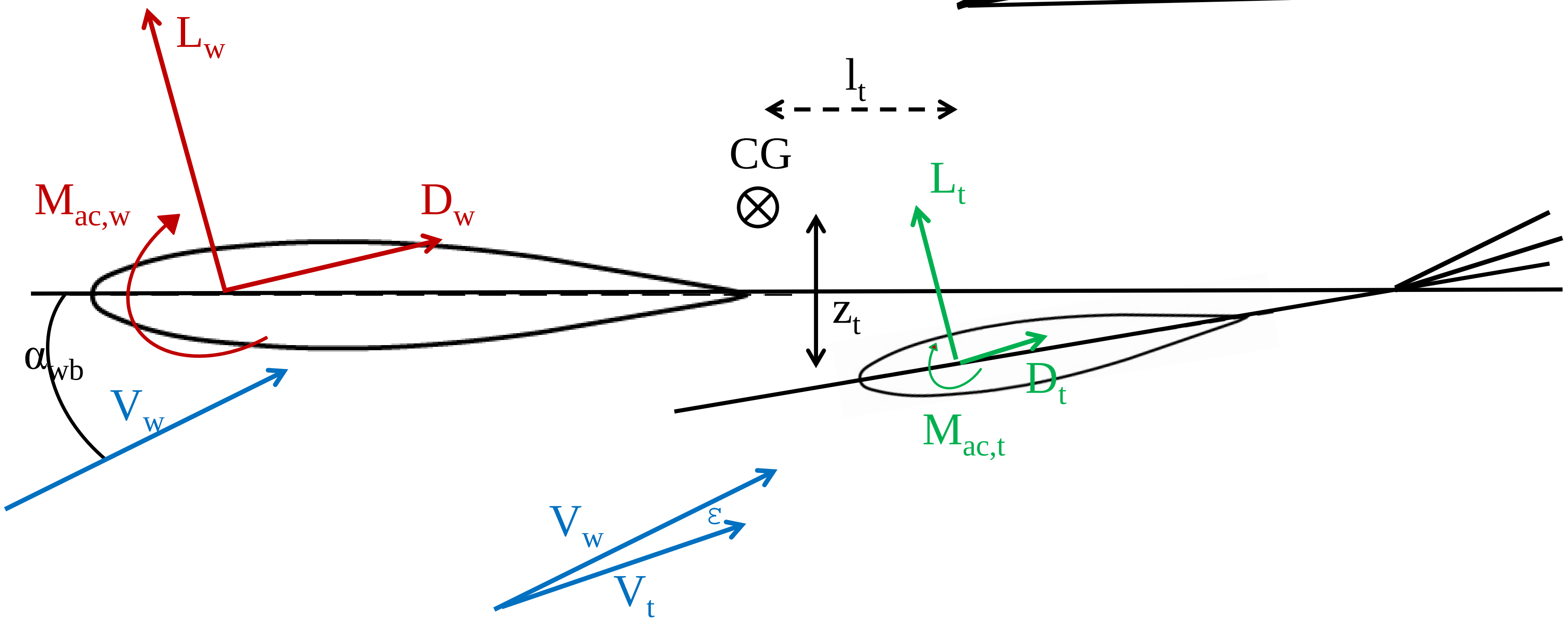
- Airflow at tail is deflected downward by downwash due to finite wing
- Because of the retarding force of skin friction and pressure drag over the wing, airflow reaching the tail has been slowed

For practical purposes it is sufficient to add tail lift directly to wing-body lift to obtain lift of complete airplane

a. Longitudinal stability

Contribution of the tail to M_{cg}:





after simplification: $$M_{cg,t} = -l_t L_t$$

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Contribution of the tail to M_{cg}: $$M_{cg,t} = -l_t L_t$$

Using tail lift coefficient: $C_{L,t} = \frac{L_t}{q_t S_t} \rightarrow M_{CG,t} = -l_t q_t S_t C_{L,t}$

$$\rightarrow \quad C_{M_{CG,t}} = \frac{M_{CG,t}}{q_w S c} = -\frac{l_t S_t}{c S}\frac{q_t}{q_w} C_{L,t}$$

$l_t S_t$: volume characteristic of the size and location of the tail

cS: volume characteristic of the wing

→ tail volume ratio $$V_H \equiv \frac{l_t S_t}{cS}$$ $\rightarrow C_{M_{CG,t}} = -V_H \frac{q_t}{q_w} C_{L,t}$

Note that in general $\frac{q_t}{q_w}$ is very close to 1 and is considered equal to 1 in the rest of the explanation

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Contribution of the tail to M_{cg}:

$$\alpha_t = \alpha_{wb} - i_t - \varepsilon \quad \text{and} \quad C_{L,t} = a_t \alpha_t = a_t\left(\alpha_{wb} - i_t - \varepsilon\right)$$

with a_t: tail lift slope

Downwash angle ε is difficult to predict theoretically and is usually obtained from experiment and is such that : $\varepsilon = \varepsilon_0 + \frac{\partial \varepsilon}{\partial \alpha_{wb}} \alpha_{wb}$

Thus $C_{L,t} = a_t \alpha_{wb}\left(1 - \frac{\partial \varepsilon}{\partial \alpha_{wb}}\right) - a_t\left(i_t + \varepsilon_0\right)$

and finally $C_{M_{cg,t}} = -a_t V_H \alpha_{wb}\left(1 - \frac{\partial \varepsilon}{\partial \alpha_{wb}}\right) + a_t V_H\left(i_t + \varepsilon_0\right)$

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Total pitching moment about the center of gravity

$$C_{M,cg} = C_{M,cg_{wb}} + C_{M,cg_{t}}$$

tail volume ratio

$$C_{M,cg} = C_{M,ac_{wb}} + C_{L_{wb}}\left(x_{cg} - x_{ac_{wb}}\right) - V_H C_{L,t}$$

$$C_{M,cg} = C_{M,ac_{wb}} + a_{wb}\alpha_{wb}\left(x_{cg} - x_{ac_{wb}} - V_H \frac{a_t}{a_{wb}}\left(1 - \frac{d\varepsilon}{d\alpha_{wb}}\right)\right) + V_H a_t\left(i_t + \varepsilon_0\right)$$

downwash angle when wing-body is at zero lift
(obtained from experimental data)

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Total pitching moment about the center of gravity

Considering that:

absolute angle of attack referenced to zero-lift line of complete airplane

= absolute angle of attack referenced to zero-lift line of wing-body combination

$$\alpha_{wb} = \alpha_a$$

$$a_{wb} = a$$

$$C_{M,cg} = C_{M,ac_{wb}} + a\alpha_a \left(x_{cg} - x_{ac_{wb}} - V_H \frac{a_t}{a} \left(1 - \frac{d\varepsilon}{d\alpha_a} \right) \right) + V_H a_t \left(i_t + \varepsilon_0 \right)$$

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Equations for longitudinal stability

$C_{M,0}$ = $C_{M,cg}$ when α_a=0

$$C_{M,0} \equiv \left(C_{M,cg}\right)_{L=0} = C_{M,ac_{wb}} + V_H a_t \left(i_t + \varepsilon_0\right)$$

Must be >0 to balance the airplane

Since $C_{M,ac\ wb}$<0 for conventional airplanes

→ $V_H a_t (i_t + \varepsilon_0)$ must be >0 and large enough

→ i_t must be >0

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Equations for longitudinal stability

Consider now the slope of the moment coefficient curve

$$\frac{dC_{M,cg}}{d\alpha_a} = a\left(x_{cg} - x_{ac_{wb}} - V_H \frac{a_t}{a}\left(1 - \frac{d\varepsilon}{d\alpha_a}\right)\right)$$

Shows powerful influence of location x of c.g. and of tail volume ratio V_H in determining longitudinal static stability

Establish a certain philosophy in the design of an airplane:

Ex: consider an airplane where location of c.g. essentially dictated by payload or other mission requirements → desired amount of static stability can be obtained simply by designing V_H large enough

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Stick fixed neutral point

Static longitudinal stability: strong function of x_{cg}

Neutral point: specific location of c.g. such that $\frac{dC_{M,cg}}{d\alpha_a} = 0$

Location of neutral point obtained from previous equation

$$x_n = x_{ac_{wb}} + V_H \frac{a_t}{a}\left(1 - \frac{d\varepsilon}{d\alpha_a}\right)$$

Established by design configuration of airplane:

for a given airplane design, neutral point is a fixed quantity: quite independent of actual location x_{cg} of c.g.

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Stick fixed neutral point

$$\frac{dC_{M,cg}}{d\alpha_a} = a\left(x_{cg} - x_n\right)$$

Stability criterion: for longitudinal stability, position of x_{cg} of c.g. must always be forward of neutral point

Recall that: aerodynamic center for a wing: point about which moments are independent of the angle of attack

Extrapolated to whole airplane, when $x_{cg}=x_n$, $C_{M,cg}$ is independent of α: neutral point can be considered as aerodynamic center of complete airplane

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Static margin (x_n-x_{cg}): direct measure of longitudinal static stability

Note that, from the expression

$$C_{M,cg} = C_{M,ac_{wb}} + V_H a_t (i_t + \varepsilon_0) + C_L (x_{cg} - x_n)$$

that can also be written as

$$C_{M,cg} = C_{M,0} + C_L (x_{cg} - x_n)$$

you can also obtain an expression for the static margin as:

$$\boxed{\frac{dC_{M,cg}}{dC_L} = (x_{cg} - x_n)}$$

1. STATIC STABILITY AND CONTROL

a. Longitudinal stability

Positive Stability

- c.g. ahead of Neutral Point
- Nose-up → Nose-down restoring moment

Neutral Stability

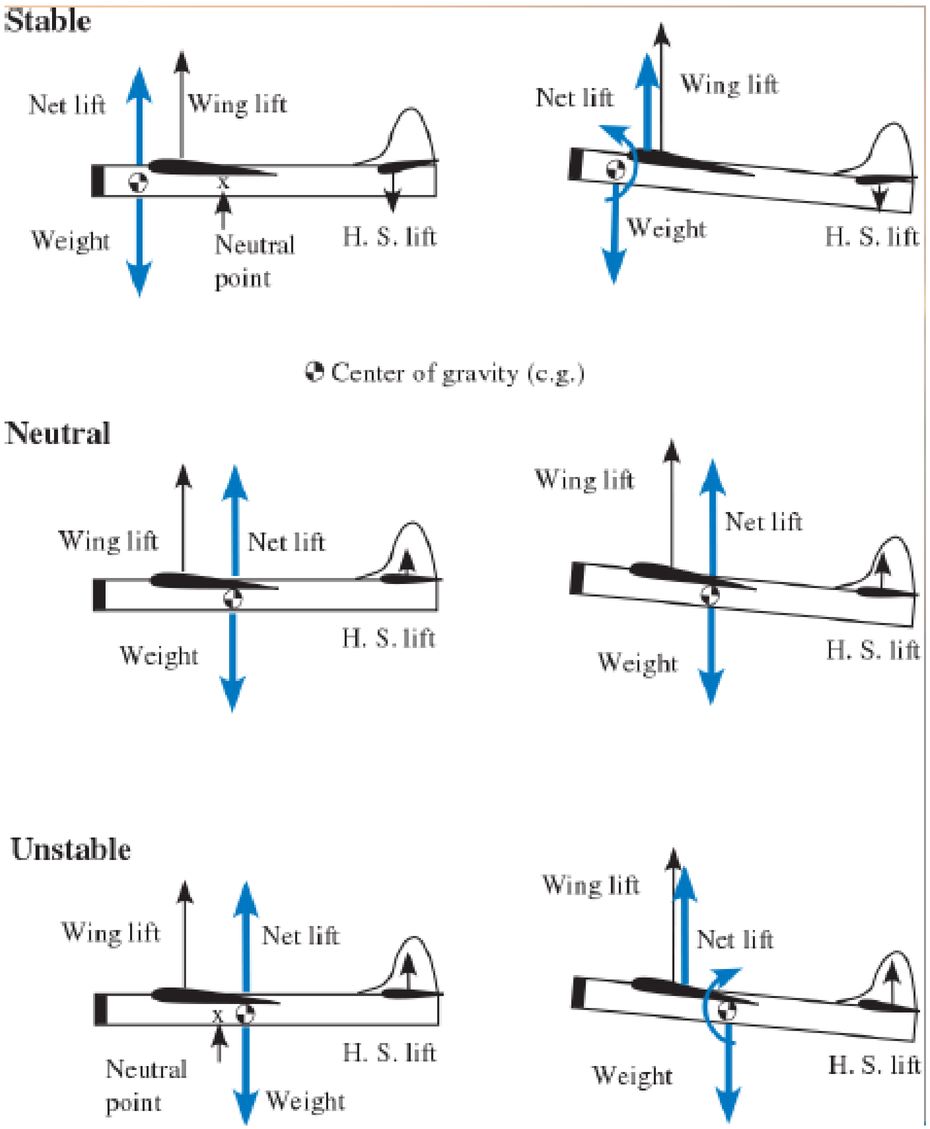
- c.g. on Neutral Point
- Nose-up → $C_{M_{CG}} = C_{M_0}$

Negative Stability (Instability)

- c.g. behind Neutral Point
- Nose-up → Nose-up moment

Stability tuning

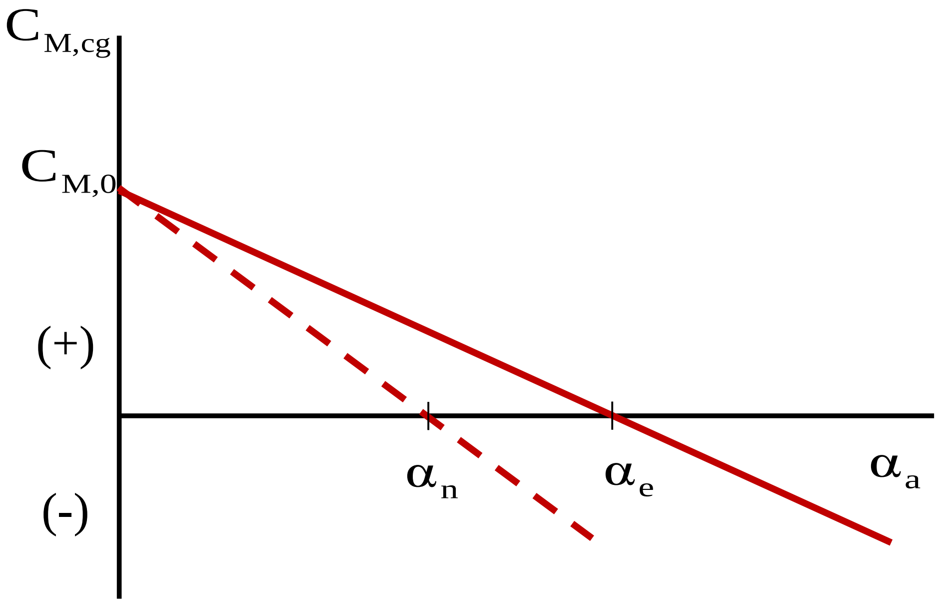
- Size / position of horizontal stabilizer
- Weight distribution

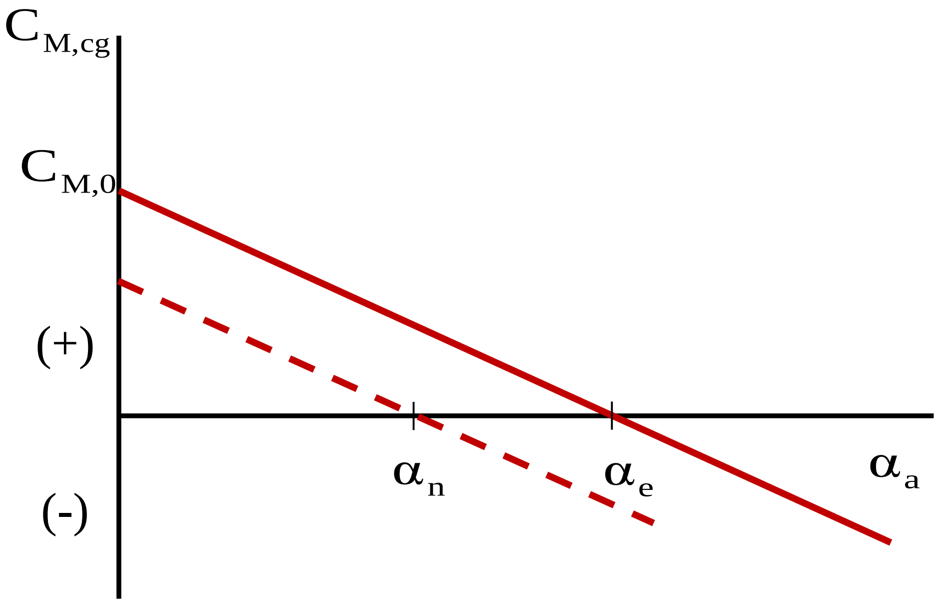


1. STATIC STABILITY AND CONTROL

b. Longitudinal control

How can we obtain a new trim angle of attack (if change of V needed)?





Change in trim angle of attack due to change in slope of moment coefficient curve

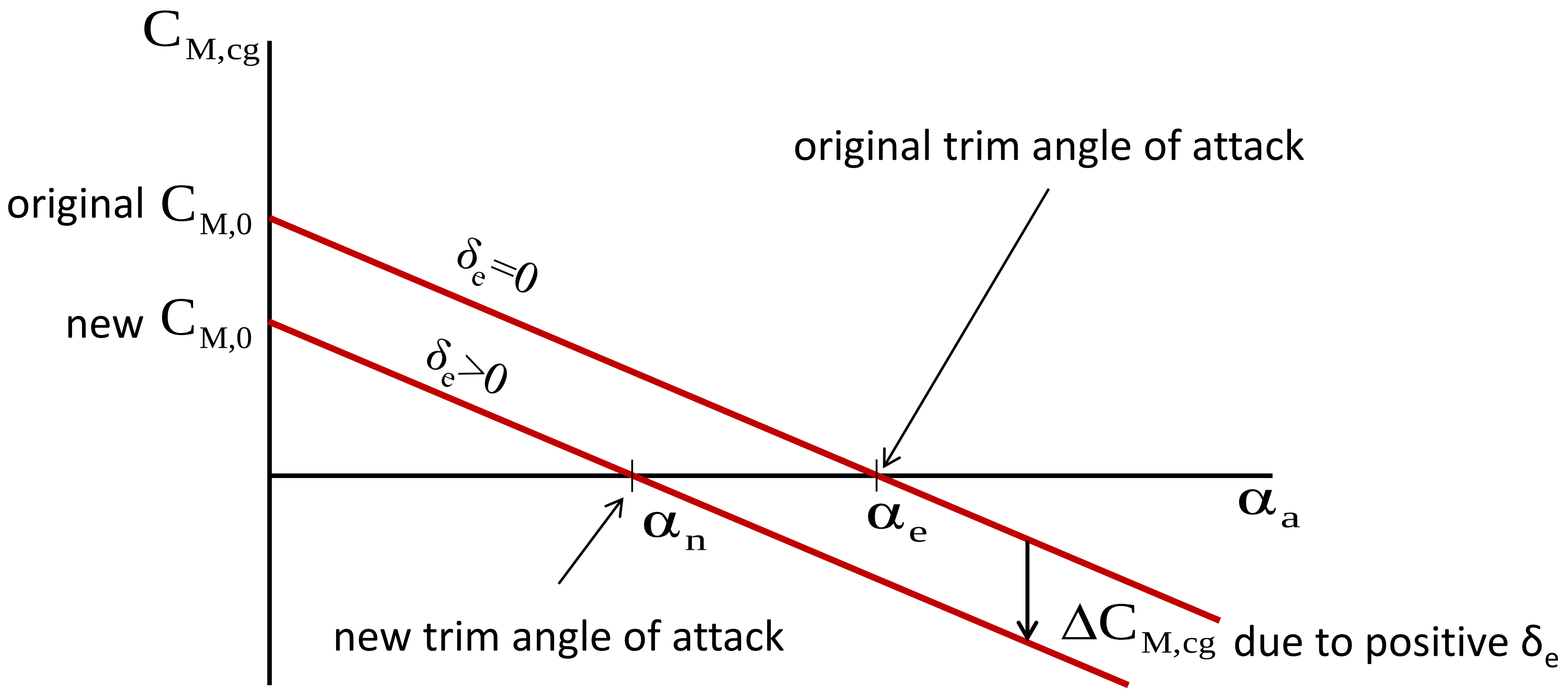
→ shift c.g.

Change in trim angle of attack due to change in $C_{M,0}$

→ deflect elevator on horizontal tail

1. STATIC STABILITY AND CONTROL

b. Longitudinal control



→ elevator can be used to change and control trim of airplane

→ this controls the equilibrium velocity of the airplane

1. STATIC STABILITY AND CONTROL

b. Longitudinal control

Deflected elevator → $C_{L,t}$ curve shifted to the left

$$C_{L,t} = \frac{dC_{L,t}}{d\alpha_t}\alpha_t + \frac{dC_{L,t}}{d\delta_e}\delta_e$$

using the tail lift slope notation: $C_{L,t} = a_t\alpha_t + \frac{dC_{L,t}}{d\delta_e}\delta_e$

and the pitch moment equation:

$$C_{M,cg} = C_{M,ac_{wb}} + C_{L_{wb}}\left(x_{cg} - x_{ac_{wb}}\right) - V_H C_{L,t}$$

we obtain:

$$C_{M,cg} = C_{M,ac_{wb}} + C_{L_{wb}}\left(x_{cg} - x_{ac_{wb}}\right) - V_H\left(a_t\alpha_t + \boxed{\frac{dC_{L,t}}{d\delta_e}\delta_e}\right)$$

1. STATIC STABILITY AND CONTROL

b. Longitudinal control

$$\rightarrow \frac{dC_{M,cg}}{d\delta_e} = -V_H \boxed{\frac{dC_{L,t}}{d\delta_e}}$$

elevator control effectiveness

and the change in pitching moment acting on the plane:

$$\Delta C_M = \boxed{\frac{dC_{M,cg}}{d\delta_e}} \delta_e = -V_H \frac{dC_{L,t}}{d\delta_e} \delta_e$$

elevator control power

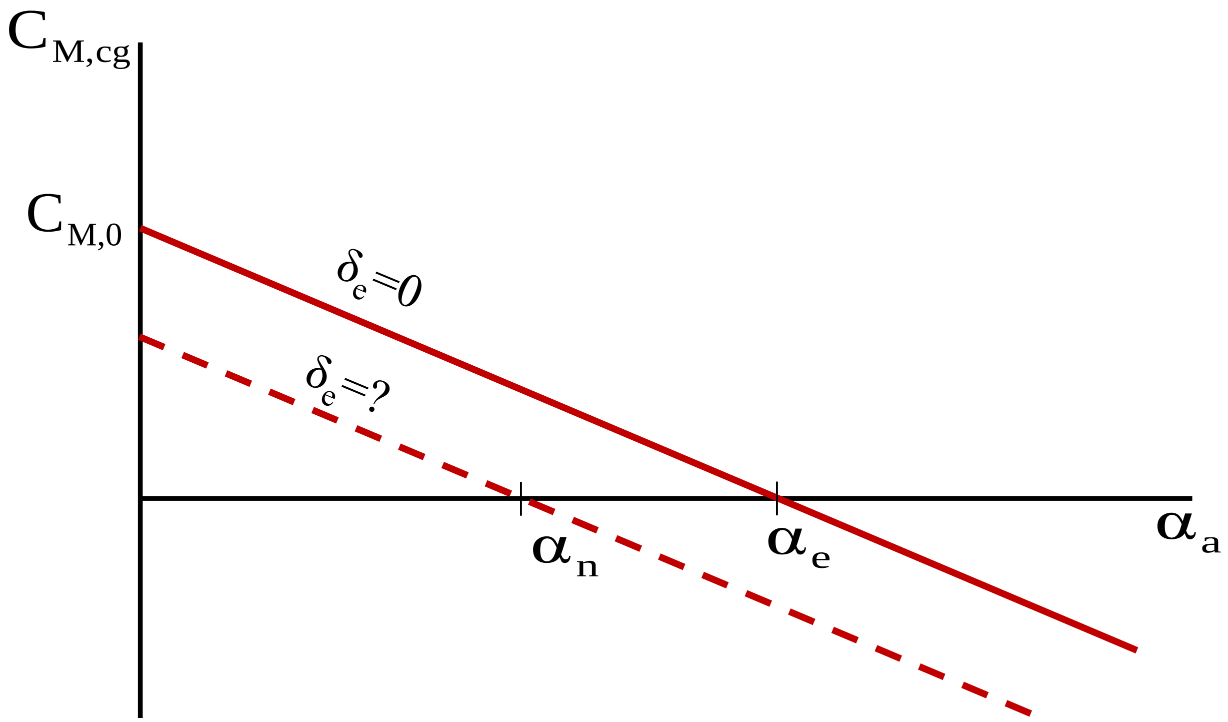
→ new **pitching moment equation**:

$$C_{M,cg} = C_{M,0} + \frac{dC_{M,cg}}{dC_L} C_L + \frac{dC_{M,cg}}{d\delta_e} \delta_e$$

or

$$\boxed{C_{M,cg} = \underbrace{C_{M,0} + \frac{dC_{M,cg}}{d\alpha}\alpha}_{\text{stability}} + \underbrace{\frac{dC_{M,cg}}{d\delta_e}\delta_e}_{\text{control}}}$$

b. Longitudinal control



$$0 = C_{M,0} + \frac{dC_{M,cg}}{d\alpha}\alpha_{trim} + \frac{dC_{M,cg}}{d\delta_e}\delta_{trim}$$

$$C_{Ltrim} = \frac{dC_L}{d\alpha}\alpha_{trim} + \frac{dC_L}{d\delta_e}\delta_{trim}$$

→ δ_{trim}

1. STATIC STABILITY AND CONTROL

Introduction to directional and lateral stability and controllability:

Contrary to the longitudinal case, where rotation only occurs around the y-axis, in the directional/lateral case 2 rotations occur around x- and z-axis.

Moments due to these rotations are coupled:

roll velocity → roll moment + yaw moment

yaw velocity → yaw moment + roll moment

1. STATIC STABILITY AND CONTROL

Introduction to directional and lateral stability and controllability:

Variables to consider in the lateral/directional movements:

β: slip angle between relative wind and roll axis (or lateral velocity v)

ψ: yaw angle or heading angle (between roll axis at equilibrium and actual roll axis)

Φ: lateral inclination angle or roll angle or bank angle (between yaw axis at equilibrium and actual yaw axis)

δ_a: aileron angle (>0 for stick left: left aileron up)

δ_r: rudder angle (>0 for rudder to the left)

1. STATIC STABILITY AND CONTROL

Introduction to directional and lateral stability and controllability:

Lateral/directional force and moments:

Lateral aerodynamic force: Y_A

Roll aerodynamic moment: L_A

Yaw aerodynamic moment: N_A

and corresponding **aerodynamic coefficients**:

$$C_Y = \frac{Y_A}{qS} \qquad C_l = \frac{L_A}{qSb} \qquad C_n = \frac{N_A}{qSb}$$

1. STATIC STABILITY AND CONTROL

Introduction to directional and lateral stability and controllability:

Total lateral force coefficient:

$$C_Y = C_{Y0} + C_{Y\beta}\beta + C_{Y\delta_a}\delta_a + C_{Y\delta_r}\delta_r \quad \text{with } C_{Y\beta} = \frac{\partial C_Y}{\partial \beta}, \ldots$$

0 for symmetric airplanes

Approximated to 0 for most practical cases of lateral control

1. STATIC STABILITY AND CONTROL

Introduction to directional and lateral stability and controllability:

Total yaw aerodynamic moment coefficient:

$$C_n = C_{n0} + C_{n\beta}\beta + C_{n\delta_a}\delta_a + C_{n\delta_r}\delta_r \quad \text{with } C_{n\beta} = \frac{\partial C_n}{\partial \beta}, \ldots$$

- C_{n0}: 0 for symmetric airplanes
- $C_{n\beta}$: directional stability with fixed control surfaces
- $C_{n\delta_a}$: adverse yaw
- $C_{n\delta_r}$: rudder control power

1. STATIC STABILITY AND CONTROL

Introduction to directional and lateral stability and controllability:

Total roll aerodynamic moment coefficient:

$$C_l = C_{l0} + C_{l\phi}\phi + C_{l\delta_a}\delta_a + C_{l\delta_r}\delta_r \quad \text{with } C_{l\phi} = \frac{\partial C_l}{\partial \phi}, \ldots$$

0 for symmetric airplanes

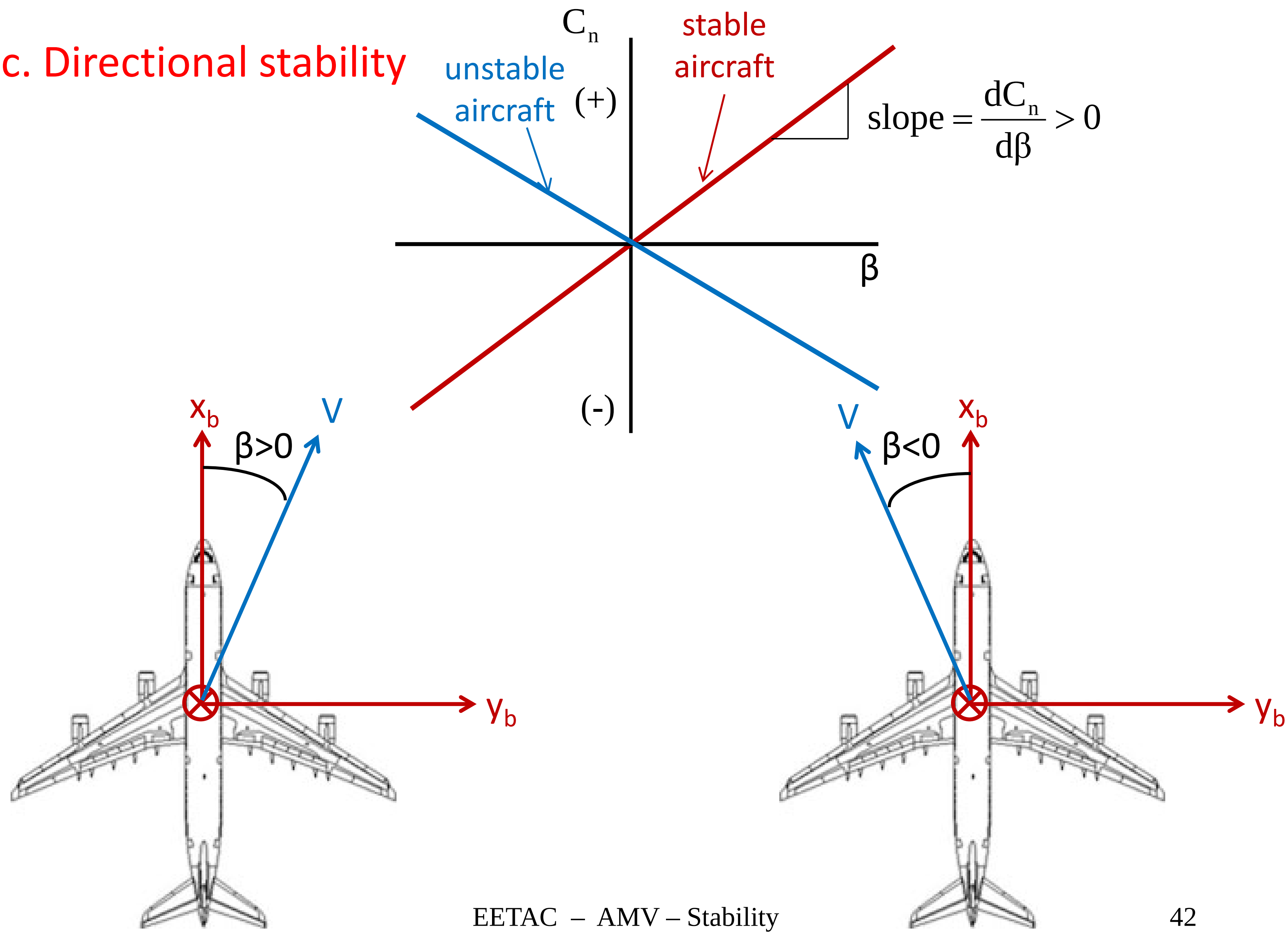
lateral stability with fixed control surfaces (dihedral effect)

aileron control power

rudder located above X axis
→ creates roll moment (small)

1. STATIC STABILITY AND CONTROL

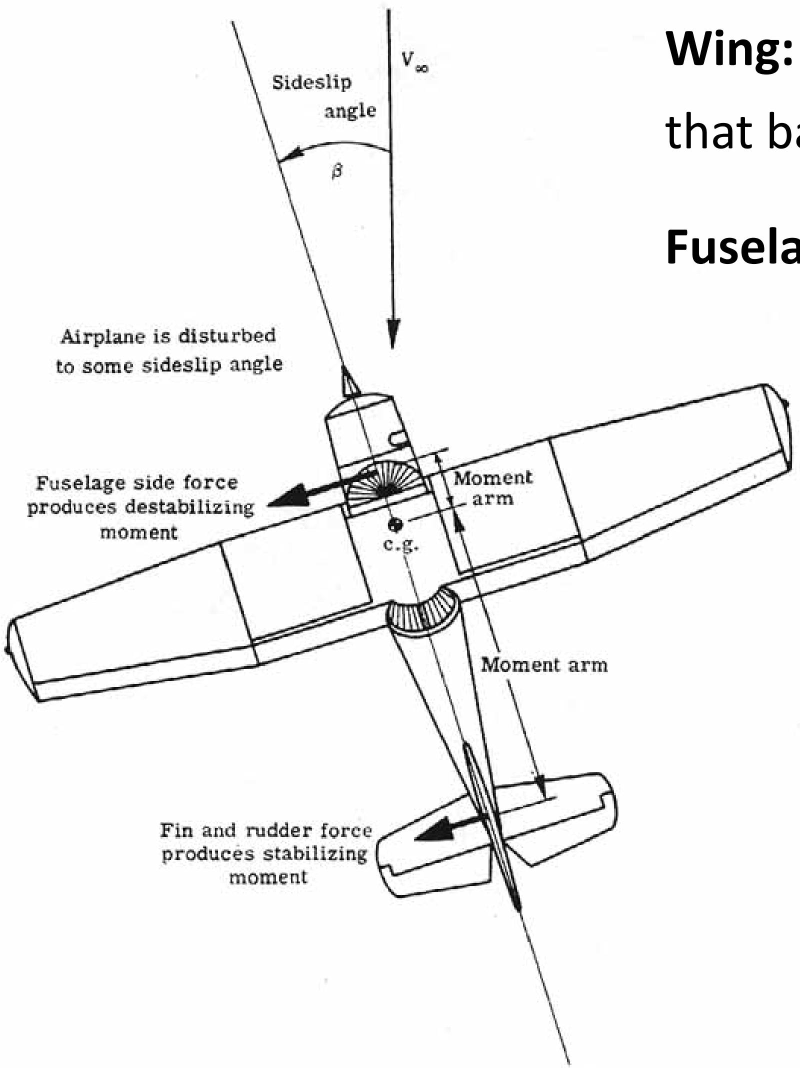
c. Directional stability



1. STATIC STABILITY AND CONTROL

c. Directional stability

Contribution of Aircraft components to directional stability:



Wing: quite small contribution (for small α). Note that backward swept wing have a stabilizing effect

Fuselage: generally destabilizing effect: because usually c.g. is behind the point of application of forces created on fuselage.

When an airplane is in a disturbed condition at a sideslip angle β, in general fuselage alone will generate a moment that tends to increase the disturbance

1. STATIC STABILITY AND CONTROL

c. Directional stability

Contribution of Aircraft components to directional stability:

Wing + fuselage contribution: calculated from empirical expression [USAF]:

$$\frac{dC_{n,wf}}{d\beta} = -k_n k_{Rl} \frac{S_{fs} l_f}{S_w b}$$

where: k_n: empirical wing-body interference factor (function of fuselage geometry)

k_{RL}: empirical correction factor (function of fuselage Reynolds number)

S_{fs}: projected side area of fuselage

l_f: length of fuselage

1. STATIC STABILITY AND CONTROL

c. Directional stability

Contribution of Aircraft components to directional stability:

Vertical tail stabilizing effect: after a perturbation creates a yaw moment that tends to rotate the airplane back to its equilibrium position

Restoring moment produced: side force acting on vertical tail:

$$Y_v = \frac{dC_L}{d\alpha_v} \alpha_v q_v S_v$$

where: subscript v refers to properties of vertical tail

$q_v = 1/2 \rho V_v^2$, dynamic pressure

$\alpha_v = \beta + \sigma$, angle of attack that the vertical tail plane will experience

σ: sidewash angle: caused by flow field distortion due to wings + fuselage

1. STATIC STABILITY AND CONTROL

c. Directional stability

Contribution of Aircraft components to directional stability: Vertical tail

Moment produced by vertical tail:

$$N_v = Y_v l_v = l_v \frac{dC_L}{d\alpha_v}(\beta + \sigma) q_v S_v$$

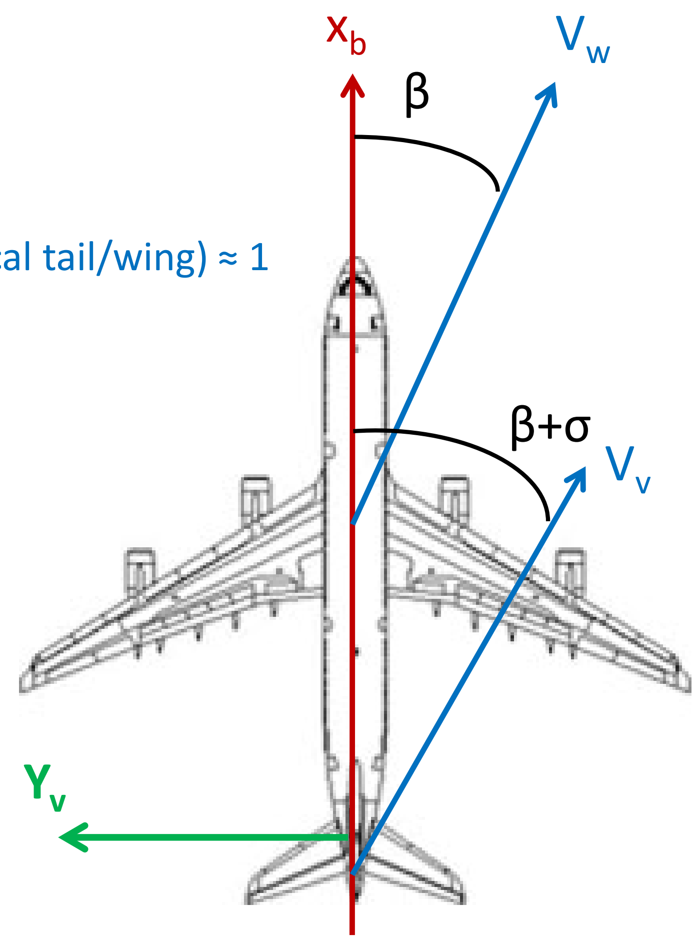
or coefficient moment: ratio of dynamic pressures (vertical tail/wing) ≈ 1

$$C_n = \frac{N_v}{q_w S b} = \frac{l_v S_v}{bS} \frac{q_v}{q_w} \frac{dC_L}{d\alpha_v}(\beta + \sigma)$$

vertical tail volume ratio

Contribution of vertical tail to directional stability:

$$\frac{dC_{n,v}}{d\beta} = \frac{l_v S_v}{bS} \frac{q_v}{q_w} \frac{dC_L}{d\alpha_v}\left(1 + \frac{d\sigma}{d\beta}\right)$$



1. STATIC STABILITY AND CONTROL

d. Directional control

achieved by rudder: its size is determined by directional control requirements:

- adverse yaw: rudder must overcome adverse yaw so that coordinated turn can be achieved. Critical condition occurs when aircraft flies slow (high C_L)
- crosswind landings: to maintain alignment with runway during crosswind landing requires pilot to fly at sideslip angle. Rudder must be powerful enough to permit pilot to trim airplane for specified crosswinds (for transport airplanes up to 15.5m/s or 51 ft/s)
- asymmetric power condition: rudder must overcome yawing moment produced by asymmetric thrust
- spin recovery: rudder must be powerful enough to oppose spin rotation

1. STATIC STABILITY AND CONTROL

d. Directional control

positive rudder deflection produces negative yawing moment:

$$N = -Y_v l_v = -l_v C_{L_v} q_v S_v$$

or in terms of yawing moment coefficients:

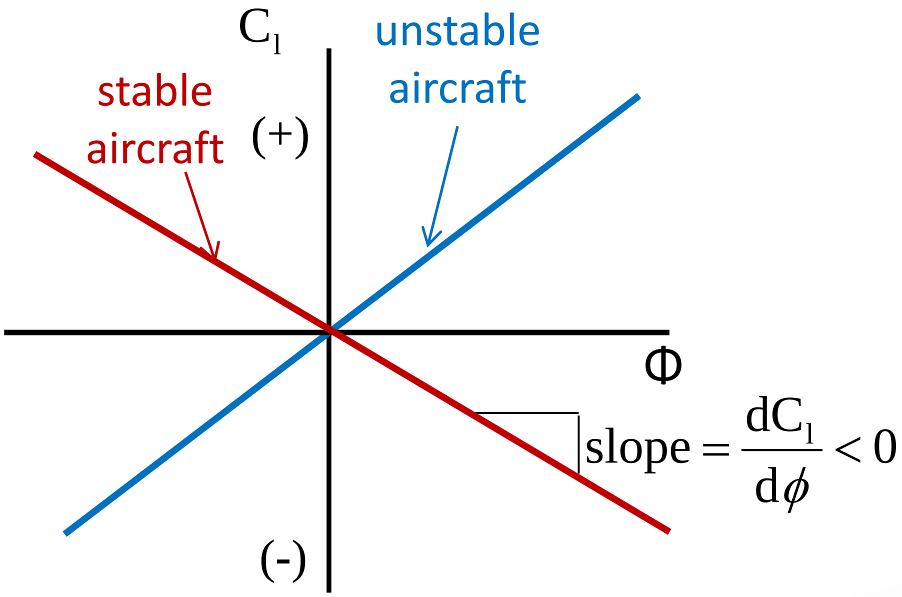
$$C_n = \frac{N}{q_w S b} = -\frac{q_v}{q_w}\frac{l_v S_v}{S b}\boxed{\frac{dC_{L_v}}{d\delta_r}\delta_r} \rightarrow C_{L_v}$$

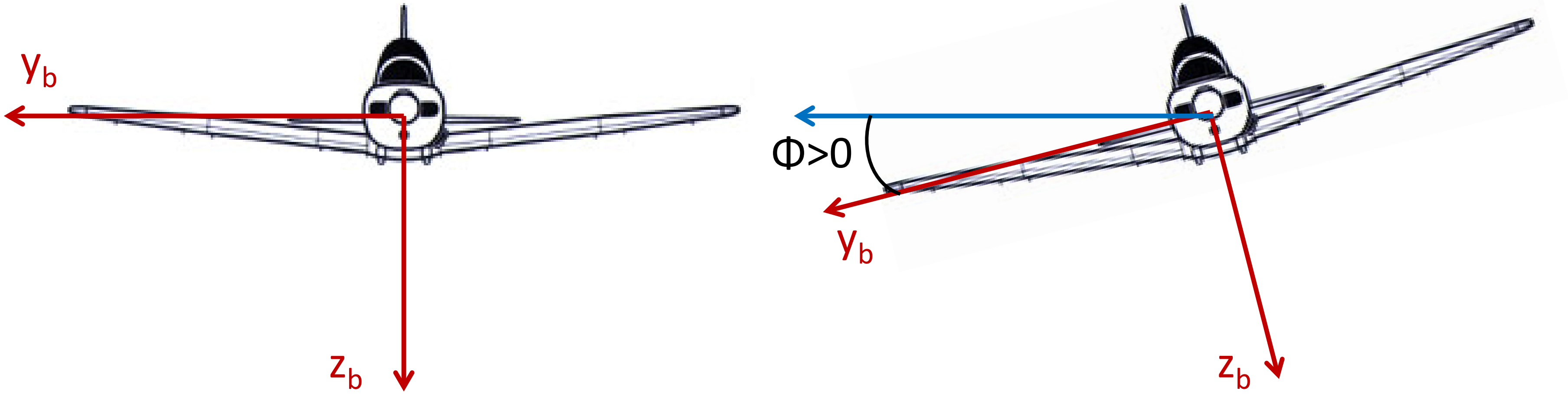
and rudder control power is defined as

$$\boxed{\frac{dC_n}{d\delta_r}} = -\frac{q_v}{q_w}\frac{l_v S_v}{S b}\frac{dC_{L_v}}{d\delta_r}$$

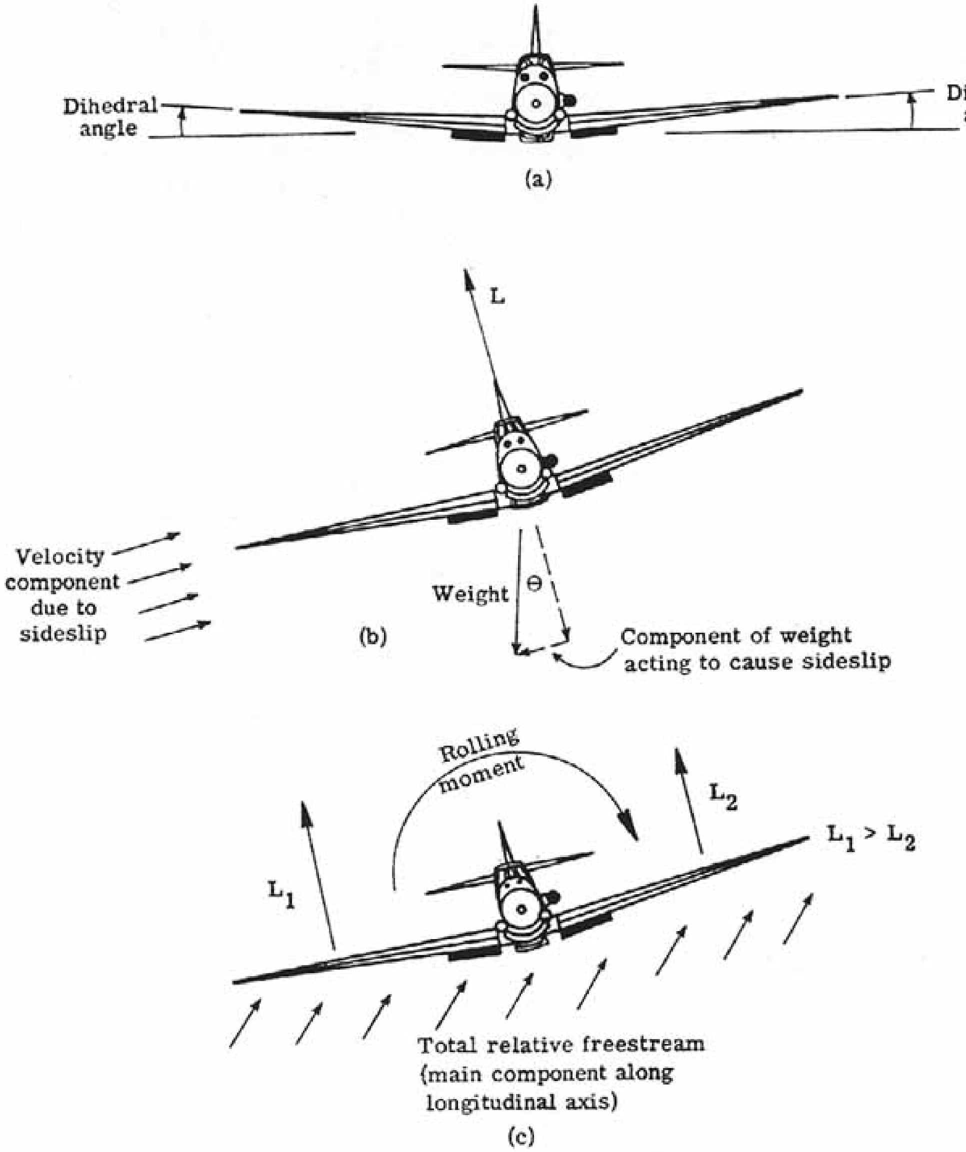
1. STATIC STABILITY AND CONTROL

e. Lateral stability





1. STATIC STABILITY AND CONTROL



e. Lateral stability:

main contributor to $\frac{dC_l}{d\phi}$:

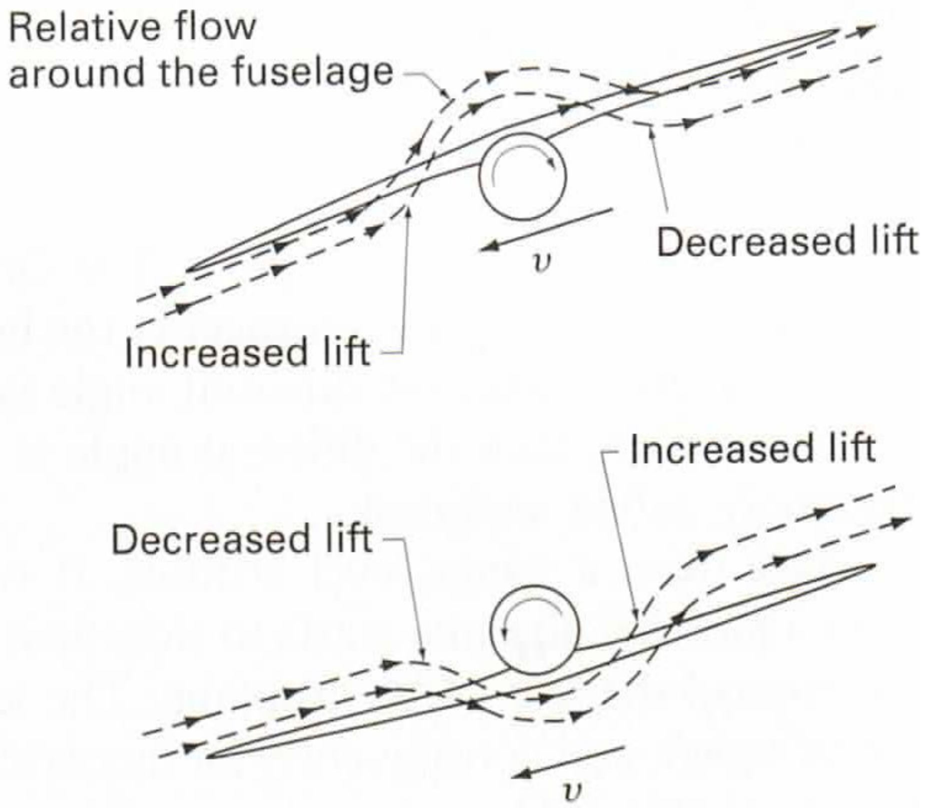
wing dihedral angle

If a disturbance causes 1 wing to drop relative to the other (b), lift vector rotates: component of the weight acting inward causes the airplane to move sideways in this direction

When wings have dihedral, wing toward the free-stream velocity (lower wing), will experience a greater angle of attack than raised wing and hence greater lift

→ net force and moment tending to reduce bank angle (c)

1. STATIC STABILITY AND CONTROL



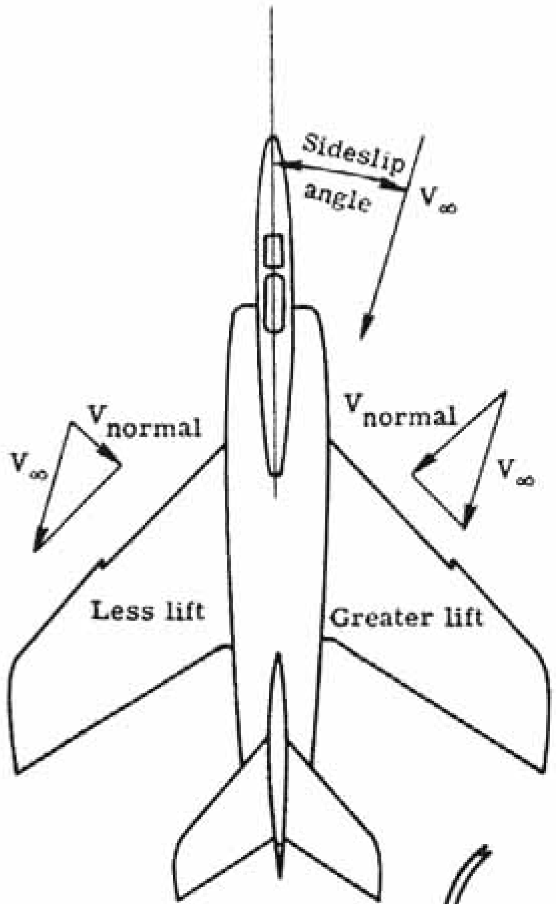
e. Lateral stability:

Effect of wing placement on lateral stability

For low-wing: fuselage contributes a negative dihedral effect

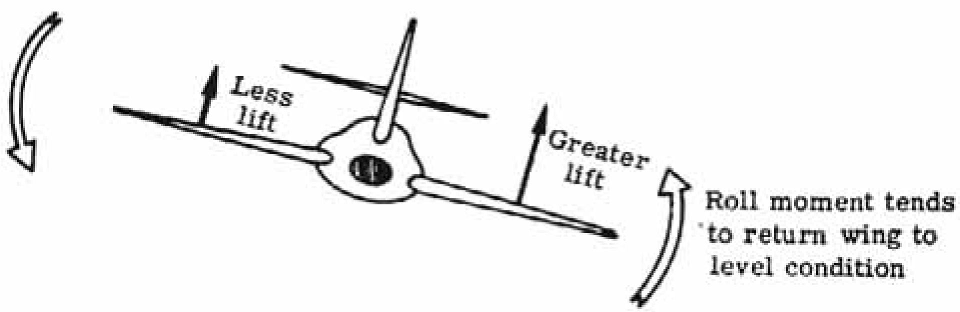
For high-wing: >0 dihedral effect

1. STATIC STABILITY AND CONTROL



e. Lateral stability:

Wing backward sweep will help promote lateral stability

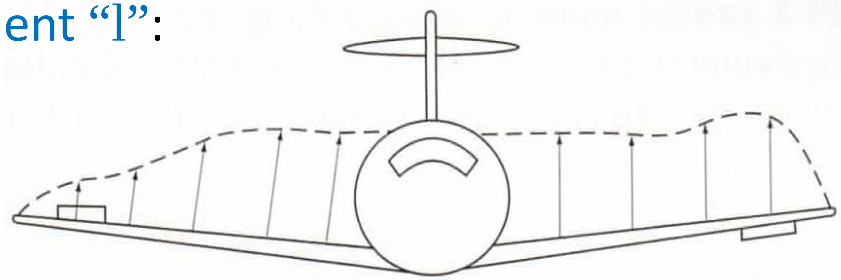


1. STATIC STABILITY AND CONTROL

f. Lateral control:

achieved by differential deflection of ailerons → modify spanwise lift distribution to create moment around x-axis

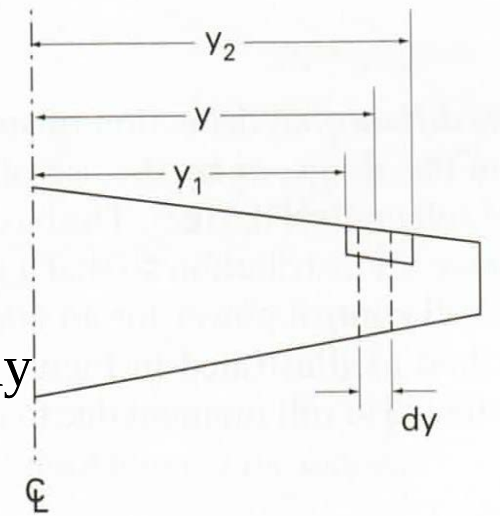
estimate roll control power obtained by a simple strip integration method, incremental change in roll moment "l":



$$\Delta l = (\Delta L)y$$

or in coefficient form:

$$\Delta C_l = \frac{\Delta l}{qSb} = \frac{q\,dy\,cC_L\,y}{qSb} = \frac{dy\,cC_L\,y}{Sb}$$

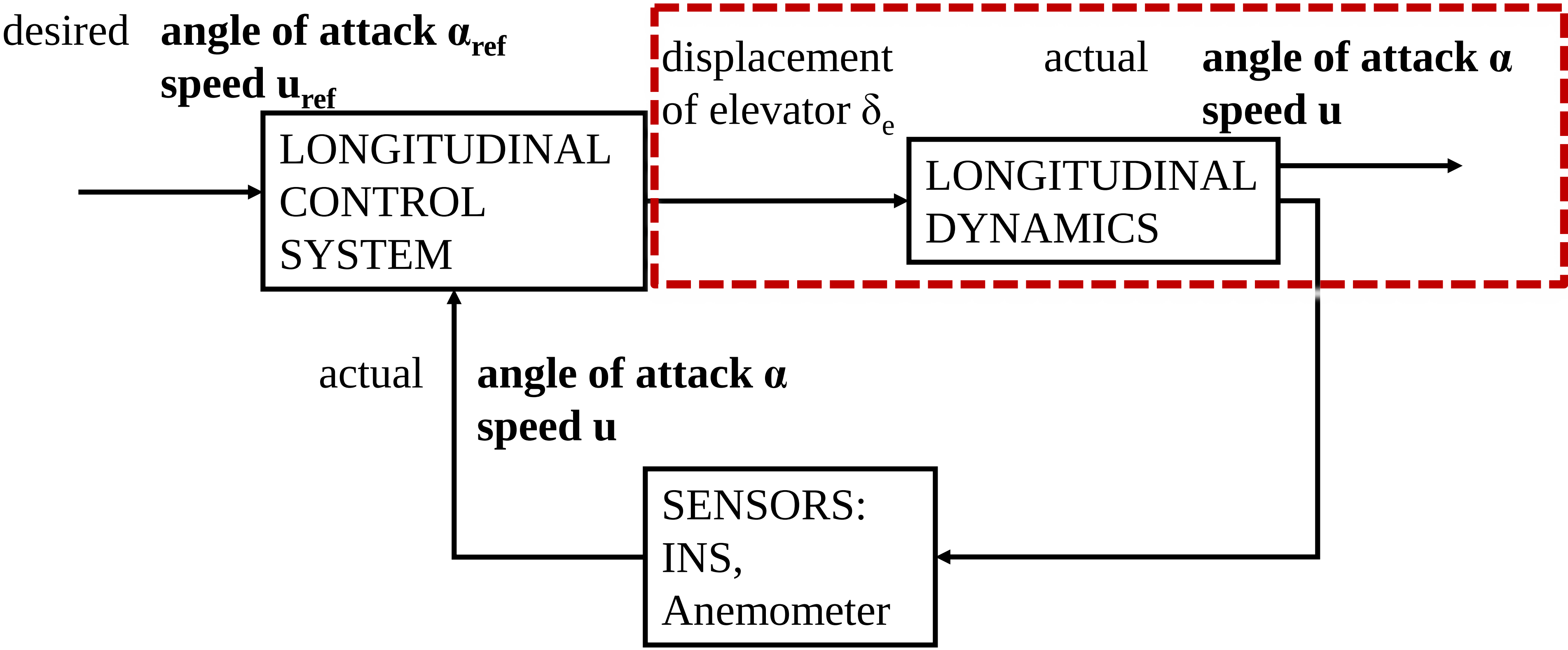


The section lift coefficient C_L on the stations containing the aileron can be written as:

$$C_L = \frac{dC_L}{d\alpha}\boxed{\frac{d\alpha}{d\delta_a}}\delta_a \xrightarrow{\text{integrating}} C_l = \frac{2\,{dC_{L,w}}\big/{d\alpha_w}\;{d\alpha}\big/{d\delta_a}\;\delta_a}{Sb}\int_{y_1}^{y_2} cy\,dy$$

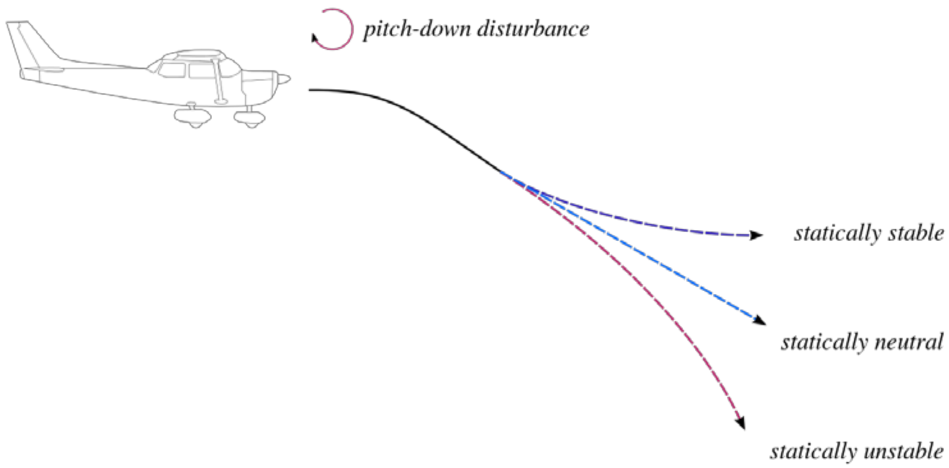
aileron control effectiveness

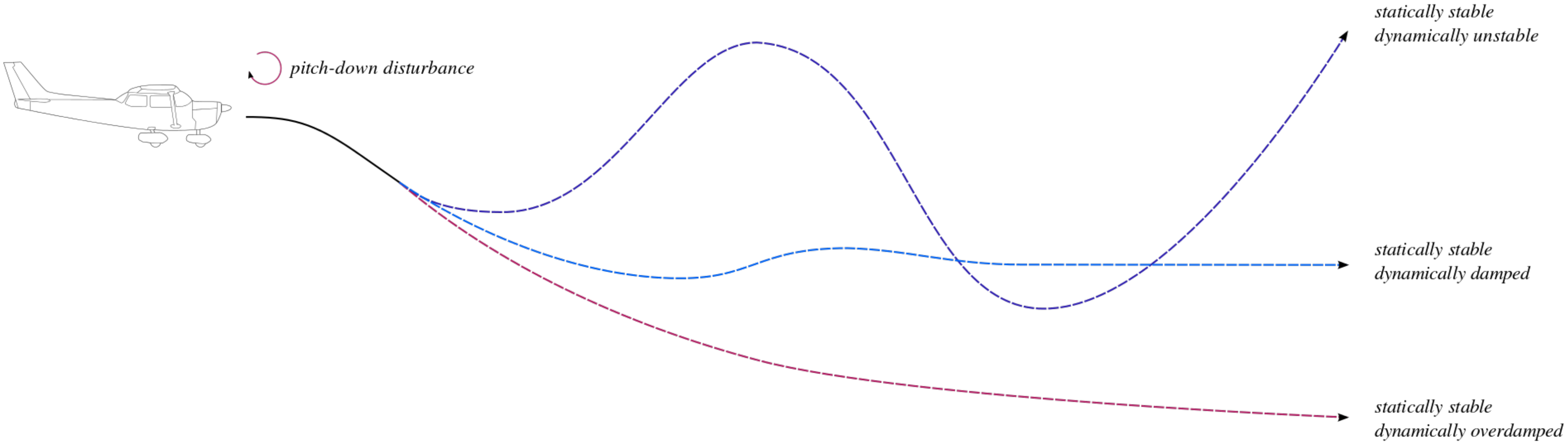
2. AIRCRAFT EQUATIONS OF MOTION



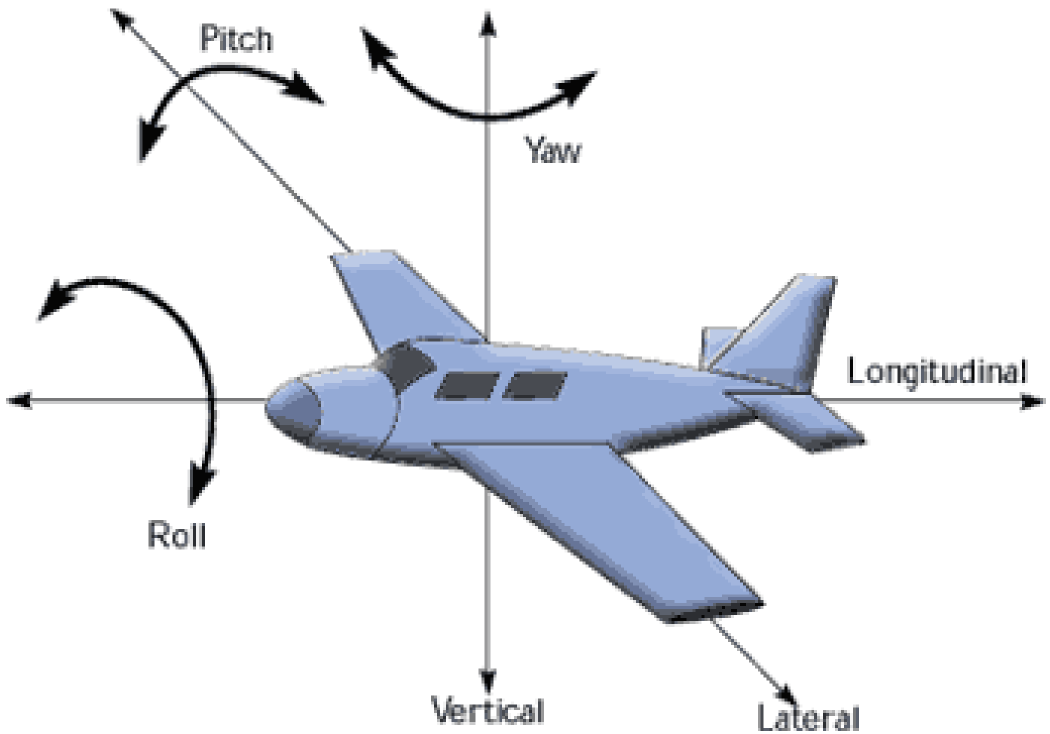
Aircraft dynamics → fundamental part of an aircraft control system

2. AIRCRAFT EQUATIONS OF MOTION





2. AIRCRAFT EQUATIONS OF MOTION



(U, V, W) speed of airplane's mass center in the referential of the airplane with respect to the referential of the ground

(P, Q, R) angular speed (rate) in the referential of the airplane with respect to the referential of the ground

(L, M, N) roll, pitch and yaw momentum

2. AIRCRAFT EQUATIONS OF MOTION

Newton second's Law:

$$\begin{cases} \sum \vec{F}_{Ext} = \dfrac{d\left(m\vec{V}_T\right)}{dt} = \sum \vec{F}_0 + \sum \Delta \vec{F} \\ \sum \vec{M}_{Ext} = \dfrac{d\vec{H}}{dt} = \sum \vec{M}_0 + \sum \Delta \vec{M} \end{cases}$$

Where ***H*** is the angular momentum.

Airplane is considered in equilibrium before perturbation occurs, thus

$$\begin{cases} \sum \vec{F}_0 = 0 \\ \sum \vec{M}_0 = 0 \end{cases} \longrightarrow \begin{cases} \sum \Delta \vec{F} = \dfrac{d\left(m\vec{V}_T\right)}{dt} \\ \sum \Delta \vec{M} = \dfrac{d\vec{H}}{dt} \end{cases}$$

2. AIRCRAFT EQUATIONS OF MOTION

Hypothesis # 1: ***X*** and ***Z*** axis are in the airplane's symmetrical axis and center of gravity = origin of the axis system

Hypothesis # 2: Constant airplane mass during any particular dynamic analysis

Hypothesis # 3: Airplane = rigid body → any 2 points on or within the airframe remain fixed with respect to each other

Hypothesis # 4: Ground = inertial referential (a free particle has a rectilinear uniform translation movement)

Hypothesis # 5: Leveled flight, non turbulent and non-accelerated

Hypothesis # 6: small equilibrium perturbations compared to equilibrium values

3. LONGITUDINAL MOTION

In case of longitudinal study:

→ there is only pitch movement **/Oy**

→ there is variation in $\mathbf{F_x}$ and $\mathbf{F_z}$ but not in $\mathbf{F_y}$ (speed **V=0**)

→ there is no roll nor yaw momentum

3. LONGITUDINAL MOTION

Stick fixed longitudinal motion

Considering a transport airplane, with 4 engines flying straight and leveled at 40,000ft with a constant speed of 600ft/sec (=355 knots)

The obtained differential system of longitudinal equations would be

$$\begin{cases} 13.78\,'\,\dot{u}(t) + 0.088\,'\,u(t) - 0.392\,'\,\alpha(t) + 0.74\,\theta(t) = 0 \\ 1.48\,'\,u(t) + 13.78'\,\dot{\alpha}(t) + 4.46\,'\,\alpha(t) - 13.78\,\dot{\theta}(t) = 0 \\ 0.619\,'\,\alpha(t) + 0.514\ddot{\theta}(t) + 0.192\dot{\theta}(t) = 0 \end{cases}$$

3. LONGITUDINAL MOTION

Stick fixed longitudinal motion

It can also be written as:

$$\begin{cases} 13.78\,\dot{u}(t) = -0.088\,u(t) + 0.392\,\alpha(t) - 0.74\,\theta(t) \\ 13.78\,\dot{\alpha}(t) = -1.48\,u(t) - 4.46\,\alpha(t) + 13.78\,q(t) \\ 0.514\dot{q}(t) = -0.619\alpha(t) - 0.192q(t) \\ \dot{\theta}(t) = q(t) \end{cases}$$

$$\begin{cases} \dot{u}(t) = -0.0064\,u(t) + 0.284\,\alpha(t) - 0.0537\,\theta(t) \\ \dot{\alpha}(t) = -0.1074\,u(t) - 0.3237\,\alpha(t) + q(t) \\ \dot{q}(t) = -1.2043\,\alpha(t) - 0.3735\,q(t) \\ \dot{\theta}(t) = q(t) \end{cases}$$

or in state-space form:

$$\begin{bmatrix} \dot{u}(t) \\ \dot{\alpha}(t) \\ \dot{q}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} -0.0064 & 0.0284 & 0 & -0.0537 \\ -0.1074 & -0.3237 & 1 & 0 \\ 0 & -1.2043 & -0.3735 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u(t) \\ \alpha(t) \\ q(t) \\ \theta(t) \end{bmatrix}$$

3. LONGITUDINAL MOTION

State-space form

Motion equations can be written as a set of 1st order differential equations called the state-space (or state variable equation) and represented mathematically as: $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$

where $\mathbf{x}$ is the state vector and $\mathbf{A}$ the matrix containing the aircraft's dimensional stability derivatives

The homogeneous solution of this 1st order eq. diff can be obtained by assuming a solution of the form: $\mathbf{x} = \mathbf{x}_r e^{\lambda_r t}$

Then substituting the solution into the 1st order eq. diff: $[\lambda_r \mathbf{I} - \mathbf{A}]\mathbf{x} = 0$

3. LONGITUDINAL MOTION

Characteristic equation

For nontrivial solution to exist, the determinant $\left|\lambda_r \mathbf{I} - \mathbf{A}\right|$ must be = 0

λ_r: characteristic roots or eigenvalues of $\mathbf{A}$

Each real eigenvalue or pair of complex eigenvalues correspond to 1 mode of the system:

- real eigenvalues correspond to aperiodic modes
- conjugated complex eigenvalues correspond to periodic/oscillatory modes

3. LONGITUDINAL MOTION

Stick fixed longitudinal motion

use Matlab to solve this matrix problem

```
% Matlab code:
A=[-0.0064 0.0284  0       -0.0537;
   -0.1074 -0.3237 1       0;
    0      -1.2043 -0.3735 0;
    0      0       1       0];
lambda= eig(A)
```

4 complex (2 pairs of conjugated) eigenvalues are obtained:

$$\lambda_{1,2} = -0.3496 \pm 1.0964j$$ → mode I

$$\lambda_{3,4} = -0.0022 \pm 0.0724j$$ → mode II

negative real part → system dynamically stable: if system were given an initial disturbance, the motion will show a damped sinusoidal movement, and frequency of the oscillation would be governed by the imaginary part of λ

3. LONGITUDINAL MOTION

Mode characterization

From: $\lambda_i = \sigma_i \pm j\omega_i$

we define the time constant: $$\tau = \frac{1}{\left|\mathrm{Re}(\lambda_i)\right|}$$

the damping factor: $$\zeta = \left|\frac{\mathrm{Re}(\lambda_i)}{\lambda_i}\right| = \frac{|\sigma_i|}{\sqrt{\sigma_i^2 + \omega_i^2}}$$

and, when the mode is oscillatory, its period can be calculated as: $$T = \frac{2\pi}{\omega_i}$$

and its natural frequency as: $$\omega_n = \left|\lambda_i\right|$$

3. LONGITUDINAL MOTION

Stick fixed longitudinal motion

From the 2 pairs of conjugated roots we can identify 2 periodic modes:

Mode I: τ_I=2.86s and ζ_I=0.30

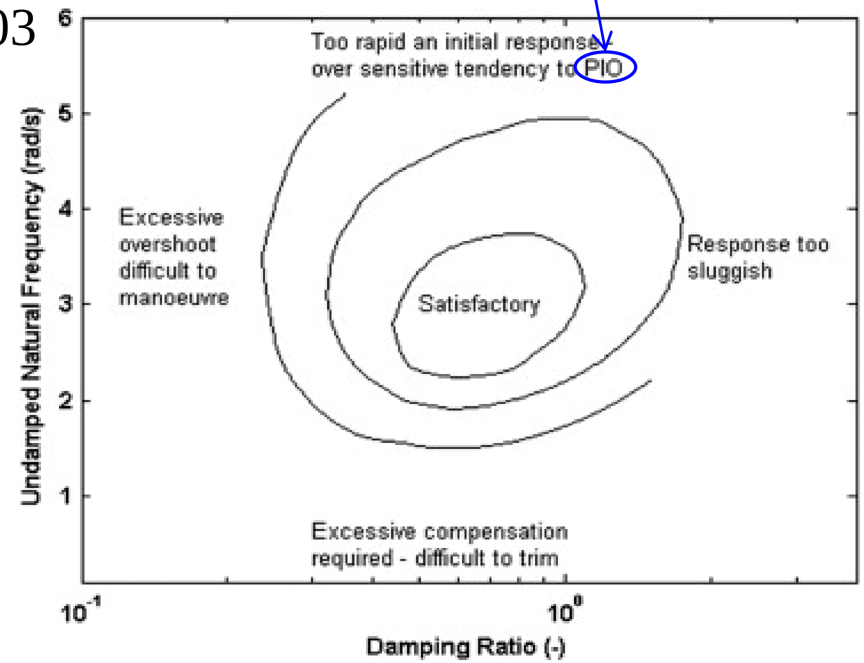
→ high frequency: short-period oscillation mode

Mode II: τ_{II}=454.55s and ζ_{II}=0.03

→ low frequency: phugoid mode

if longitudinal eigenvalues do not meet handling quality specifications, airplane difficult to fly and unacceptable by pilots

PIO: Pilot Induced Oscillations



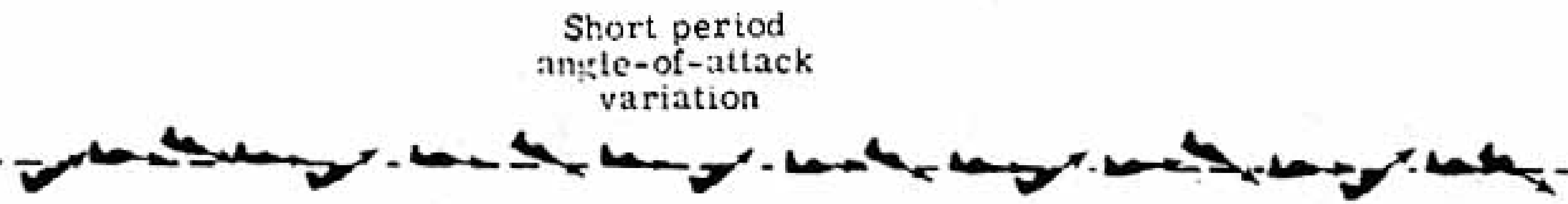
3. LONGITUDINAL MOTION

Stick fixed longitudinal motion

short-period oscillation mode:

• variations of α y θ, with little change of speed u

• if ζ is too low, we need a feedback control system to increase the damping factor ζ



(b) Short-period longitudinal oscillation.

3. LONGITUDINAL MOTION

Stick fixed longitudinal motion

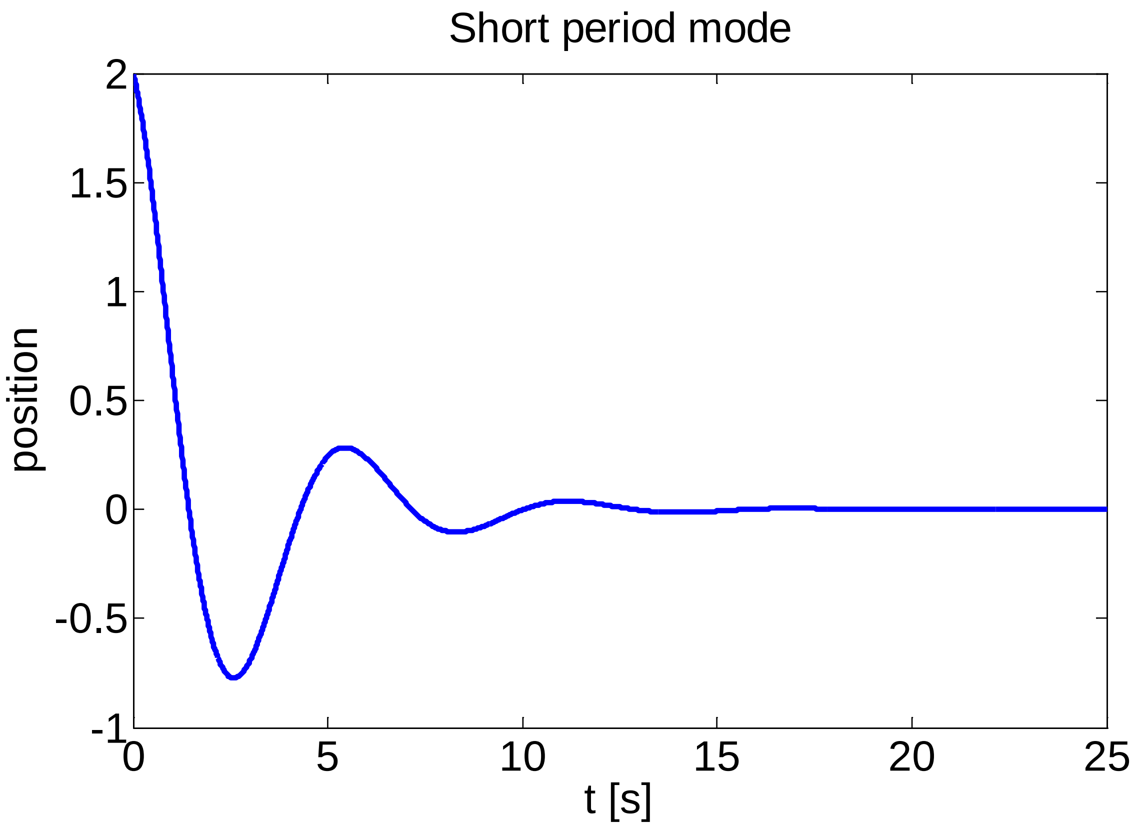
short-period oscillation mode:

• low period: de 0.6 a 6s

• difficult to know its existence: cause can be a wind burst or a sudden activation of flight controls

• fast damping without effort from pilot

```
% Matlab code:
t1=[0:0.01:25];
xsp=exp(lambda(1)*t1)+exp(lambda(2)*t1);
plot(t1,xsp)
```

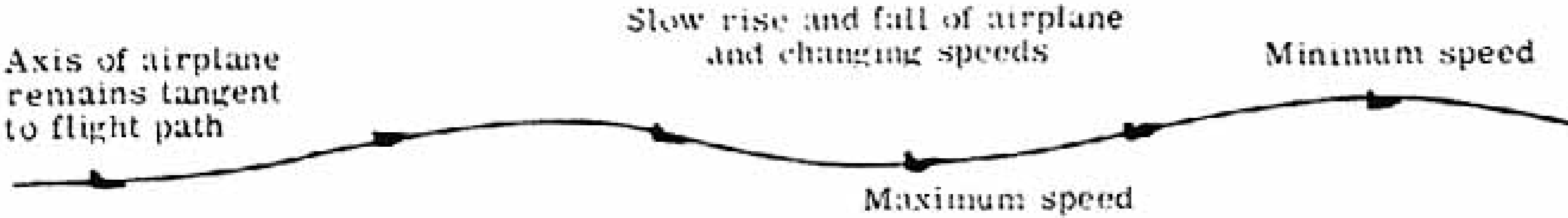


3. LONGITUDINAL MOTION

Stick fixed longitudinal motion

phugoid mode:

- variations of u and θ, with α nearly constant
- kinetic and potential energy exchange
- airplane tends to a sinusoidal flight
- values of period and ζ depend on the airplane and its flight conditions



(a) Phugoid longitudinal oscillation.

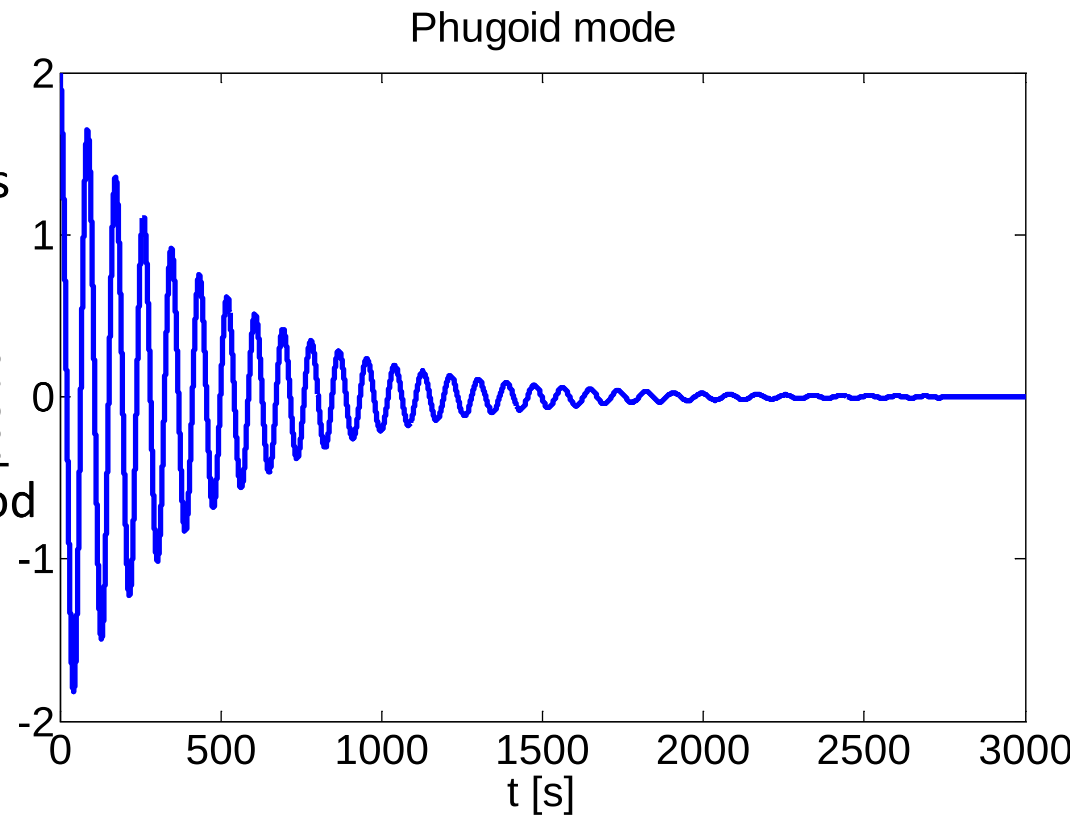
3. LONGITUDINAL MOTION

Stick fixed longitudinal motion

```
% Matlab code:
t2=[0:0.01:3000];
xf=exp(lambda(3)*t2)+exp(lambda(4)*t2);
plot(t2,xf)
```

phugoid mode:

- phugoid period varies between 25s at low speed to several minutes at high speeds
- low damping
- easy to control by pilot (high period → more time to react and activate flight controls)



3. LONGITUDINAL MOTION

Stick fixed longitudinal motion

Amplitude, oscillation period and damping of the longitudinal modes depend on:

- aircraft (C coefficients…)
- altitude (air density)
- airspeed

phugoid period ↗ with speed, and ↘ with altitude at fixed Mach number

short-period oscillation period does the opposite: ↘ with speed and ↗ with altitude

3. LONGITUDINAL MOTION

With a displacement of the elevator

δ_e: elevator deviation (rad), $\delta_e > 0$: elevator goes down,

the new system of 1st order differential equations is

$$\begin{cases} 13.78\,\dot{u}(t) = -0.088\,u(t) + 0.392\,\alpha(t) - 0.74\,\theta(t) \\ 13.78\,\dot{\alpha}(t) = -1.48\,u(t) - 4.46\,\alpha(t) + 13.78\,q(t) - 0.246\,\delta_e(t) \\ 0.514\dot{q}(t) = -0.619\alpha(t) - 0.192q(t) - 0.710\,\delta_e(t) \\ \dot{\theta}(t) = q(t) \end{cases}$$

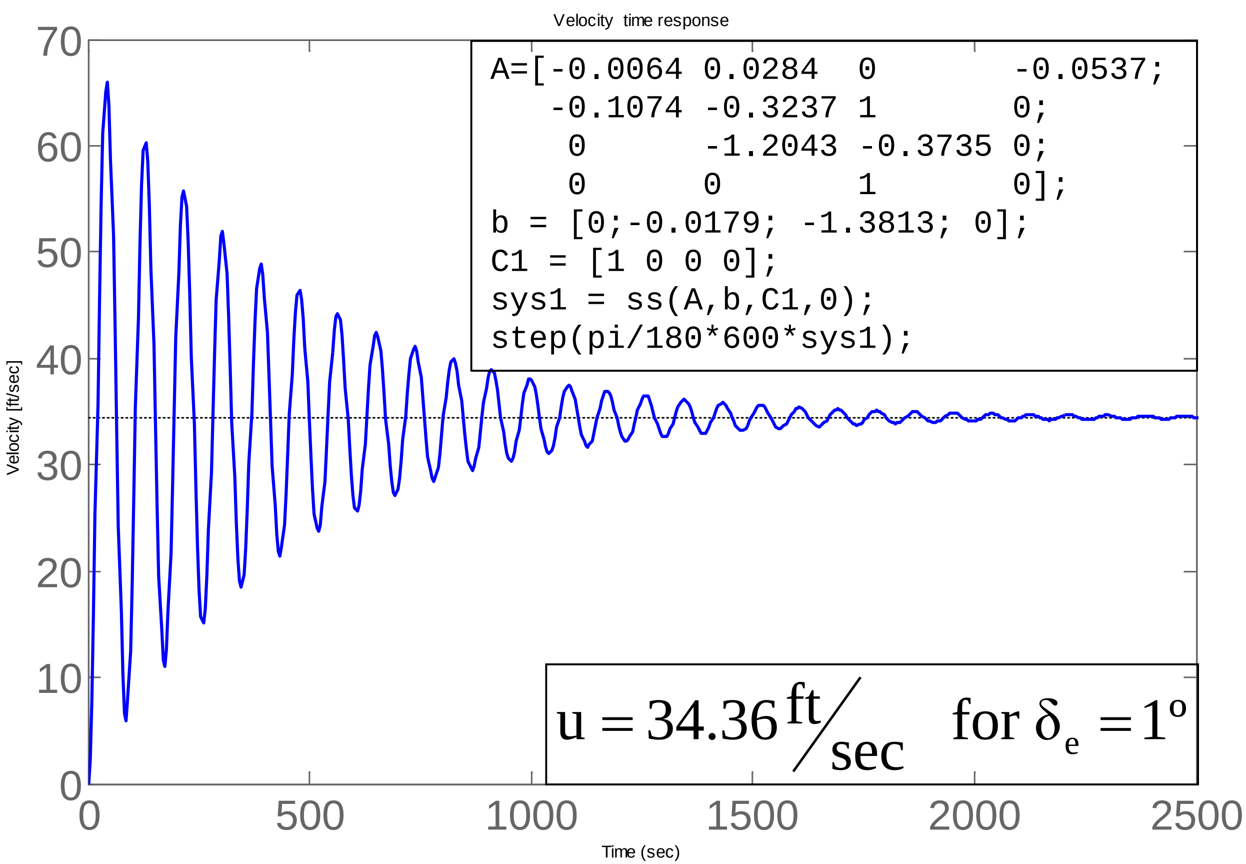
or in state-space form:

$$\begin{bmatrix} \dot{u}(t) \\ \dot{\alpha}(t) \\ \dot{q}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} -0.0064 & 0.0284 & 0 & -0.0537 \\ -0.1074 & -0.3237 & 1 & 0 \\ 0 & -1.2043 & -0.3735 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u(t) \\ \alpha(t) \\ q(t) \\ \theta(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -0.0179 \\ -1.3813 \\ 0 \end{bmatrix} \delta_e(t)$$

3. LONGITUDINAL MOTION

With a displacement of the elevator

for δ_e=1º, find the variation in u velocity (output here):



state-space form:

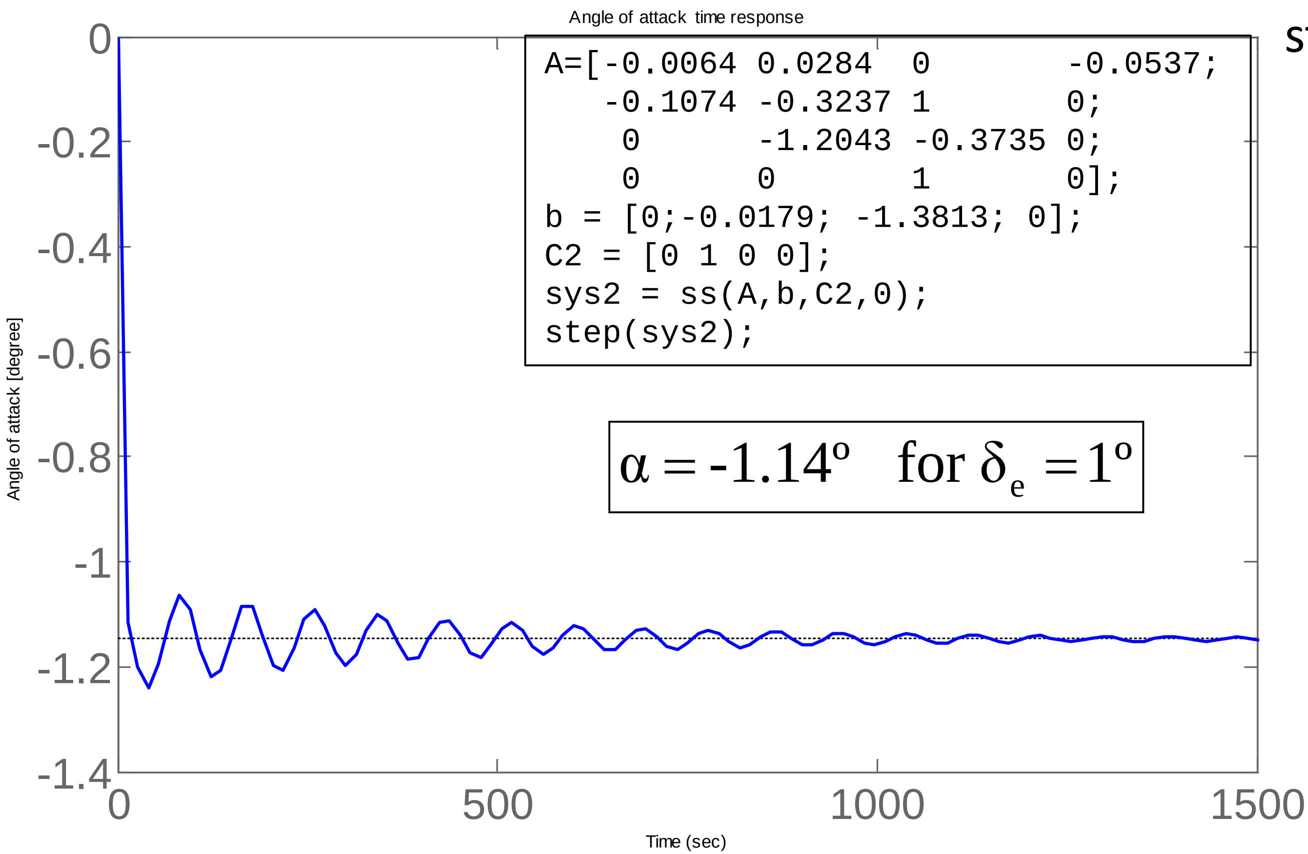
$$\begin{bmatrix} \dot{u}(t) \\ \dot{\alpha}(t) \\ \dot{q}(t) \\ \dot{\theta}(t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} u(t) \\ \alpha(t) \\ q(t) \\ \theta(t) \end{bmatrix} + \mathbf{b}\delta_e(t)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u(t) \\ \alpha(t) \\ q(t) \\ \theta(t) \end{bmatrix} + [0]\delta_e(t)$$

3. LONGITUDINAL MOTION

With a displacement of the elevator

for δ_e=1º, find the variation in angle of attack (output here):



state-space form:

$$\begin{bmatrix} \dot{u}(t) \\ \dot{\alpha}(t) \\ \dot{q}(t) \\ \dot{\theta}(t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} u(t) \\ \alpha(t) \\ q(t) \\ \theta(t) \end{bmatrix} + \mathbf{b}\delta_e(t)$$

$$y = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} u(t) \\ \alpha(t) \\ q(t) \\ \theta(t) \end{bmatrix} + [0]\delta_e(t)$$

4. LATERAL MOTION

Stick fixed lateral motion

In general we will find that the roots of the lateral/directional characteristic equation are: 2 real roots & a pair of complex roots

The airplane response can be characterized by the following motions:

- spiral mode: a slowly convergent or divergent motion (long time constant → easily controlled by pilot)
- roll mode: highly convergent motion (small time constant → airplane's response to an aileron movement)
- Dutch roll: a lightly damped oscillatory motion having a low frequency

4. LATERAL MOTION

Stick fixed lateral motion

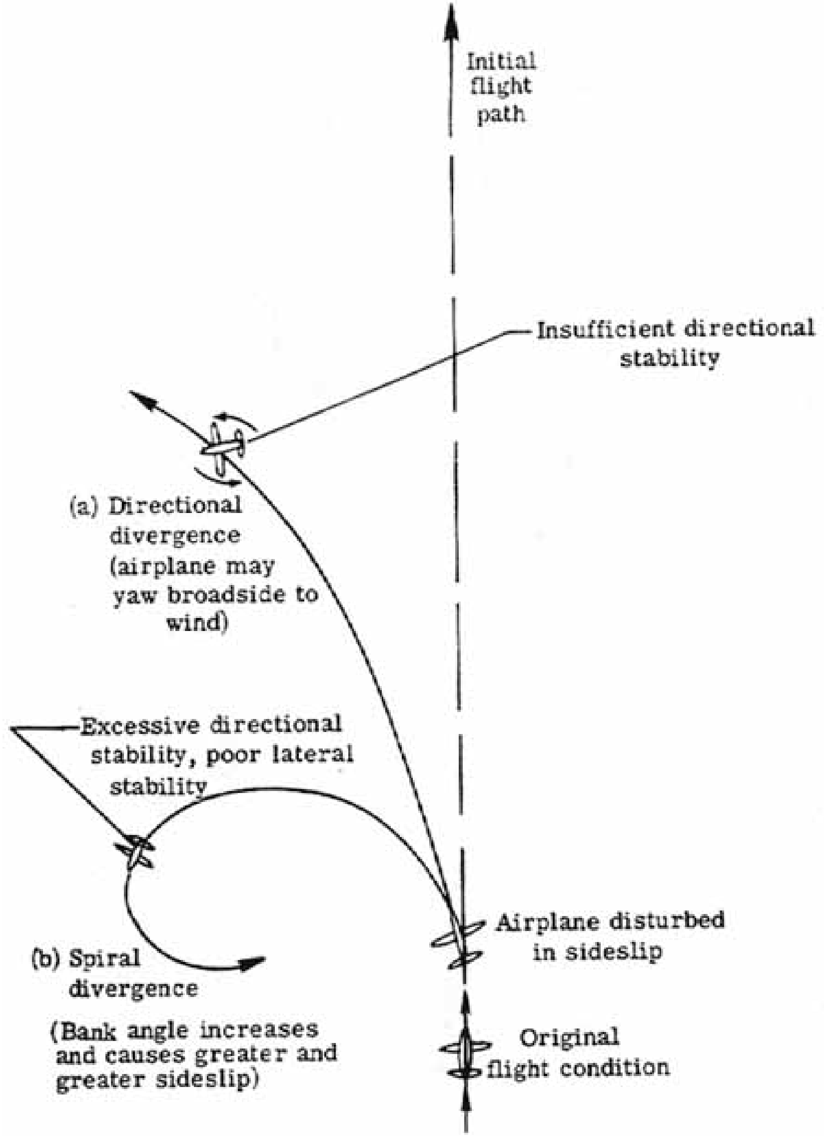
Directional and spiral divergence

Aircraft has much directional static stability and small dihedral

Perturbation turns downward the left wing and turns left

Dihedral: left wing goes up

If dihedral is too small no time to recover horizontal position



4. LATERAL MOTION

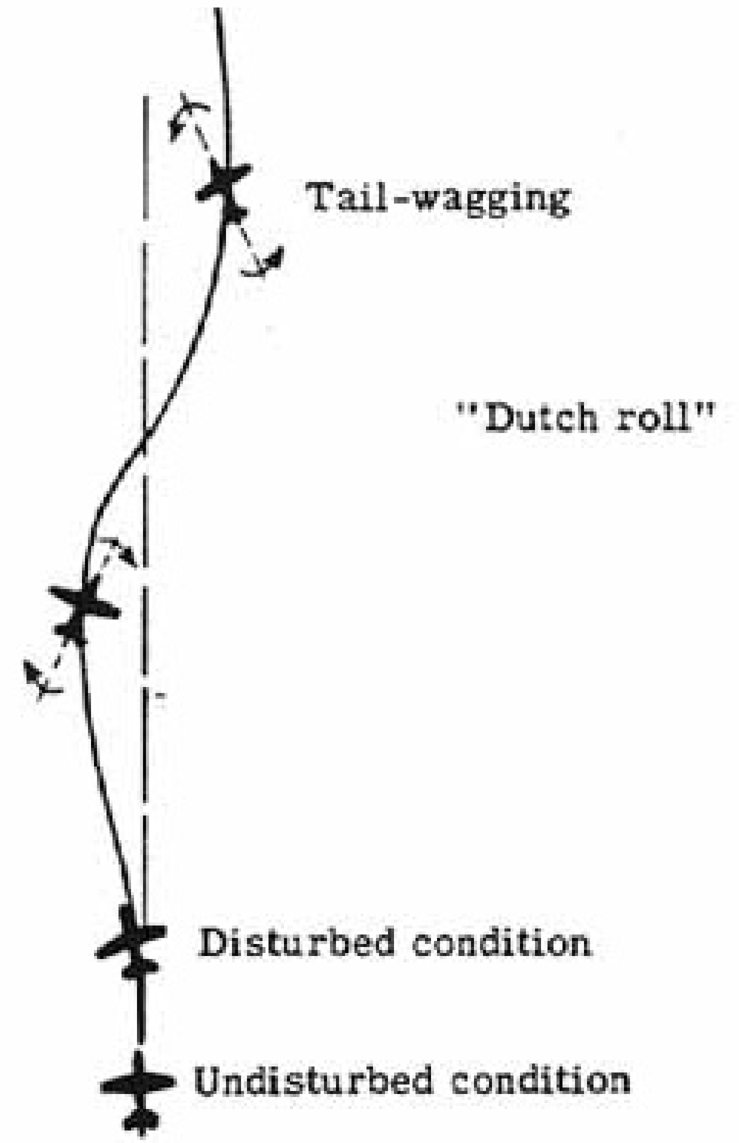
Stick fixed lateral motion

Dutch roll

Characteristics of both divergences:

- strong lateral stability
- low directional stability

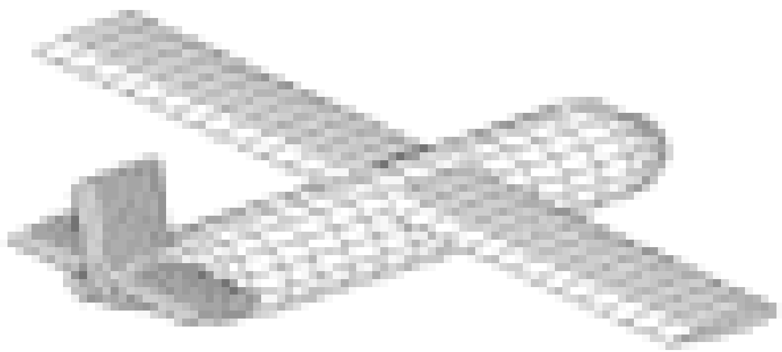
Needs artificial damper if natural damper is too low (yaw damper)

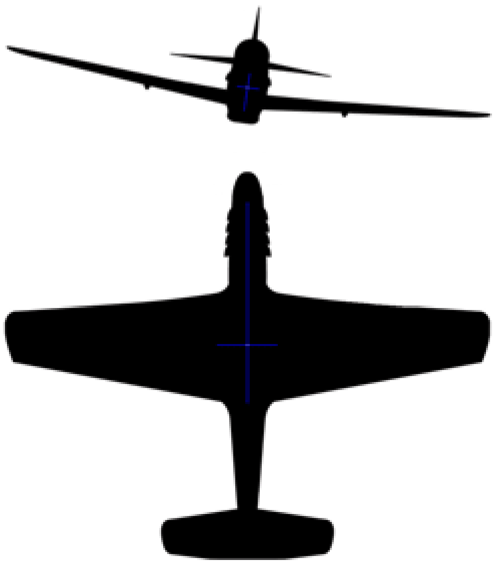


4. LATERAL MOTION

Stick fixed lateral motion

Dutch roll





If slip occurs, airplane has a yaw movement in a given direction and a roll movement in the opposite direction

5. CROSSED COUPLING

= when a turn movement or a maneuver over an axis produces movement over a different axis

Under hypothesis of small perturbations: movement can be separated, the only coupling is lateral/directional:

- rudder movement → lateral turn
- elevator deflection → pitch only

With higher angles of attack,

- pitch can generate roll and yaw (and the opposite)
- roll maneuver → pitch and yaw (divergent)

→ pilot training

→ installation of roll speed limiters and mechanism that increases angular damping (within automatic control systems)

CONCLUSION

When studying airplane stability and control: 2 classes of stability

- **inherent stability**: property of the basic airframe with either fixed or free controls. Mild inherent instability can be *tolerated* if it can be controlled by pilot (such as slow divergence)

- **synthetic stability**: provided by an automatic flight control system and vanishes if the control system fails. Closed loop system *must be stable* in its response to atmospheric disturbances or to commands

Automatic control systems are capable of stabilizing an inherently unstable airplane or simply improving its stability

→ Control & Guidance course (3B)

REFERENCES

- *Flight Stability and Automatic Control,* R. Nelson, 2nd Edition, Mc Graw-Hill, 1998
- *Dynamics of Flight, Stability and Control,* B. Etkin, L. Reid, 3rd Edition, John Wiley & Sons, Inc, 1996
- *Automatic control of Aircraft and Missiles*, J. H. Blakelock, John Wiley & Sons, Second Edition, 1991
- *Aircraft Performance and Design*, J. Anderson, Mc Graw-Hill, New York, 1999
- *Introduction to Flight*, J. Anderson, Fifth Edition, Mc Graw-Hill, New York, 2005
- *Mecánica del vuelo*, M.A. Gómez Tierno, M. Pérez Cortés, C. Puentes Márquez, 2ª Edición, Garceta, 2012
- *USAF Stability and Control Datcom,* Flight Control Division, Air Force Flight Dynamics Laboratory, Wright Patterson Air Force Base, Fairborn, OH